A FORMAL STATISTICAL TREATMENT OF A TURBULENT FLOW

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Abstract. *In this article a formal statistical approach for the treatment of turbulence generated by large eddy computer simulations is presented. A model for compressible flows at large Reynolds numbers and low Mach numbers is used for simulating a backward facing step air flow. A scaling analysis has clearly shown that the internal energy transport due to turbulent velocity fluctuations and the work done by the pressure field are the relevant mechanisms needed to modeling subgrid-scale flows. From the numerical simulations, the velocity temporal series collected for ten different positions in the flow domain, are statistically treated. The statistical approach is based on probability averages of the flow quantities evaluated over several realizations of the simulated flow. In order to define these realizations a long-time record of a turbulent velocity signal is cut up into pieces of length T, where T is much longer than the characteristic relaxation time occurring in the flow. These pieces are then treated as observations of different responses in an ensemble of similarly simulated flows. The underlying assumption here is the so-called ergodic hypothesis. The ergodic hypothesis is verified, and it is shown that in some regions of the flow the standard statistical approach of time averaging is not appropriated to characterize the turbulence. The ergodic deviations are compared with theoretical predictions given by scaling arguments and a good agreement is observed. Results for velocity signals, auto-correlation functions, probability distributions, as well as skewness and flatness coefficients are presented. The statistical approach explored in this work has been used as a potential tool for estimating computer simulation time scale in order to produce flow recordings with sufficient information for a long time statistical description of turbulent velocity fluctuations.*

Keywords: *large eddy simulation, scaling analysis, ergodicity hypothesis, memory scales, statistical treatment*

1. Introduction

Large-eddy simulation (LES) is an important technique in the study of turbulent flows. In LES the governing equations are spatial averaged allowing the large-scale motion to be solved. In other hand, from this averaging process, the so-called subgrid terms remains and constitutive models are needed to its calculations (Sagaut, 1988). LES requires less computational effort than direct numerical simulations (DNS), which attempts to solve all scales present in the turbulent flow (Vreman, 1995). Other important characteristic is the unsteady feature of LES. This implies that a statistical treatment is needed in order to permit an accurated characterization of the simulated turbulent flow. The purpose of this paper is to perform a statistical treatment of a flow resulting from large-eddy simulations. The simulation method is not focused, but a study of the magnitude order of the subgrid terms are performed and a statistical treatment that characterizes the memory time scales and evaluates the ergodicity of the investigated turbulent flow is presented and applied.

In a general case, a formal statistical treatment is based on probability averages evaluated over an ensemble of several realization of the same process, which defines a stochastic set. From ergodic process, the probability average can be replaced by a temporal average, and the statistical analysis is more feasible. Nevertheless, when the turbulence is dominated by large structures, typically strongly correlated, the ergodic hypothesis cannot be assumed and only a probability analysis may correctly describe the statistical features of the flow (Ifeachor, 1993). In a LES context, the total time of simulation needs to be long enough to ensure the ergodicity of the process or to allow a probability analysis. In a non-homogeneous flow, this total time may be different for each region in the domain.

In this work, a large-eddy simulation model in the limit of high Reynolds number and low Mach number compressible flows is presented. A scale analysis is performed in order to evaluate the relative importance of the subgrid terms when the flow obeys the $Re_\infty \gg 1$ and $M_\infty \ll 1$ limit. A scaling analysis is also used to the estimation of the error ϵ due to the ergodicity hypothesis. A turbulent flow over a backward-facing step is simulated and a correlation analysis is made in order to quantify the memory time scales for ten different position on the domain. Based on this correlation time a stochastic set is build and a probability analysis is done. Its results are confronted with the results of a temporal analysis and the differences found are used to validate the ϵ parameter predicted by the scaling. In addition, turbulent intensities, skewness and flatness factors are presented. All of this statistical quantities are calculated using the probability approach and a confidence interval for its values are obtained. This paper is organized as follows. The mathematical approach, specifying the averaging process of the governing equations, including the scale analysis for subgrid terms and constitutive relations for the remaining, is found in §2 . Statistical formulation is presented in §3. The simulation setup is shown is §4. Results are the subject of §5. A summarization of the main conclusions of the work is presented in §6.

2. Mathematical approach

2.1. Averaging governing equations

Let a generic flow property that can be a function of the space and time $\phi(\mathbf{x},t)$. The spatial average $\overline{\phi}(\mathbf{x},t)$ is defined as,

$$\overline{\phi}(\mathbf{x},t) = \int_\Omega \phi(\mathbf{r},t) G(\mathbf{x}-\mathbf{r}) d\mathbf{r}, \tag{1}$$

where $\mathbf{x}$ is the position vector and $\mathbf{r}$ is the displacement vector regarding to $\mathbf{x}$. The function $G(\mathbf{x}-\mathbf{r})$ is a filter function $G : \Re \to [0,1]$ and satisfies

$$\lim_{|\mathbf{x}-\mathbf{r}|\to\infty} G(\mathbf{x}-\mathbf{r}) = 0 \qquad\qquad \int_D G(\mathbf{x}) d\mathbf{x} = 1, \tag{2}$$

where D is the flow domain. This averaging process still regards the linearity and the commutability with the spatial and temporal differentiations

$$\overline{\phi+\psi} = \overline{\phi} + \overline{\psi}, \qquad\qquad \overline{\frac{\partial \phi}{\partial s}} = \frac{\partial \overline{\phi}}{\partial s}, \tag{3}$$

where $s = \mathbf{x}, t$. The properties 3 are derived from the continuity of ϕ and the properties of the filter function presented in 2 (Sobral & Cunha, 2002). A density weighted average process is more appropriated for compressible models. This process corresponds to the well known Favre filtering (Favre, 1983), defined as

$$\widetilde{\phi} = \frac{\overline{\rho\phi}}{\overline{\rho}}, \tag{4}$$

where ρ is the density of the fluid. Note that according Eq.(4), $\overline{\rho\phi} = \overline{\rho}\widetilde{\phi}$. This identity is largely applied for the averaging of the governing equations. In 5 and 6 it is shown the averaged mass and momentum balance equations:

$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial}{\partial x_i}\left(\overline{\rho}\widetilde{u}_i\right) = 0, \tag{5}$$

$$\frac{\partial}{\partial t}\left(\overline{\rho}\widetilde{u}_i\right) + \frac{\partial}{\partial x_j}\left(\overline{\rho}\widetilde{u_i u_j}\right) = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}(2\mu \overline{S}_{ij}), \tag{6}$$

where

$$\overline{S}_{ij} = \left(\overline{D}_{ij} - \frac{1}{3}\frac{\partial \overline{u}_k}{\partial x_k}\delta_{ij}\right), \qquad \overline{D}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right). \tag{7}$$

Here, u_i are the components of the velocity vector $\mathbf{u}$, p is the mechanical pressure, μ the dynamical viscosity coefficient and D_{ij} are the components of the strain rate tensor $\mathbf{D}$. Since velocity co-variance is defined as $\sigma_u = \widetilde{u_i u_j} - \widetilde{u}_i \widetilde{u}_j$, the averaged product of velocities that appears in Eq.(6) can be written as $\widetilde{u_i u_j} = \widetilde{u}_i\widetilde{u}_j + \sigma_u$. If we use that $\overline{S}_{ij} \sim \widetilde{S}_{ij}$, in terms of scaling, the averaged momentum equation becomes

$$\frac{\partial}{\partial t}\left(\overline{\rho}\widetilde{u}_i\right) + \frac{\partial}{\partial x_j}\left(\overline{\rho}\widetilde{u}_i\widetilde{u}_j\right) = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}(2\mu \widetilde{S}_{ij}) - \frac{\partial}{\partial x_j}\left(\widetilde{\rho}\sigma_u\right). \tag{8}$$

Now, defining the tensor $\Sigma_{ij} \equiv -\widetilde{\rho}\sigma_u$, a modified Cauchy equation can be written as follows

$$\overline{\rho}\frac{D\widetilde{\mathbf{u}}}{Dt} = \nabla \cdot \widetilde{\mathbf{T}}, \tag{9}$$

where the new constitutive equation for the stress tensor is

$$\widetilde{\mathbf{T}} = -\overline{p}\mathbf{I} + 2\mu\left[\widetilde{\mathbf{D}} - \frac{1}{3}(\nabla \cdot \widetilde{\mathbf{u}})\mathbf{I}\right] + \mathbf{\Sigma}. \tag{10}$$

Where $\mathbf{I}$ is the identity tensor. In this formulation, Σ represents the momentum transport, by velocities fluctuations, in the subgrid scales. The same averaging process is applied to the energy equation leading to

$$\begin{aligned}\frac{\partial}{\partial t}\left(\overline{\rho}\widetilde{e}_T\right) + \frac{\partial}{\partial x_j}\left(\overline{\rho}\widetilde{u}_j\widetilde{e}_T\right) &= -\frac{\partial}{\partial x_j}\left(\overline{p}\widetilde{u}_j\right) + \frac{\partial}{\partial x_j}\left(2\mu\widetilde{S}_{ij}\widetilde{u}_i\right) - \frac{\partial \widetilde{q}_j}{\partial x_j} \\ -\frac{\partial}{\partial x_j}\underbrace{\left[\overline{\rho}(\widetilde{u_j e_T} - \widetilde{u}_j\widetilde{e}_T)\right]}_{I} - \frac{\partial}{\partial x_j}\underbrace{\left(\overline{pu_j} - \overline{p}\widetilde{u}_j\right)}_{II} &+ \frac{\partial}{\partial x_j}2\mu\underbrace{\left(\overline{S_{ij}u_i} - \widetilde{S}_{ij}\widetilde{u}_i\right)}_{III}.\end{aligned} \tag{11}$$

where q_i are the components of the heat flux vector $\mathbf{q}$, given by $\mathbf{q} = -\mathbf{k}\nabla\mathbf{T}$, and e_T is the total energy given by $e_T = e + \rho\mathbf{u}\cdot\mathbf{u}/2$, where e is the internal energy. The terms on the right hand side identified by II and III represents the work done by the shear stress in the subgrid scale, whereas term I is the convective transport of total energy and can be decomposed into new contributions expressed bellow,

$$\overline{\rho}(\widetilde{u_j e_T} - \widetilde{u}_j\widetilde{e}_T) = \underbrace{\overline{\rho}(\widetilde{u_j e} - \widetilde{u}_j\widetilde{e})}_{IV} + \underbrace{\frac{\overline{\rho}}{2}(\widetilde{u_j u_k u_k} - \widetilde{u}_j\widetilde{u}_k\widetilde{u}_k)}_{V}. \tag{12}$$

Using the equation of state for perfect gases, terms II and IV are directly related by the following expression:

$$\overline{pu_j} - \overline{p}\widetilde{u}_j = (\gamma - 1)(\overline{\rho e u_j} - \overline{\rho e}\widetilde{u}_j) = (\gamma - 1)\overline{\rho}(\widetilde{eu_j} - \widetilde{e}\widetilde{u}_j). \tag{13}$$

In Eq.(13), $\gamma = c_p/c_v$, where c_p and c_v are the specific heat at constant pressure and volume, respectively . Adding terms II and IV it is defined the vector Q_j that represents the transport of internal energy in the subgrid scales, namely

$$Q_j = \gamma\overline{\rho}\left(\widetilde{eu_j} - \widetilde{e}\widetilde{u}_j\right). \tag{14}$$

Terms III, IV and the vector Q_j, resulting from averaging process of energy equation, needs to be modeleted in a general case.

2.2. A scaling analysis

Before to propose a model to the subgrid terms, some scaling analysis can be performed in order to evaluate the relative importance of each subgrid mechanism. First, note that the vector Q_j can be expressed in terms of temperature diffusion in the subgrid scale

$$Q_j = \gamma\overline{\rho}\left(\widetilde{eu_j} - \widetilde{u}_j\widetilde{e}\right) = \overline{\rho}c_p\left(\widetilde{Tu_j} - \widetilde{T}\widetilde{u}_j\right). \tag{15}$$

This diffusion mechanism is promoted by the velocity fluctuation transport in this scale. In such case, a typical scale of these velocity fluctuations is given by $U^* = \sqrt{\mathbf{u}'\cdot\mathbf{u}'}$, where $\mathbf{u}' = \mathbf{u} - \widetilde{\mathbf{u}}$. From a perfect gas, the temperature can be related to sound speed c in the form $T = c^2/\gamma R$. Then, a typical scale for the vector Q_j is given by

$$Q_j \sim \frac{\overline{\rho}c_p U^* c^2}{\gamma R}. \tag{16}$$

Where R is the gas constant given by Carnot formula like $R = c_p - c_v$ and L is a typical lenght scale of the turbulent flow. Similarity to the Q_j we have

$$\mu\left(\overline{S_{ij}u_i} - \widetilde{S}_{ij}\widetilde{u}_i\right) \sim \frac{\mu(U^*)^2}{L}, \tag{17}$$

$$\overline{\rho}(\widetilde{u_j u_k u_k} - \widetilde{u}_j\widetilde{u}_k\widetilde{u}_k) \sim \overline{\rho}(U^*)^3, \tag{18}$$

where L is a characteristic length scale of the large eddies. Comparing the magnitude of the term III and V with the vector Q_j and supposing a high Reynolds number and a low Mach number flow, one obtains:

$$\left| \frac{(\overline{S_{ij}u_i} - \widetilde{S}_{ij}\widetilde{u}_i)}{Q_j} \right| \sim \frac{\gamma R}{cp}\frac{M_\infty^2}{Re_\infty} \ll 1, \tag{19}$$

$$\left| \frac{\overline{\rho}(\widetilde{u_j u_k u_k} - \widetilde{u}_j\widetilde{u}_k\widetilde{u}_k)}{Q_j} \right| \sim \frac{\gamma R}{cp} M_\infty^2 \ll 1. \tag{20}$$

Where $Re_\infty = \rho LU/\mu$, $M_\infty = U/c$ and U is a typical velocity scale of the non disturbed flow. By this way, it is clear that in low Mach and high Reynolds numbers, the work done by shear stress and the kinetic energy transport done by subgrid eddies are small when comparing then with the transport of internal energy Q_j. The scalings are supported by Knight *et al.*(1998), who haves evaluated the subgrid terms remained from energy equation using direct numerical simulation. In conclusion, the only two terms to be modeleted under the limit $Re_\infty \gg 1$ and $M_\infty \ll 1$ in these situations are the subgrid stress tensor Σ_{ij} and the subgrid internal energy transport vector Q_j.

2.3. Constitutive relations for the remaining subgrid terms

The model used to the evaluation of the subgrid stress tensor is the well-known Smagorinsky model (Smagorinsky, 1963). It suggests a constitutive relation for $\boldsymbol{\Sigma}$ tensor in the form

$$\boldsymbol{\Sigma} = 2\mu_t\widetilde{\mathbf{D}}, \tag{21}$$

Where the coefficient μ_t is a turbulent viscosity is calculated under conditions of inertial equilibrium sub-range of turbulence (Landau, 1995) namely

$$\mu_t = \overline{\rho}(C_S\lambda)^2||\widetilde{\mathbf{D}}||. \tag{22}$$

Here $||\widetilde{\mathbf{D}}||$ is the norm of strain rate tensor defined like $||\widetilde{\mathbf{D}}|| = (2\widetilde{\mathbf{D}} : \widetilde{\mathbf{D}})^{1/2}$. The filter width λ is set equal to $2h$, where h is the grid spacing. It indicates that the smallest eddies are represented by two grid points. The factor C_S are known as Smagorinsky constant. Several values have been proposed for this constant and its value varies in $0.1 - 0.2$ range (Lilly, 1987; Deardorff, 1970). In the present work is used $C_S = 0.20$, as suggested by Deardorff (1970). Thus, the model for the subgrid-stress tensor takes the form

$$\boldsymbol{\Sigma} = 2\overline{\rho}(C_S\lambda)^2||\widetilde{\mathbf{D}}||\widetilde{\mathbf{D}}. \tag{23}$$

The subgrid internal energy transport tensor Q_j is related to the diffusion of temperature in the subgrid scales due to velocities fluctuations and may be modelated as being a diffusive heat transport given by a modified Fourier law in the form

$$Q_j = -k_t\frac{\partial \widetilde{T}}{\partial x_j}. \tag{24}$$

The turbulent heat conductivity k_t may be written in terms of a turbulent Prandtl number, Pr_t (Lesier, 1993)

$$k_t = \frac{c_p}{Pr_t}\mu_t. \tag{25}$$

For the edge of a turbulent boundary layer $Pr_t = 0.6$ (Fulachier & Dumas, 1976) and this value has been used in the simulations. The set of governing equation can also be non-dimensionalized by using a characteristic length L and the properties of the non disturbed flow. From this point through all over work we will omit any superscript notation and assume that all properties are dimensionless averaged quantities. The final model to be simulated is given by the continuity principle, written in the Eq.(5), and the momentum and energy averaged equations written as

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_j u_i) = -\frac{\partial p}{\partial x_i} + \frac{1}{Re_\infty}\frac{\partial}{\partial x_j}\left[2(\mu + \mu_t)S_{ij}\right] \tag{26}$$

$$\frac{\partial}{\partial t}(\rho e_T) + \frac{\partial}{\partial x_i}(\rho e_T u_i) = -\frac{\partial}{\partial x_i}(p u_i) + \frac{1}{Re_\infty}\frac{\partial}{\partial x_i}(2\mu S_{ij}) + \frac{1}{(\gamma - 1)Pr_\infty M_\infty^2 Re_\infty}\frac{\partial}{\partial x_i}\left[(k + k_t)\frac{\partial T}{\partial x_i}\right], \tag{27}$$

where Pr_∞ are the Prandtl number of non disturbed flow, given by $Pr_\infty = c_p \mu_\infty / k$.

3. Statistical formulation

The main goal of this work is to treat statistically turbulent flows from numerical simulations. In this context, the flow is considered a stochastic process given by $u = u(t, \alpha)$, where $\alpha = 1..N$ are the realizations of the process (see Fig.(1)). The randomic function $u(t_o, \alpha)$, for a fixed time $t = t_o$, may be any flow property like pressure or velocity fluctuations. A temporal average $\underline{u}(\alpha)$ of a realization of the process is given by

$$\underline{u}(\alpha) = \lim_{T\to\infty} \frac{1}{T}\int_0^T u(t, \alpha)dt. \tag{28}$$

In the other hand, if each realization has the same probability to occur, a statistical (or probability) average is defined as being

$$\langle u(t)\rangle = \lim_{N\to\infty}\frac{1}{N}\sum_{\alpha=1}^{N} u(t, \alpha). \tag{29}$$

A fluctuation about the probability average, is defined as $u'(t) = u(t) - \langle u\rangle$. Using this definition, the velocity fluctuation correlation function of the process is given by

$$R\left(u'(t), u'(t+\tau)\right) = \frac{\langle u'(t)u'(t+\tau)\rangle}{\langle u'(t)u'(t)\rangle}. \tag{30}$$

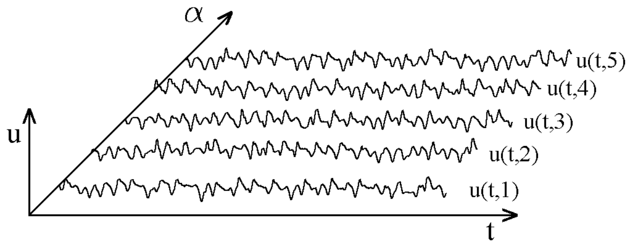

Figure 1: Stochastic process $u(t, \alpha)$. Each temporal series is a realization of the process.

In the case which the probability average and the correlation function do not vary with time we simply write $\langle u\rangle$ and $R(\tau)$ and the process is said statistically stationary. A process which the temporal and the probability averages do not differ is said to be an *ergodic process*. In order to evaluate the ergodicity of a stochastic process, we take the variance between the two averages. Also using the definition of correlation function given in Eq.(30) is possible to shown that

$$\sigma^2(T) = \left\langle (\underline{u} - \langle u\rangle)^2 \right\rangle = \frac{2\langle u'^2\rangle}{T}\int_0^T \left(1 - \frac{\tau}{T}\right) R(\tau)d\tau. \tag{31}$$

Equation (31) is an important result that relates the correlation function with the variance σ^2. Note that if $T \to \infty$ leads to $\sigma^2 \to 0$, that implies the ergodicity condition. Usually, in a process in which the correlation function decays rapidly for a relatively short time T, the ergodic condition is verified. In particular, in the case of a homogenous and isotropic turbulence, the correlation function is closely to one of a random walk process, say $R(\tau) \sim e^{-\tau/\Theta}$, where Θ is a relaxation time associated to an interval in which the events are weakly correlated. The value of Θ can be estimated using the integral scale L/U_∞ . If we define the error due to non-ergodicity of the process as

$$\epsilon = \sigma / \langle u\rangle \tag{32}$$

and use the exponential decay for the correlation function, the integral in the Eq.(31) may be performed and gives an estimation of ϵ, namely

$$\varepsilon^2 \sim \frac{2\langle u'^2\rangle\Theta}{T\langle u\rangle^2} = \frac{2I^2\Theta}{T}, \tag{33}$$

where $I = \sqrt{\langle u'^2\rangle}/\langle u\rangle$ is the turbulent intensity of the flow. Equation (33) gives also an estimation of the long time T necessary to hold the ergodicity of the process. In this case, the time average approach is sufficient to describe statistically the ergodic process. In this work, the result expressed in the Eq.(33) is tested by a direct calculation of the variance σ^2.In order to build a stochastic process from the numerical simulations a large temporal series is dropped into smaller series corresponding now to the realizations of the process (see Fig.(2)). The short temporal series are independent events of the turbulent flow, since the time scale involved are long enough for the complete decaying of the correlation function. It means that a series has no memory on the events that occur in the preceding time. This fragmentation procedure is equivalent to starts a new simulation from a different initial condition which has no correlation with the first one.

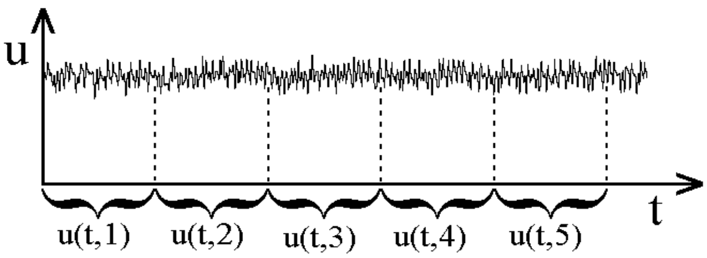

Figure 2: Fragmentation of the simulated turbulent signal in several independent realizations.

Another important quantity of the flow memory is the Taylor time scale. Using a Taylor series to expand $u'(t+\tau)$ in a neighborhood of t and supposing a stationary statistical process, the correlation function based on a time average may be written as

$$R(\tau) = \underline{u'(t)u'(t+\tau)}/\underline{u'^2} = 1 - \frac{\tau^2}{2!}\frac{1}{\underline{u'^2}}\underline{\left(\frac{\partial u'}{\partial t}\right)^2} + \mathcal{O}(\tau^3), \tag{34}$$

or

$$R(\tau) \sim 1 - \frac{\tau^2}{\lambda_\tau^2}, \qquad \text{where} \qquad \frac{1}{\lambda_\tau^2} = \frac{1}{2\underline{u'^2}}\underline{\left(\frac{\partial u'}{\partial t}\right)^2}. \tag{35}$$

From Eq.(35) it is clear that λ_τ is the short time scale of the correlation process. Using a second degree polynomial function in order to fit the correlation function for short times it is possible to estimate the λ_τ. Typically, the Taylor scale is larger than a dissipative time scale, but is not related to the integral scale observed in the macroscopic flow, i.e. $\lambda_0^2/\nu \ll \lambda_\tau \ll L/U_\infty$, where λ_0 is the Kolmogorov length scale. Effectively, the Taylor scale is a memory characteristic time of the flow. If t is the present time, we can say that the flow has an intense dependence of the events that occur in the interval $(t - \lambda_\tau, t)$.

4. Simulation setup

The turbulent flow over a bi-dimensional backward facing-step is simulated in this work. Figure (3) shows the flow domain. The inflow velocity profile imposed is uniform. The Reynolds number based on the step height is $Re_H = 38000$, ($Re \gg 1$) and the Mach number is $M_\infty = 0.03$, ($M_\infty \ll 1$). The numerical method used to discretize the filtered governing equation is the MacCormack method written to a finite volume formulation (Hirsch, 1990). This procedure leads to a second order precision discretization, in the space and time differentiations. The initial transients, corresponding to 10^6 iterations, i.e. $2,3s$ of physical time are ignored. A convective time scale of the flow i.e. $t_c \sim H/U$, was about 5×10^{-3} the longest simulation time, i.e. $T \cong 21s$. This wide interval is necessary because the velocity signal is fragmented into smallest temporal series to define the stochastic set. It will be shown that in some regions of the simulated flow even this long time interval is not appropriated for a statistical description of the flow based on temporal series. Figure (3) shows the position of the points in the flow and the streamlines of the mean turbulent field.

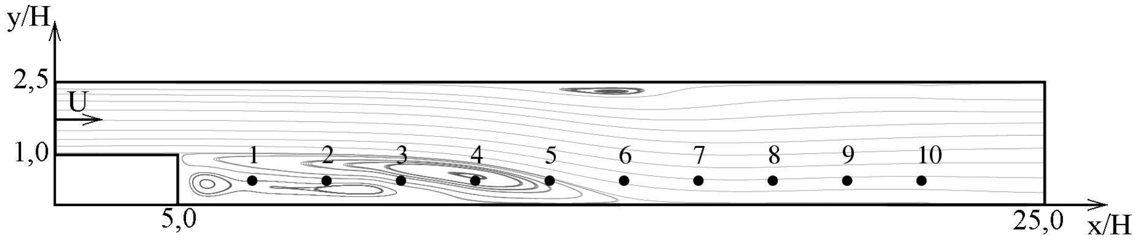

Figure 3: Flow domain and position of velocities probes in the flow. Equally spaced probes from x/H=1.5 (probe 1) to x/H=15.0 (probe 10). The height of all probes is y/H=0.5. $H = 5.08cm$, $U = 11.63m/s$

5. Results

The first step in the statistical treatment is to define the stochastic set in the probes locations. The temporal series for each probe must be fragmented into smaller ones with lengths are sufficiently large to define an independent realization. In order to evaluate this interval, a no-memory time scale are estimated by the interval in which the correlation function is already null. In this context, the correlation function is evaluated by an temporal averages approach. Figure (4) shows the normalized velocity fluctuation correlation function behavior for probes in two different dimensionless positions x/H.

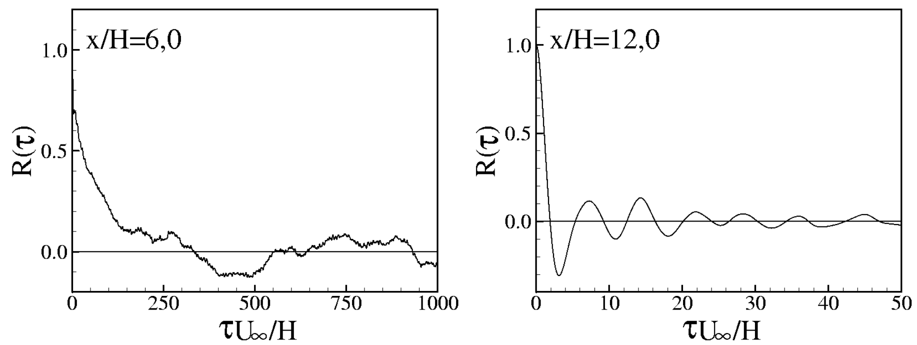

Figure 4: Normalized velocity fluctuation correlation function, $R(\tau) = \overline{u'(t+\tau)u'(t)}/\overline{u'^2}$, for probes 4 and 8.

From the graphs in Fig.(4), it can be observed that there is a remarkable difference between the memory scales for the probes 4 and 8. In probe 4, the memory time scale has an order of magnitude of 500 $\tau U_\infty/H$, since in probe 8 this time scale has order 10 $\tau U_\infty/H$. The temporal series in the probe positions are divided into fragments in which it is possible to consider the process non correlated. Table (1) shows the length of this fragments $\Theta U_\infty/H$, and the number of fragments (realizations) for each probe. By this way, a probability analysis have been carried out. All statistical quantity has an error associated that defines a confidence interval. Figure (5) shows the average velocity fluctuations in the probes 4 and 8. It is seen temporal oscillations in the value of the mean velocity fluctuation, that is more intense in the probe 4. This oscillation is a direct consequence of the difference between temporal and probability averages, what is related to a non-ergodic behavior. A variance between the averages processes gives a direct measure of the error ϵ, defined in the Eq.(32). This error can also be estimated by the relation proposed in the Eq.(33). Table (2) shows the predicted and the calculated error for each probe in the turbulence. The purpose of relation 33 is to give an estimation of the order of magnitude of the error introduced when the probability analysis is replaced by a temporal analysis. The results presented in Tab.(2) shows a very good agreement between the calculations. By this way, Eq.(33) can be used in order to estimate the appropriated long time T needed to a temporal analysis promotes a precise statistical description of the turbulent flow. In the case of non steady numerical simulations, it also possible from our approach to estimate the total time of simulation required for a correct statistical characterization of the flow.

In the case of probes 4 and 5, an exponential decay does not fit the decaying behavior of the normalized correlation function. It indicates that the turbulence in this region has a quite different behavior of a random walk process. The dispersion process of momentum transport by velocity fluctuations seems to characterize an anomalous diffusion. In that case, the integral in the Eq.(31) is evaluated numerically. The probe 4 shows a strong non-ergodic behavior, suggesting that in this region a temporal analysis cannot be used to describe the

Table 1: No memory time and number of realizations for each probe

Probe	$\Theta U_\infty/H$	Number of realizations
1,2,3	100	40
4,5	500	9
6	200	20
7,8,9,10	10	100

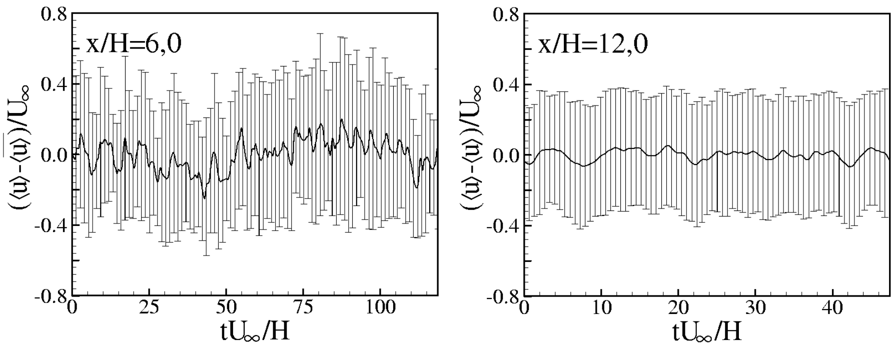

Figure 5: Averaged velocity fluctuation signal in the probes 4 and 8.

Table 2: Comparison between ϵ predicted by relation 33 and directly evaluated from numerical simulation data using the variance between the temporal and probability averages.

Probe	x/H	Directly evaluated error	Predicted error
1	$1,5$	16,3 %	20 %
2	$3,0$	12,1 %	20 %
3	$4,5$	13,8 %	20 %
4	$6,0$	4975 %	3400 %
5	$7,5$	24,5 %	13 %
6	$9,0$	9,6 %	10 %
7	$10,5$	2,5 %	3 %
8	$12,0$	3,2 %	3 %
9	$13,5$	3,5 %	3 %
10	$15,0$	3,2 %	3 %

local turbulence. It is possible to infer that all probes inserted into the recirculation bubble shown in Fig.(3) presents a significant deviation from the ergodic condition. Consequently, the flow in this regions persist strongly correlated for a long time as shown in Tab.(1). It means that large turbulent structures dominate the flow in the recirculation bubble. In probes 7 to 10, in the other hand, the turbulence is characterized by structures of small scales with short memory intervals and behaviors closer to randomic motions. In this case, a temporal analysis describes precisely the flow. The correlation functions for probes 4 and 8 are shown in Fig.(6). From this plots, it is possible to evaluate the memory level of the process. In probes 4 and 5 the correlation function decays very slowly with respect to the other ones. Their shapes are also different and an exponential or parabolic fit are not appropriated. For the all other probes, an exponential fit can be used for determining the no-memory intervals.

The statistical distribution of the process is shown in Fig.(7). A non-gaussian distribution is clear in probe 4, whereas in the probe 8, the behavior of the probability density function is closer to the normal distribution. The behavior of the statistical distribution are quantified by the skewness and flatness factors, defined as $\varphi = \langle u'^3\rangle/\xi^3$ and $\kappa = \langle u'^4\rangle/\xi^4$, where $\xi^2 = \langle u'^2\rangle$, respectively. These factors and the turbulent intensities for each probe are listed in Tab.(3). A normal statistical process has $\varphi = 0.0$ and $\kappa = 3.0$. It is possible to infer that all process display some non-gaussian behavior. The turbulent intensity at probe 4 has the order of 2×10^4%, whereas between the others the grater value of this parameter has the order of 1×10^2 (see probe 1, Tab.(3)). That fact strongly contributes to the non-ergodicity of probe 4. The interval of confidence represented by the error bars in the plots shown in Fig.(7) and by the associated errors to the quantities in

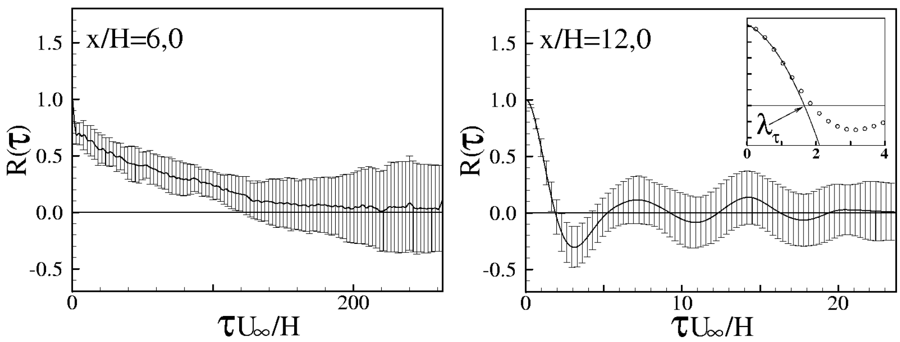

Figure 6: Normalized velocity fluctuations correlation function for the probes 4 and 8. Attempt to different time scales used in the plots. In probe 8 it is possible to fit a second degree polynomial function and estimate the Taylor time scale as $\lambda_\tau \sim 2$ dimensionless time unities.

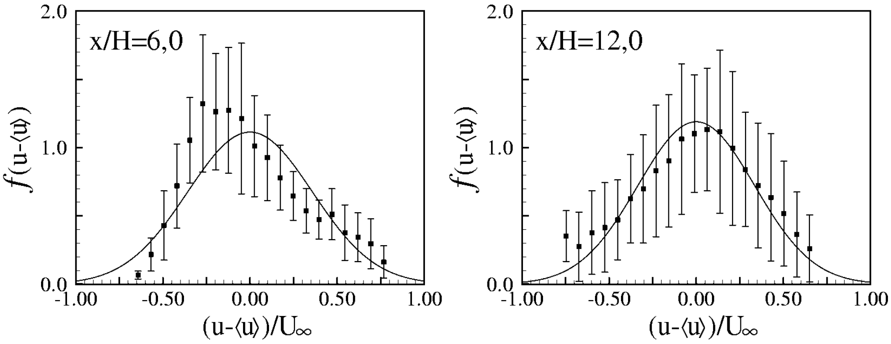

Figure 7: Probability density function for probes 4 and 8. The full line shows the corresponding gaussian process.

Table 3: Turbulent intensities, skewness and flatness factors for each probe.

Probe	x/H	Turbulence intensity	φ	κ
1	$1,5$	113 ± 38 %	$-0,3 \pm 0,6$	$2,7 \pm 0,8$
2	$3,0$	77 ± 26 %	$-0,2 \pm 0,5$	$2,7 \pm 0,5$
3	$4,5$	82 ± 16 %	$0,2 \pm 0,3$	$2,7 \pm 0,5$
4	$6,0$	$(2,1 \pm 0,4) \times 10^4$ %	$0,5 \pm 0,3$	$2,6 \pm 0,6$
5	$7,5$	88 ± 16 %	$0,0 \pm 0,3$	$2,3 \pm 0,3$
6	$9,0$	43 ± 6 %	$-0,3 \pm 0,3$	$2,5 \pm 0,2$
7	$10,5$	37 ± 4 %	$-0,5 \pm 0,4$	$2,9 \pm 0,7$
8	$12,0$	37 ± 4 %	$-0,2 \pm 0,5$	$2,9 \pm 0,5$
9	$13,5$	39 ± 3 %	$0,0 \pm 0,3$	$2,8 \pm 0,4$
10	$15,0$	41 ± 3 %	$0,0 \pm 0,2$	$2,7 \pm 0,4$

Tab.(3) are relatively large. It suggest that for a better characterization of these parameters more realizations are required, consequently it demands more computational effort in order to simulate larger time intervals.

6. Conclusion

In this article a formal statistical approach for the treatment of turbulence generated by large eddy computer simulations has been presented. A scaling analysis has been shown that the only two significant subgrid terms in the averaged energy equation are the internal energy transport, due to turbulent velocity fluctuation, and the work done by the pressure field in the subgrid scale. A large-eddy simulation of the turbulent air flow over a backward-facing step has been realized. From the numerical simulations, ten different points in the flow domain have been statistically treated using a probability approach. The realizations of the statistical ensemble was defined by the cut up of a long-time velocity record into pieces of a length much longer than the characteristic relaxation time. For the definition of this no-memory time scale an analysis of the correlation function has been made. The ergodicity of the turbulence was investigated. The deviations of this condition was compared with theoretical predictions given by scaling arguments and a good agreement was observed. It suggests that the error due to ergodicity hypotheses may be used as a tool to predict the total time of simulation required for a well statistical characterization of the flow. Probability functions, skewness and flatness coefficients have been shown a deviation of a gaussian behavior in all analyzed positions.

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