BOX PLOT CONTROL CHART FOR INTERACTIVITITY OF INFORMATION IN STATISTICAL PROCESS CONTROL

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Abstract. *This paper suggests a procedure for evaluation of data generated in an independent basis of quality control supposed to function as a check for internal quality control activities and as intermediate between manufacturing and marketplace for a Brazilian brick industry. The procedure aims to establish a continuous quality monitoring system for final products being delivered: providing feedback concerned to the levels of processes adjusting, and giving general information whether the results obtained are in conformity or not with Standard Specifications. It uses a box plot display, first with the purpose of Exploratory Data Analysis (EDA) and, moreover, according to Statistical Process Control (SPC) principles, when it evolves into a box plot control chart. The scheme of box plot control chart provides more interactivity of information than the conventional scheme of Shewhart's control chart. The advantages of using the procedure are illustrated by a presentation of an example using empirical data. It is shown that specific characteristics of box plot and its statistic computation enable to present measures of position and variability together with trends and skewness, sample by sample of results representatives of products being delivered, and, furthermore, enables to plot specification lines, statistical control limits, and outliers for statistical inference, when these parameters become expressive for analysis of data behavior.*

Keywords: *Box plot Control Chart, Statistical Process Control, Process Improvement, Performance Measurement, Ceramic Block.*

1. Introduction

Following a proposal of modernization of Brazilian Brick Industry, a group of manufacturers of ceramic blocks (red clay bricks) worry about the development of procedures for evaluation of final products being delivered in order to check conformity with Standard Specification and to reach more favorable operational performance at internal processes adjusting activities. They also aim to improve the way they make proof of quality at the market place using records of data. With the support of Associação Brasileira de Normas Técnicas (ABNT) they search "drastically to increase the respect to the official Standard in vigor" (Anicer, 2002) to their products, at the same time they are reviewing these same standard themselves and creating new ones necessary to the modernization of industry.

At this context, several acceptance criteria for products have been established which will reflect widely on the management of two fronts of actions: (i) that of improvement of internal quality control activities, and (ii) that of external confidence on products, which also depends on maintain reliable records of data concerned to issues of identification and certification. It is expected, with the inspections and assays accomplishment in independent basis of quality control, to obtain useful information for these both proposes simultaneously.

2. Procedure Outline

A reliable way to maintain structured information for performance measurement of manufacturing processes frequently includes statistical quality control techniques as support for data processing. These techniques are useful; both at the regular basis of internal quality control or at an independent basis of quality control. At the regular basis of internal quality control it is mainly expected that the results of parameters being controlled, in each point of the internal part of process chain, follow according to manufacturing specification in long-term runs with maximum of efficiency. At the independent basis of quality control it is mainly expected that the products been delivered are according to official Standards Specifications and specific exigencies established by contractors, and also to produce feedback to internal process adjusting.

The main idea, using the figure of box plot to monitoring statistical data from the independent basis of quality control, is that it is possible to make inference about relationships of results of final products concerned with some results reached at the regular basis of internal quality control activities, in order to improve performance in many stages of production. The application of box plot display, as proposed here, may produce exploratory and inferential information for statistical position and variability analysis of samples being submitted to evaluation in the independent basis of quality control since the beginning of its introduction, as if it were a signature of key-characteristics of lots of products being delivered associated with their correspondent period of production.

Questions may be posed concerned to the formation of lots for evaluation in the independent basis of quality control and to the validity of results for feedback to the internal quality control activities. The adequate management of this passage, with the correct correspondence to the periods of production, is the key factor to be managed, in order to give consistency to reflexions and inferences realized. For reason of focus, details on this management will not be treated here.

2.1 The box plot display

Original box plot *(or box and whiskers)* display was developed by Tukey (1977) to support an approach to statistics known as Exploratory Data Analysis (EDA). As the name suggests, the main intent of EDA is to be " 'exploratory' – to suggest evidence for decision making" - in contrast "to 'confirmatory' approaches that are designed to give statistical decisions" (White & Schroeder, 1987). Using *box plot display* to compose first a *basic box plot panel* and moreover a *box plot control chart*, one can associate the main intent of EDA with those features of Statistical Process Control (SPC), uncovering the benefits of both simultaneously. This is done, in this paper, with the figure providing interactivity of information since the beginning of data evaluation in the independent basis of quality control in the clay brick industry.

This paper introduces a variant of the standard box plot display as defined by Tukey (1977). It adopts modifications in computation of statistics and appearance with the propose of improve efficiency during evaluation of independent samples taken for indirect observation of lots of products. The pair of statistics *median of the data set* and *range of the middle half of the ordered data*, used to observe position and variability respectively with standard box plot, were changed by a pair of statistics *mean of the middle half of ordered sample* and *range of the middle half of the ordered sample* computed from random sample of size $n \geq 4$. Studies of statistical efficiency done by Iglewicz & Hoaglin (1987) pointed to the convenience of applying these statistics in box plot control charts.

Given an ordered sample for evaluation, the box plot display is derived from 5 notable order statistics: $\overline{X}_f$ (*mean of the middle half of ordered sample*); R_f (*range of the middle half of the ordered sample*); $F_L=X_{(f)}$ (*lower hinge*); $F_U=X_{(n+1-f)}$ (*upper hinge*); $X_{(min)}$ (*the minimum observation of the ordered sample*) and $X_{(max)}$ (*the maximum observation of the ordered sample*), where n is the sample size; $f = [(n+3)/2]/2$ for j=1, 2, ,...., n and [x] denotes the greatest integer less than or equal to x.

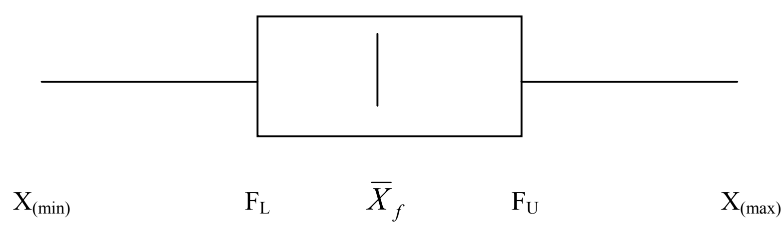


Figure 1 - Basic box plot figure

Figure (1) shows the box plot display, used to construct the *basic box plot panel***,** which derives from the 5 notable order statistics described above. It is composed by: a central box which length represents R_f , a transverse line in the central box that represents $\overline{X}_f$, the left extreme of central box that represents F_L , the right extreme of central box that represents F_U , and two prolonged lines at the extremes of central box representing observations out of internal middle half of sample. Details about the computation of $\overline{X}_f$ and R_f will be presented in Section 3.

2.2 The basic box plot panel for EDA

Using only a single configuration, sequences of box plot display correspondent to samples submitted to evaluation in the independent basis of quality control, from the first one, may be related to a Cartesian axis, in order to compose a *basic box plot panel* without concerning of theoretical SPC principles. At this stage, the vertical axis plots a scale of individual units for control of characteristic X_i under observation, the horizontal axis marks the sample sequence for process investigation and the two dotted lines mark the limits of specification for control of non-conformity of individuals. The positioning and variability among box plots representative of samples checked are the general information that enable to take conclusion about behavior of processes accordingly to EDA. Because limits of specification are plotted at the graphic panel, the criterion for outliers marking is not included in this stage of construction.

Figure (2) shows a *single box plot graphic panel* plotted for real data of the *length dimension* checked in 30 samples of a type of clay brick evaluated in an independent basis of quality control. This independent basis was organized with the aim of provide feedback for process adjustment. The type of the brick submitted to evaluation had the nominal value of length sat up on 290 mm and specification limits plotted at 287 mm and at 293 mm, respectively, as indicated at the panel.

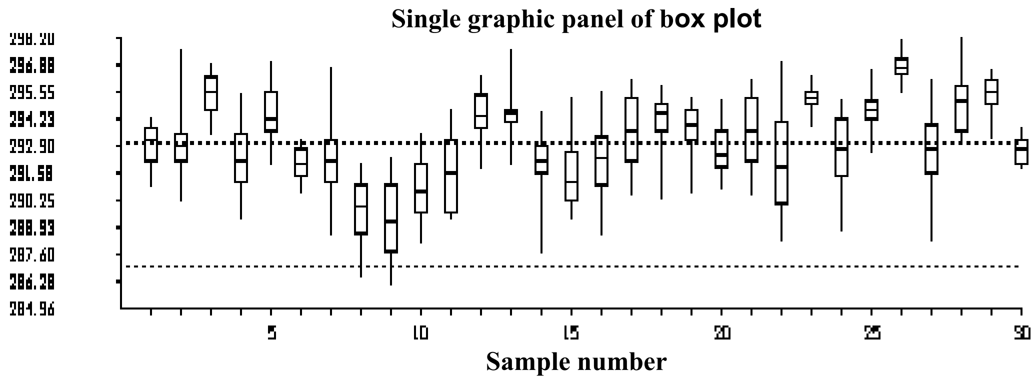


Figure 2 – Single graphic panel of box plot showing results of real data taking from length measurement on clay bricks

A brief exploratory data analysis using the graphic panel of Fig. (2) points immediately to a serious disagreement between various isolated lots manufactured and the correspondent specification limits plotted. In addition shows a significant fluctuation in pattern of position and variability among the lots suggesting a lack of statistical control.

To improve our analysis, Fig. (3) shows a correspondent panel graph constructed to represent hypothetical results of products coming from the same manufacturing process as if it were developed under statistical control with small fluctuation. It was supposed that the data follow a normal distribution with the nominal value of 290 mm taken as a historical arithmetic mean and a standard deviation of 1mm. The range correspondent to the interval 287mm to 293mm, plotted on the positions of the specification limits, equals 6 σ. The data set was generated using computer simulation and sample of size 13.

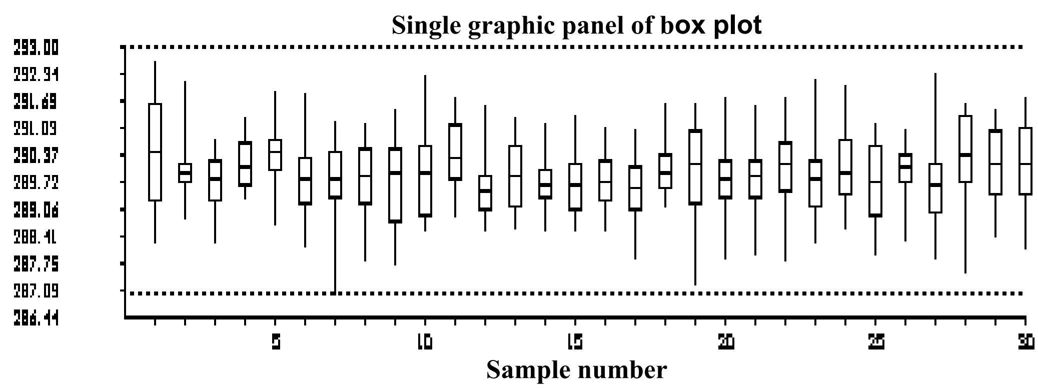


Figure 3 - Single graphic panel of box plot displaying simulated results of data for length measurement on clay bricks

Comparing the preceding graphic panels it is possible to make some inference about the general efforts necessary to adjust the process chain of industry and meet exigencies concerned to the evaluation of conformity accordingly to Standard Specification, nevertheless the amount of fluctuation in patterns of position and variability of the process could be better evaluated using SPC principles.

2.3 Scheme of box plot control chart $\overline{X}_f$ - R_f for SPC

To complete the procedure for evaluation of data generated in an independent basis of quality control, applying SPC principles, the scheme of box plot control chart $\overline{X}_f$ - R_f - briefly presented in this Section - is primarily supported by two previous works of Peixoto (1992, 1993), which were founded in Tukey (1977), White & Schroeder (1987), and Hoaling & Iglewicz (1987). Beyond the basic features of the *single box plot graphic panel*, presented in previous section, other important characteristics like plotting statistical control limits (for control of short and long terms drifts in processes) and displaying of outliers are included at the graphic panel committed with SPC principles. Specific details are:

(a) The *central line* for control of $\overline{X}_f$ is marked with heavy line and *control limits* (for the same statistic $\overline{X}_f$) with dashed lines;

(b) A dashed box on panel indicates that the value of R_f is out of statistical control;

(c) For each ordered individual observation $X_{(i)}$, located in the interval ($X_{(min)} \leq X_{(i)} \leq F_L$) or ($F_U \leq X_{(i)} \leq X_{(max)}$), if its respective numeric value is less than LCL (low control limit for individuals) or greater than UCL (upper control limit for individuals), then the observation is marked with an *asterisk* (*) in each box plot display. Contrarily, if its respective numeric value is greater than LCL or less than UCL, the observation is indicated at the prolongation of the whiskers of box plot display. Note that low and upper limits for individuals are not plotted at panel.

(d) The X marks on horizontal axis indicates those samples throw out during computation of control limits, attempting SPC principles.

Details about the computation of central lines and control limits for $\overline{X}_f$ and R_f, and LCL and UCL for individuals will also be presented in Section 3.

To illustrate the preceding information, Fig. (4) presents a complete box plot control chart designed to statistical control of $\overline{X}_f$; R_f and individual values for samples presented in Fig. (2).

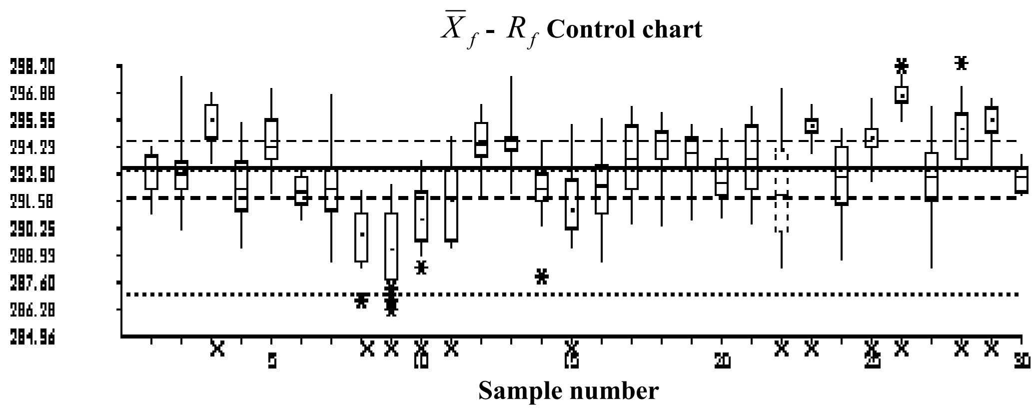


Figure 4 - Box plot control chart $\overline{X}_f$ - R_f using results of real data of length measurement of clay bricks

Table 1- Values of central lines and control limits (3-sigmas limits) for $\overline{X}_f$ - R_f Box Plot Control Charts of Fig. (4) and Fig. (5), and $\overline{X}$ - R Control Charts of Fig. (6).

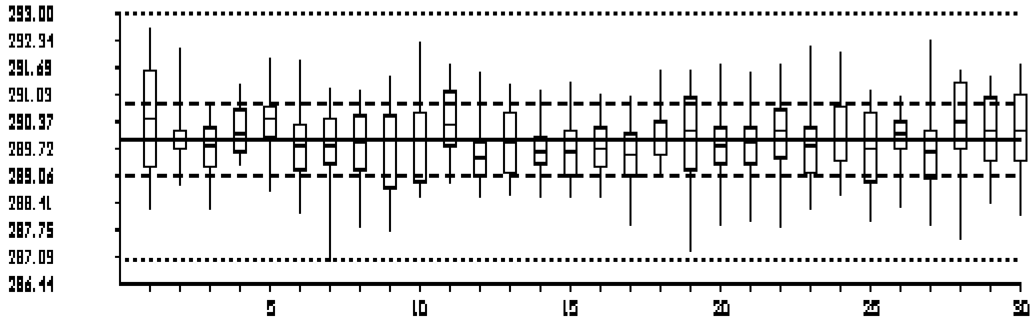
Statistics	$\overline{X}_f$	R_f	X_i	$\overline{X}_f$	R_f	X_i	$\overline{X}$	R
	For Fig. (4)			For Fig. (5)			For Fig. (6)	
Central Line (CL)	293.151	1.897	293.151	289.934	1.195	289.934	293.029	5.602
Low Cont. Limit (LCL)	291.753	0	288.471	289.045	0	286.964	291.634	1.720
Upper Cont. Limit (UCL)	294.549	3.763	297.831	290.824	2.394	292.900	294.424	9.483

Table (1) presents results of central line and control limits computed accordingly to development of Section 3.

Beyond disagreement with specification limits pointed before, from the point of view of SPC, there are many alarms of lack of statistical control: eleven (11) of position and one (1) of variability (sample number 22). There are also a great amount of outliers that indicate a possible instability in the adjusting of cutting process.

To improve the analysis, Fig (5) presents the same data generated for Fig. (3) shown a correspondent panel graph constructed to represent hypothetical results of products coming from the same manufacturing process developed under statistical control.

$\overline{X}_f$ - R_f Control chart



Sample number

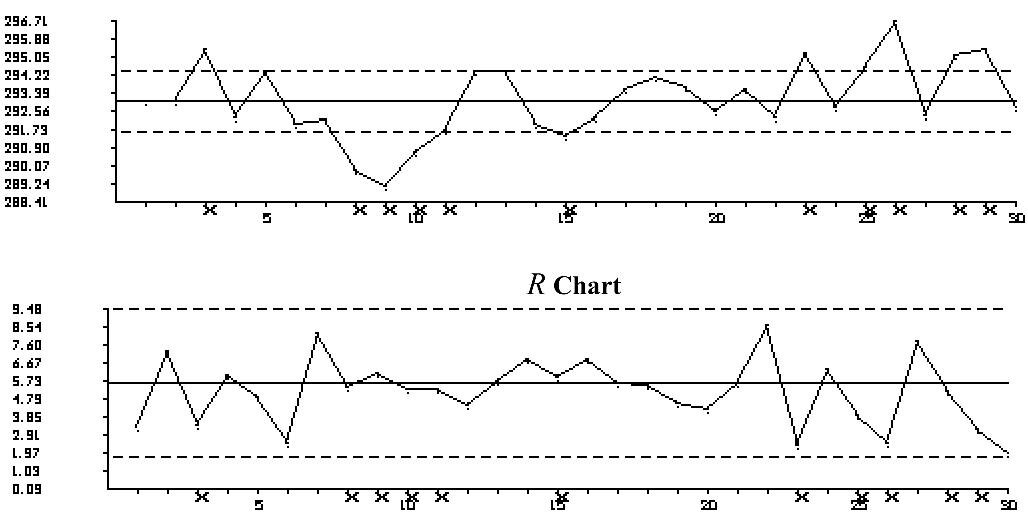
Figure 5 - Box plot control chart $\overline{X}_f$ - R_f using simulated data for length measurement of clay bricks

2.4 Comparison with the scheme of Shewhart's control chart

The conventional scheme of Shewhart' control chart for $\overline{X}$ and range R, is presented here as counterpart for evaluation of interactivity of information generated in Figure (4). White & Schroeder (1987) and Hoaling & Iglewicz (1987) also followed this procedure in their proposals of box plot control chart.

Differently of $\overline{X}_f$ - R_f scheme, the $\overline{X}$ - R scheme uses an associated pair of control charts for control of sample means and sample ranges separately. $\overline{X}$ and R are shown as single points plotted at each graphic panel. Only an abstract conjugation of both control charts in the Scheme of Shewhart enables additional information about interactivity among points of position and variability in short and long term runs.

$\overline{X}$ Chart



Sample number

Figure 6 – Control chart $\overline{X}$ - R displaying results of real data of length measurement of clay bricks Presented in Fig. (2)

Unless n=1, it is not recommended, in Shewhart`s Scheme, to present limits of specification marked at $\overline{X}$ chart because of difference in scale plotting $\overline{X}$ and the values of individual units X_i. In $\overline{X}_f$ - R_f scheme this restriction is suppressed because order statistics computation enables individual units to be presented. In particular, the ability for displaying individual units value in form of outliers is of enormous importance in analyzing data behavior, because discrepant observations can be immediately identified and traceable in their origin in order to determine cause of variation without influencing the stability of displaying alignment and further computation of control limits for SPC. The use of the resistant statistics $\overline{X}_f$ and R_f (see Section 3), for determination of long-term parameters, is also an advantage despite the small loss of sensitivity of box plot scheme compared to Shewhart`s Scheme (Hoaling & Iglewicz, 1987). Additionally, to display individual units can be helpful in three sigma (or other rule) control of non-conformance data, depending on how limits of specification are fixed and indices of process capacity are required (PEIXOTO, 1992).

3. Statistical Background

For observations X_1, X_2, ... , Xn, the order statistics are the ordered observations $X_{(1)} \leq X_{(2)} \leq .... \leq X_{(n)}$, where each position $X_{(.)}$ can be assumed by anyone X_i. In SPC, the standard pair of estimators of μ and σ are the mean $\overline{X}$ and the range R, that can be derived as linear combination of order statistics given respectively by,

$$\overline{X} = (X_{(1)} + X_{(2)} + ... + X_{(n)})/n \text{, and } R = X_{(n)} - X_{(1)} .$$

Another way of estimating μ and σ, among many other possibilities, is computing $\overline{X}_f$ and R_f. These estimators involve only computation toward the middle half of the n ordered observations of samples left away extremes values and other adjacent values depending on size of n. $\overline{X}_f$ and R_f belong to a class of estimators known as *resistant estimators*, because they are few affected by deviant values of sample distributions. To simplify, "resistant estimators prove to be quite efficient when the data come from distributions with heavier tails than the normal" Hoaling & Iglewicz (1987). This characteristic enables to stabilize control limits against sporadic small drift in position or variability of processes, or against a combination of both.

If there is a set of ordered observations of $n \geq 4$:

a) $\overline{X}_f$ is defined as follow:

$\overline{X}_f = \overline{X}$, for n=4

(1)

$$\overline{X}_{(f)} = \sum_{j=f}^{n+1-f} X_{(j)} /(n+2-2f) \text{, for f integer}$$

(2)

$$\overline{X}_{(f)} = \{X_{(f)} + X_{(n+1-f)} + \sum_{j=i+2}^{n+1f} X_{(j)} \}/(n-2i) \text{, for } f = i + 1/2$$

(3)

where $f = [(n+3)/2]/2$, j=1, 2, ,...., n and [x] denotes the greatest integer less than or equal to x.

b) R_f is defined as:

$$R_{(f)} = X_{(n+1-f)} - X_{(f)}$$

(4)

I lustration 1

For

$f = [(n+3)/2]/2$

If n=5

$f = [(5+3)/2]/2 = [4]/2 = 4/2 = 2$; where [4] =4 (the greatest integer equal to 4), i=2, and $n+1-f = 4$

Thus $\overline{X}_f = (X_{(2)} + X_{(3)} + X_{(4)})/3$, and $R_{(f)} = X_{(4)} - X_{(2)}$

If n=8

$f = [(8+3)/2]/2 = [5.5]/2 = 5/2 = 2.5$; where [5.5]=5 (the greatest integer less than 5.5), i=2 and $n+1-f = 8+1-2{,}5 = 6.5$

Thus,

$$\overline{X}_f = (X_{(2.5)} + X_{(6.5)} + \sum_{j=4}^{5} X_{(j)})/8-4$$

$$= \{1/2(X_{(2)} + X_{(3)}) + 1/2(X_{(6)} + X_{(7)}) + X_{(4)} + X_{(5)} \}/4$$

$$R_{(f)} = X_{(6.5)} - X_{(2.5)} = 1/2(X_{(6)} + X_{(7)}) - 1/2(X_{(2)} + X_{(3)})$$

Table (2) gives the linear combination for R_f estimator for $4 \le n \le 15$. To derive correspondent values of $\overline{X}_f$, one may change the sign (-) by (+) and complete the expression with interpolation of absent elements.

n	Expression of Rf	d_4(n)	A2F	D3F	D4F
4	$0.5(X_{(3)}+ X_{(4)}- X_{(1)} - X_{(2)})$	1.326	1.131	0.000	2.325
5	$X_{(4)} - X_{(2)}$	0.990	1.444	0.000	2.723
6	$X_{(5)} - X_{(2)})$	1.283	1.003	0.000	2.378
7	$0.5(X_{(5)}+ X_{(6)}- X_{(2)} - X_{(3)})$	1.110	1.074	0.000	2.226
8	$0.5(X_{(6)}+ X_{(7)}- X_{(2)} - X_{(3)})$	1.325	0.840	0.000	2.072
9	$X_{(7)} - X_{(3)}$	1.143	0.939	0.000	2.225
10	$X_{(8)} - X_{(3)}$	1.312	0.770	0.000	2.082
11	$0.5(X_{(8)}+ X_{(9)}- X_{(3)} - X_{(4)})$	1.190	0.816	0.000	2.005
12	$0.5(X_{(9)}+ X_{(10)}- X_{(3)} - X_{(4)})$	1.329	0.696	0.084	1.916
13	$X_{(10)} - X_{(4)}$	1.205	0.774	0.000	2.003
14	$X_{(11)} - X_{(4)}$	1.323	0.649	0.080	1.920
15	$0.5(X_{(11)}+ X_{(12)}- X_{(4)} - X_{(5)})$	1.230	0.681	0.128	1.872

Table 2- Important information to design $\overline{X}_f$ - R_f schemes of control charts.

3.1 Control Limits Computation

To compute 3-sigms control limits for $\overline{X}_f$ - R_f scheme, it is assumed that sample data consist of groups of n independent normal observations coming from a normal distribution with mean μ and variance σ^2, and that an estimator of μ can be derived as:

$$\hat{\mu} \cong \overline{\overline{X}}_f ,$$

where, $\overline{\overline{X}}_f$ is the mean of the mean of the middle half measures of the samples, given by:

$$\overline{\overline{X}}_f = (1/k)\sum_{i=1}^{k} \overline{X}_{f_i}$$

K is the number of samples taken for observation and $\overline{X}_{f_i}$ is each mean of the middle half measures of samples.

The estimate of σ, the standard deviation of population is given by:

$$\hat{\sigma} = \overline{R}_f / d_{4(n)}$$

where $\overline{R}_f$ is the mean of the middle half of the samples given by:

$$\overline{R}_f = 1/k \sum_{i=1}^{k} R_{f_i}$$

and R_{f_i} represents each range of the middle half of samples.

Finally, $d_{4(n)}$, presented in Table 2, is derived of order statistic theory (Peixoto,1992).

Computation of control limits for $\overline{X}_f$ and R_f uses the factors denoted by A2F, D3F and D4F, given by Hoaling & Iglewicz (1987), which are also included in Table 2. A especial characteristic introduced by Peixoto(1992) is the procedure for determination of outlier in sample observations. Instead of boundaries of F_L-1,5R_f and F_L+1,5R_f, adopted by Hoaling & Iglewicz (1987), outliers are marked in relation to control limits for individual units X_i based on $\pm 3\hat{\sigma}$, as is defined in conventional $\overline{X}$ - R scheme of Shewhart for n=1. So doing, only after control limits computation, the box plots display is enable to include outliers in representation of ordered samples. Table 3 resumes the formulas used to derive the control limits used in designing the complete $\overline{X}_f$ - R_f scheme.

Statistics	$\overline{X}_f$	R_f	X_i
Central Value	LC= $\overline{\overline{X}}_f$	$\overline{R}_f$	$\overline{\overline{X}}_i$
UCL	$\overline{\overline{X}}_f$ +A2F($\overline{R}_f$)	D4F($\overline{R}_f$)	$\overline{\overline{X}}_i + 3\hat{\sigma}$
LCL	$\overline{\overline{X}}_f$ - A2F($\overline{R}_f$)	D3F($\overline{R}_f$)	$\overline{\overline{X}}_i - 3\hat{\sigma}$

Table 3- Values of central lines and control limits (3-sigmas limits) for $\overline{X}_f$ - R_f box plot control chart and control of outliers.

3.2 Interactivity of information

An accurate inspection of displaying of $\overline{X}_f$ - R_f and $\overline{X}$ - R schemes show that for almost all alarms violation of nominal process condition of operation, at one graphic panel, there is a correspondent alarm at the other. However, $\overline{X}_f$ - R_f scheme can be seen as more informative in the sense that it shows more information interactively than its counterpart.

3.3 Sensitizing Rules for $\overline{X}_f$ - R_f Scheme of Box Plot Control Charts

In analyzing information of $\overline{X}_f$ - R_f scheme, the several criteria applied simultaneously in analysis of Shewhart's control charts, given by Montgomery (1991), are primarily applied to $\overline{X}_f$ investigation. These criteria correspond to rules (i) to (vi) below. To complete the analysis of R_f and individual units, rules (vii) to (ix) are included. The entire set of rules is:

i. One or more point outside of the control limits (the basic criterion);
ii. A run of at list eight, points, where the type of run could be either a run up or down, a run above or bellow the center line, or a run above or below the median;
iii. Two of three consecutive points outside the 2-sigma warning limits but still inside the control limits;
iv. Four of five consecutive points beyond the 1-sigma limits;

v. An usual or nonrandom pattern in the data;
vi. One or more points near a warning or control limits;
vii. One or more dashed box in box plot display indicate R_f outside of the control limits (the basic criterion to analyzing R_f);
viii. Points marked with asterisk, inside or outside limits of specification for $\overline{X}_f$, indicate that one or more outlier is present in the sample given indication of possible variation in position or variability of process for individuals results;
ix. The symmetry (or asymmetry) of a box plot display gives information about distribution of values in ordered samples.

4. CONCLUSION

The main purpose hitherto was to suggest a procedure for evaluation of data generated at an independent basis of quality control that supports a Brazilian Brick Industry, with the aim of establishing interfaces between this site, the external exigencies and the internal quality control activities. In this way, both the basic box plot panel and its evolution to a box plot control chart proved to be of great utility, finding accomplishment for performance results of final products associated with standard specification and feedback information toward in-process adjustment. Nevertheless, it was pointed that the success of this accomplishment depends heavily on correct management of statistical techniques. Here it is convenient to add that the diffusion of box plot control chart through internal quality control activities is important to match key-parameters along all manufacturing process chain.

Concerning to statistical efficiency of the techniques presented, as a general orientation, in the situations where the use of $\overline{X}$ - R scheme is possible and the capture of small shifts in process is not mandatory, the alternative use of $\overline{X}_f$ - R_f may be considered. In particular, as the box plot display is more informative than single points plotted in the graphic panel, the box plot control chart can be more adequate when there is interest in control of individual units, and their conformance to specification.

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