MODAL CONTROL APPLICATIONS IN INTELLIGENT TRUSS STRUCTURES

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Abstract. Many control strategies and techniques have been used in active vibrations control, like LQG control, H_2 and H_∞ methodologies. Usually, these methodologies involve distributed sensors and actuators and a control law to minimize a selected objetive function. In special, vibrations control in truss structures has great practical interest. Light structures are, also, usually lightly damped, which cause large amplitude vibration, and any disturbances can degrade the demanded performance. Therefore, the main purpose of this paper is to use independent modal space control (IMSC) applied in light truss structures. IMSC is characterized by controlling several modes independently and it's been used by its efficiency and for reducing the spillover effects, which can result from ignoring higher order modes when implemented the active feedback control. Others advantages of this methodology can be perceived for reducing and simplifying the control systems with complex dynamics. The control force can be obtained by actives members, as PZT wafer stacks substituting bars, that accomplish an axial force. The piezoelectric effect of actuator is ignored in the proposed mathematical model. The analytical model is obtained using finite elements methods and classical modal analysis. The results show the vibration reduction in a structure by controlling some modes and the improvement of the performance system.

Keywords: Active Vibrations Control, IMSC, Intelligent Truss Structures, PZT wafer stacks.

1. Introduction

Vibrations control in truss structures has great practical interest, mainly in modern structures of huge space vehicles and aircraft. Two demands essences are requested in designs of such structures. The first one is the excellent dynamic behavior, in order to guarantee the stability of the structure and high precision pointing. The second one is the necessity of to obtain light structures, in order to reduce the cost. However, these two requirements are often contradictory, because light structures have low degrees of internal damping, which hinder the accuracy requirements, Yan and Yam (2002), Lammering et al. (1994).

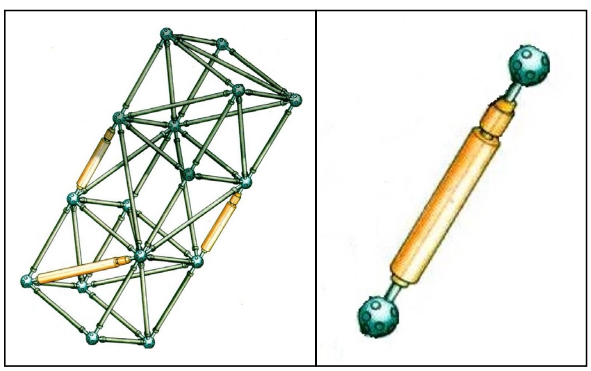


Figure 1. Truss structure example, Schütze et al. (1998) and Piezelectric actuator stacks in detail.

These difficulties can be overcome by applying recently developed advanced materials, as for instance piezoelectric materials, Brennan et al. (1994). Several researcher have proved that piezoelectric material can be effective in active vibration control. In truss structures the control force can be accomplished by piezoelectric active members, known as "PZT wafer stacks", that are mechanically linked in series producing an axial force in the bar that are positioned, Fig. (1).

Many strategies and approaches have been used to model and to design control flexible structures, for instance: Shibuta et al. (1992) present the control of a truss structure using LQG/LTR, while Liu and Zhang (2000) use IMSC in a truss with 96 bars, and Moreira et al. (1999) use techniques of robust control (H_∞ control). A control strategy very used for system with large number of degrees of freedom is based in a modal representation. This technique is known as independent modal space control (IMSC), Meirovitch and Baruh (1982), Meirovitch (1990). The IMSC method is characterized by controlling various modes independently, reducing the spillover effects, and it is numerically efficient. Moreover, the obtained approach is mathematically elegant.

2. Independent Modal Space Control (IMSC)

The mathematical model of flexible structure, for an undamped system, can be given by,

$$[M]\{\ddot{u}\}+[K]\{u\}=f(t) \tag{1}$$

where {u} is the displacements, f(t) the vector of applied forces, M and K are mass and stiffness matrices, of order n.

Making the realization, we can rewritten Eq. (1) in state space representation,

$$\{\dot{x}\}=\begin{bmatrix} [0] & [I] \\ -[K][M]^{-1} & [0] \end{bmatrix}\{x\}+\begin{bmatrix} [0] \\ [M]^{-1}[D] \end{bmatrix}\{u_c\} \tag{2}$$

where {x} is the space state variable, [D] is the actuator placement matrix and $\{u_c\}$ is the input vector.

The usual structure models by finite elements methods (FEM) demands large number of degrees of freedom (dof), and not all n modes need to be controlled. Then, Eq. (2) cannot be used directly for designing the control system, Wang et al. (1999). The control system can be designed in reduced modal space employing the transformation $\{u\}=[\Phi]\{q\}$. So, Eq. (1) can be written in modal coordinates as,

$$\{\ddot{q}\}+[\Omega]\{q\}=[\Phi]^T[D]\{u_c\}=\{f_{mc}\} \tag{3}$$

where $[\Omega]=\mathrm{diag}\{\omega_1^2, \omega_2^2, \ldots\omega_{nc}^2\}$ is the eigenvalue matrix, f_{mc} is the input modal force, and $[\Phi]$ is a nc x nc normalized mode matrix, used to cast Eq. (1) into the modal space representations. This matrix transforms the coupled equations of motion into the uncoupled form Eq. (3). Equation (3) can be rewritten as a state space equation for each controlled mode as,

$$\{\dot{z}_i\}=[A_{mi}]\{z_i\}+[B_{mi}]\{f_{mci}\} \qquad i=1,2,..,nc \tag{4}$$

in which,

$$[A_{mi}]=\begin{bmatrix} 0 & 1 \\ -\omega_i^2 & 0 \end{bmatrix},\ [B_{mi}]=\begin{bmatrix} 0 \\ 1 \end{bmatrix} \text{ and } \{z_i\}=\begin{Bmatrix} q_i \\ \dot{q}_i \end{Bmatrix} \tag{5}$$

The linear feedback control law for each controlled mode can be expressed as,

$$\{f_{mci}\}=-[G_{mpi} \quad G_{mvi}]\{z\}=-(G_{mpi}q_i+G_{mvi}\dot{q}_i) \tag{6}$$

so that the ith modal control force is proportional to both the ith modal displacement and ith modal velocity. The ith modal gains G_{mpi} and G_{mvi} can be obtained by minimizing the following quadratic performance index,

$$J_i=\int_0^\infty\left(\{z_i\}^T[Q_i]\{z_i\}+\{u_{ci}\}^T[R_i]\{u_{ci}\}\right)dt \tag{7}$$

where $[Qi]=\mathrm{diag}\{\omega_i^2, 1\}$ and $[R_i]=[r_i]$ are weighting matrices for the ith modal state vector and modal control vector, respectively. The ith optimal modal control force can be written by,

$$\{f_{mci}\}=-[R_i]^{-1}[B_i]^T[P_i]\{z_i\} \tag{8}$$

where $[P_i]$ is the matrix 2 x 2

$$[P_i]=\begin{bmatrix} P_{11}^i & P_{12}^i \\ P_{12}^i & P_{22}^i \end{bmatrix} \tag{9}$$

that satisfies Riccati equations given by,

$$[P_i][A_{mi}]+[A_{mi}]^T[P_i]+[Q_i]-[P_i][B_i][R_i]^{-1}[B_i]^T[P_i]=[0] \tag{10}$$

There are many algorithms to solve Eq. (10), but the most used is the Potter's Method. When working with IMSC it is possible to obtain analytical solution to Riccat equations by,

$$P_{12}^{i} = -r_i \omega_i{}^2 + r_i \omega_i \sqrt{\omega_i^2 + r_i{}^{-1}}$$

$$P_{22}^{i} = \sqrt{r_i - 2\omega_i^2 r_i{}^2 + 2\omega_i r_i \sqrt{\omega_i^2 r_i{}^2 + r_i}} \tag{11}$$

$$P_{11}^{i} = \sqrt{\omega_i^{-2} - r_i - 2\omega_i^2 r_i{}^2 + 2\omega_i{}^{-1} r^{-1} \left(\omega_i^2 r_i{}^2 + r_i\right)^{3/2}}$$

The minimization of Eq. (7) leads to the following modal gains, (Meirovitch, 1990)

$$G_{mpi} = \omega_i \sqrt{\omega_i{}^2 + r_i^{-1}} - \omega_i^2 \quad \text{and} \quad G_{mvi} = \sqrt{r_i^{-1} - 2\omega_i{}^2 + 2\omega_i \sqrt{\omega_i{}^2 + r_i^{-1}}} \tag{12}$$

It is very important to give the relationship between the physical control force and the modal control force. This can be given by, (Liu and Zhang, 2000)

$$\{u_c\} = \left([\Phi]^T [D]\right)^{-1} \{f_{mc}\} \tag{13}$$

The relationship between physical gains and modal gains can be written by

$$\begin{aligned} [G_P] &= \left([\Phi_i]^T [D]\right)^{-1} [G_{mpi}][\Phi_i]^T [M] \\ [G_V] &= \left([\Phi_i]^T [D]\right)^{-1} [G_{mvi}][\Phi_i]^T [M] \end{aligned} \tag{14}$$

where $[\Phi_i]$ is the ith mode. This methodology is valid only when the number of controlled modes is equal to that of actuators. In the design of the IMSC controller, the requirement for an infinite number of sensors can be approximately fulfilled by using a sufficient number of sensors at appropriate location to capture enought modal information for a specific dynamic system, Su (1993). Others alternatives for this limitation were discussed in full detail in Jia (1990).

3. Numerical Results

In this section is proposed the control design of a truss plane structure with 33 bars, Fig. (2). At the principle, each of the truss members can be replaced by an active member. The sensor is collocated at the node 10. The nodes 1 and 18 are clamped. The physical and geometric properties are shown in Tab. (1). The tubes are made of aluminum with diameter of ∅18x2mm and length L. It is considered that the damping is worthless. The model is obtained through a program developed in Matlab® using FEM, Kwon and Bang (1997).

Table 1. Material properties.

Properties	Values
Young's Modulus (N/m^2)	7.27 x 10^{10}
Density (kg/m^3)	3100
Length (m)	0.3

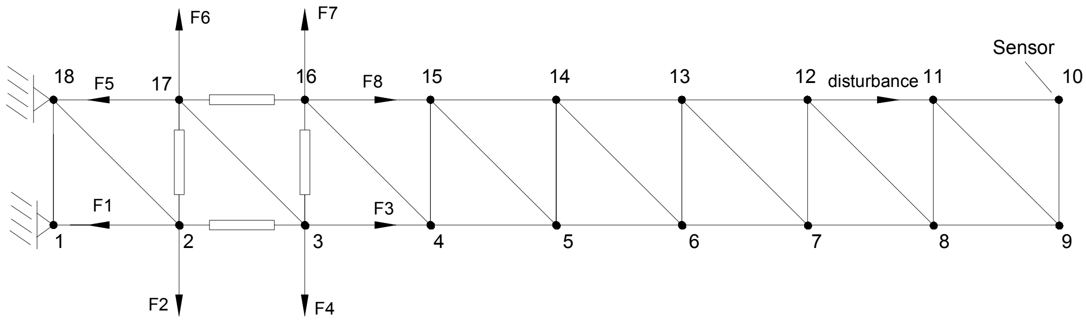
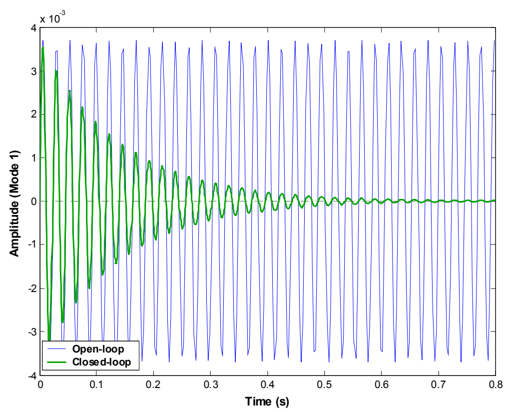


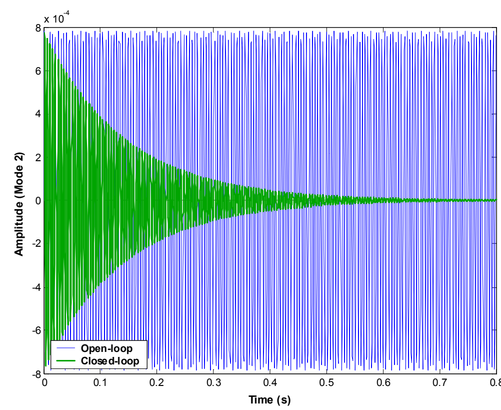
Figure 2. Truss structure controlled.

Each node has two degrees of freedom, translation in x and y direction, so, the truss structure has 32 active dofs, and the model in the form of states space results in order of 64. Obviously, for pratical and numerical limitations it's impossible to design a 64 order controller for this structure. Numeric algorithms for the solution of Riccati equation don't work well for systems of high order. For this reason in the nominal model was considered the control of the first eigth modes of the structure, and R=32-8=24 remaining modes are considered as residual dynamics, or uncertainties in the model. The actuator positions are shown in Fig. (2).

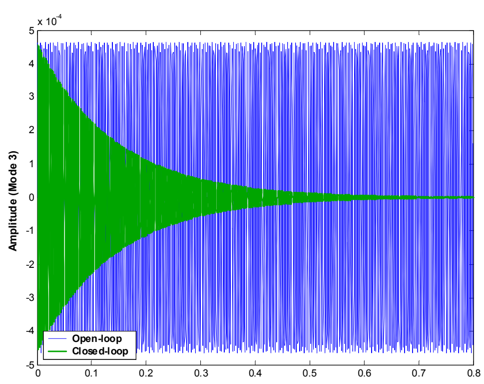
Figure (3) shows the impulse response (input disturbance vector on node 12 - horizontal direction) for open-loop and closed-loop system. This figure demonstrates that the modal responses can be successfully supressed after using the control methology described in section 2.



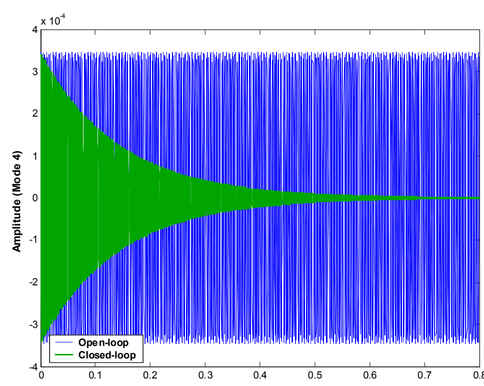
(a) first modal coordinate displacement



(b) second modal coordinate displacement



(c) third modal coordinate displacement



(d) fourth modal coordinate displacement

Figure 3. Modal response for an unitary impulse.

The closed loop natural frequency and damping ratio for controlling the first eigth modes are shown in Tab. (2), which are compared with open loop.

Table 2. Control Results.

Mode n.º		1	2	3	4
Open loop	Freq. (Hz)	42.9	203	343	458
	Damping ratio (%)	0.0	0.0	0.0	0.0
Closed Loop	Freq. (Hz)	42.9	203	343	458
	Damping ratio (%)	0.0262	0.0056	0.0033	0.0025

Figure (4) (input and output showed in Fig. (2)) shows the frequency response function (FRF) of the controlled and uncontrolled systems. The disturbance vector is considered an unitary impulse and measuring the response at node 10.

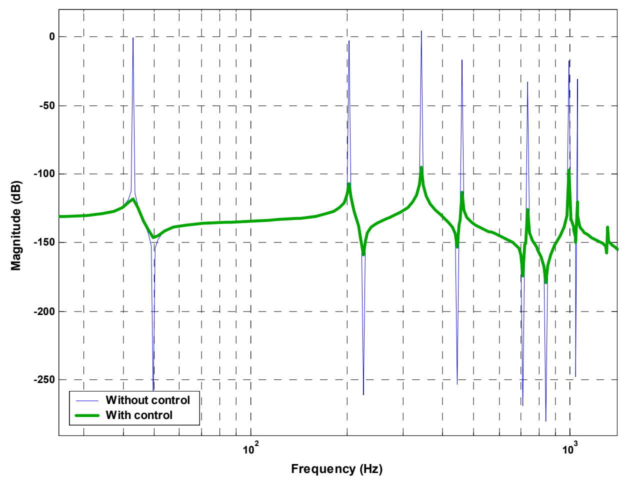
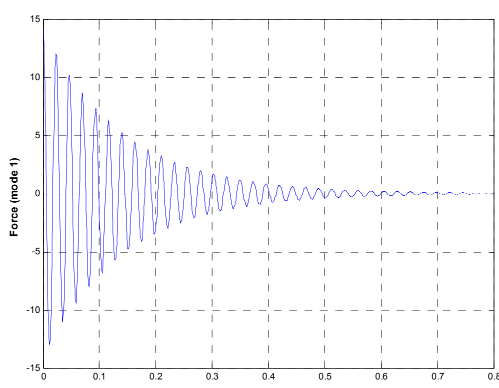


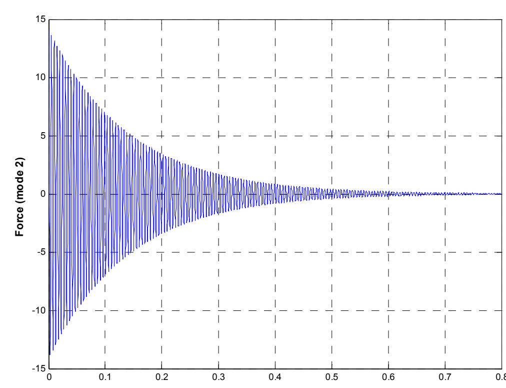
Figure 4. FRF for system with control and system without control.

When the system is controlled, the open loop value of the first mode attenuation reaches 117.25 dB. While the second mode an attenuation of 104.42 dB is achieved. The third and fourth mode an attenuation of 99.64 dB and 97.50 dB, respectively. It is important to note that the FRF for the open-loop was considered for undamped system. For practical situation, there exist some damping in the structure, consequently, the attenuation is smaller.

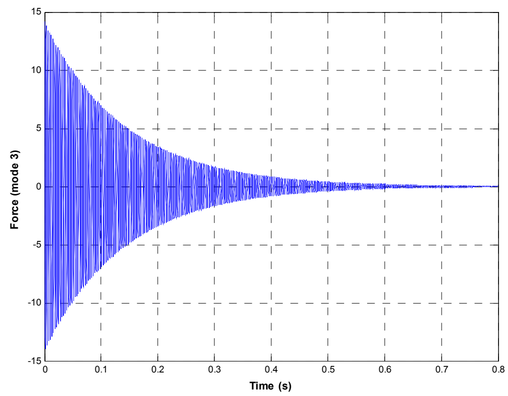
Figure (5) shows the first, second, third and fourth modal control forces. It's necessary to convert this forces into phisycal control forces, Eq. (13), to be applied into the structure. As illustrateded below, the transient reaction is attenuated quickly (within 0.6 s).



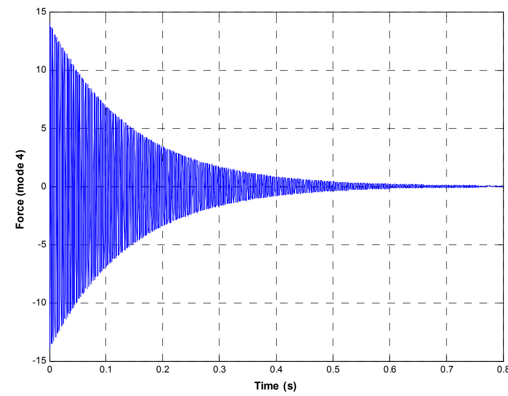
(a) first modal control force



(b) second modal control force



(c) third modal control force



(d) fourth modal control force

Figure 5. Modal control force response for an unitary impulse.

The vibration of the structure excited by the disturbance forces can be efficiently suppressed by using the IMSC. But, there is still some steady-state vibration. It includes vibration of the controlled modes, which cannot be cancelled out entirely by the control, and the vibration of uncontrolled modes. The uncontrolled modes are excited by the disturbance forces and the control forces. The responses of the uncontrolled modes induced by the control forces are called control spillover. The portion of small and high frequency oscillation in the responses of the truss structure is the effect of the control spillover.

4. Conclusion

An IMSC based LQR feedback control strategy was used actively to control the vibration of a truss structure. As example was proposed the control design of a plane structure with 33 bars, Fig. (2). Each of the truss members can be replaced by an active member. The sensor was collocated on node 10. Nodes 1 and 18 were clamped. The physical and geometric properties are shown in table 1. The tubes are made of aluminum with diameter of ∅18x2mm and length L. It is considered that the damping is worthless. The model is obtained through a program developed in Matlab® using FEM.

Active suppression of a truss plane structure with 33 bars with an impulsive disturbance force applied was reached by using the adaptive control approach in modal space. Only a set of differential equations of size 2 x 2 needs to be solved for each controlled mode by using the Independent Modal Control System scheme. In the first eigth modes, frequencies below 1400 Hz, the attenuations of the amplitude achieved were very high, for the situation considered in this paper.

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6. References

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