ANALYSIS OF INTERFACIAL CRACK IN BIMATERIAL LAMINATES USING BOUNDARY ELEMENT METHOD

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Abstract. *This paper presents a procedure for analysis of problems of linear elastic fracture mechanics in bimaterial laminates. Using this boundary element procedure, it is possible to evaluate stress intensity factors in plane problems (2D) due to the presence of interfacial cracks between the laminae that constitute the laminate. The problem is modeled using the subregion technique to describe each one of the different subdomains, represented by each material. According to this method the domain is divided in two or more sub-regions and conditions of displacements continuity and tractions equilibrium are imposed at the interface, except in the crack region. The singular behavior presented by the stress field near the crack tip is modeled by traction singular quarter point. Numerical examples of problems with in-plane loading are presented. Some of these examples have correspondents in literature, and were used for comparisons with the obtained results. A good agreement between them is obtained. Mesh convergence analyses are also presented, showing little dependence on the discretization even when coarse meshes were used.*

Keywords*. Boundary Elements Method, Fracture Mechanic, Composite Materials, Interfacial Cracks.*

1. Introduction

Structures composed of bonded layers of dissimilar materials are becoming more and more common in a variety of applications. Recently the quantitative evaluation of the strength of metal/metal, metal/ceramic and composite bonded joints have become very important. Dissimilar bonded materials are also applied extensively in microelectronic devices, particularly in the area of data storage and processing. Bonded layers are used in the construction of integrated circuit devices, and they occur as thin film/substrate combinations in magnetic tapes and hard disk drives. Dissimilar bonded materials also exist in welding, soldering and protective coating applications.

In many such structures failures originating from production processes, overload, aging or waste of the structure can be observed. These failures can lead to initiation of the interface debonding, which occurs through the growth of an interfacial crack, with serious implications on the structural reliability of bimaterial systems.

To evaluate the strength of these materials, the stress intensity factors for an interfacial crack is needed, because the fracture is commonly observed near the interface. And the study of interfacial crack of a dissimilar isotropic bimaterials with different elastic properties is becoming important in the analysis of laminates. The stress intensity factor at the interfacial crack arises from geometric discontinuities and a materials discontinuity. These discontinuities induce both the opening and the sliding mode. The stress intensity factors of opening mode (K_I) and sliding mode (K_{II}) are coupled (Williams, 1959).

The concepts of the linear elastic fracture mechanics for interfacial cracks in isotropic bimaterials were reviewed by Rice (1988), who introduced the bases for its interpretation in a similar sense of linear elastic fracture and established procedures for analyses like as homogeneous bodies when the mechanical contact zone or non linear material response in the crack tip is small.

The presence of a crack in a structure usually induces stress concentrations in the crack tip. The boundary element method is known as a particularly appropriate numeric technique applied to analysis of interfacial cracks in bimaterial components as isotropic as anisotropic.

Some works using the boundary element method on analysis of interfacial cracks in isotropic bimaterial components were presented by Tan and Gao (1990, 1991), Yuuki and Xu (1994) and recently by Portilho de Paiva (2000) and Portilho de Paiva and Sollero (2001). Other works, on the application of the method to analysis of cracks in the fiber/matrix interface, have been presented by Paris et al. (1996), Beldica and Botsis (1996) and Selvadurai (1996). More recently, the application of the method in analysis of interfacial cracks in anisotropic bimaterial components has been presented by Pan and Amadei (1999).

In this work a boundary element program was developed in which the continuous quadratic element was implemented. It was also implemented the method of the sub-regions for the modeling of the subdomain of the laminated bimaterial, besides the quarter-point element, to describe the displacements and stress fields at the crack tip.

2. Fracture mechanics in bimaterial laminate

The presence of cracks in structures usually induces stress concentrations at crack tip. Fracture mechanic in bimaterial laminate is getting increasing attention because of its direct applications to composite laminates. It is well known that laminated structures are prone to defects such as broken fibers, cracks in the matrix and interface delaminating. Premature failure due to the existence of delamination is one of the most common failure modes in composite materials and bonded joints. Analytic relationships for displacement and stress fields in the neighborhood of interfacial cracks among dissimilar materials are well documented (Murakami et al., 1987 and Rice, 1988).

The total stress intensity factor K_0 for interfacial cracks is defined as

$$K_0 = |K| = \sqrt{K_I^2 + K_{II}^2} \tag{1}$$

The stress intensity factor can be calculated in several ways. The more direct way uses the nodal values of tractions, namely

$$K_0 = \frac{\sqrt{2\pi\, l}}{\cosh(\pi\zeta)} \sqrt{(t_1)^2 + (t_2)^2} \tag{2}$$

where ζ is a function of material properties, t_1 and t_2 are tractions calculated at boundary nodes of crack tip and l is the quarter-point element length, according shown in Figure 1.

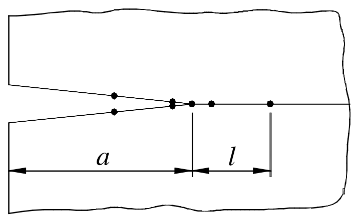

Figure 1. Crack tip elements and its characteristic dimension.

3. The boundary element method

The boundary element method has been developed for several decades and it has shown as an appropriated computational tool for analysis of fracture mechanic problems. An extensive revision of its application in fracture mechanics has been presented by Aliabadi (1997) and Cruse (1998). The boundary element formulation, relating the displacements u_i and the tractions t_i in the boundary Γ of an elastic homogeneous domain, is described by the well-known boundary integral equation, which is given by (Brebbia and Domínguez, 1989 and Kane, 1994)

$$-c_{ij}(x^*)u_j(x^*) + \int_\Gamma [t_i(x)U_{ij}(x,x^*) - u_i(x)T_{ij}(x,x^*)]d\Gamma(x) = 0 \tag{3}$$

where c_{ij} is a function of the source point x^*, and T_{ij} and U_{ij} are the fundamental solutions of tractions and displacements, respectively, for in-plane bidimensional elastostatic problems which are given by

$$T_{ij} = -\frac{1}{4\pi r(1-\nu)}\{\frac{\partial r}{\partial n}[(1-2\nu)\delta_{ij} + 2r_{,i}r_{,k}] + (1-2\nu)(n_i r_{,j} - n_j r_{,i})\} \tag{4}$$

$$U_{ij} = \frac{1}{8\pi\mu(1-\nu)}[(3-4\nu)\ln\frac{1}{r}\delta_{ij} + r_{,i}r_{,j}] \tag{5}$$

where μ is material property, r is the distance between source point x^* and field point x and n is the outside normal unitary vector at field point.

In real problems it can be difficult found an analytic solution for Eq. (3). In this case a numeric solution, where the boundary Γ is split in n boundary elements Γ_e, is used. So, the boundary integral equation is transformed into the following linear system of algebraic equations

$$\mathbf{Hu} = \mathbf{Gt} \tag{6}$$

where $\mathbf{H}$ and $\mathbf{G}$ matrix contain the fundamental solutions of tractions and displacements, respectively, and the vectors $\mathbf{u}$ and $\mathbf{t}$ contain the boundary displacements and tractions values, even known or not. After some algebraic manipulations the unknowns are isolated in a vector $\mathbf{f}$ so that the system shown in Eq. (6) can be represented by

$$\mathbf{A\,x} = \mathbf{f} \tag{7}$$

where, using an algorithm for solution of linear equation system, it is obtained the unknown values of displacements and tractions.

Quadratic continuous boundary elements were implemented in the program. These elements have as characteristic to model curved geometries and high gradients of displacements, tractions and stresses in an efficient way. But standard quadratic elements cannot represent the crack tip singular behavior very well unless using very refined meshes. Thus the traction singular quarter-point element was also implemented, once they represent the singular behaviour of tractions and displacements in the crack tip and present little dependence of the used discretization (Martínez and Domínguez, 1984 and Smith, 1988).

3.1. Sub-region formulation

A laminate is formed from two or more laminae of the same or different materials bonded together to act as single-layer structural element. The bond between adjacent lamina in a laminate is assumed to be perfect, that is, infinitesimally thin and not shear deformable. Thus the laminae cannot slip over each other, and the displacements remain continuous across the bond.

In order to model the laminate one need divide the domain in two or more areas due to presence of cracks in the structure and to existence of different materials in the domain.

Let one consider a domain divided in two sub-regions Ω_1 and Ω_2, with boundary Γ_1 and Γ_2 separated by an interface I delimited by $\Gamma_1{}^I$ and $\Gamma_2{}^I$ (Fig. 2). Displacements $\mathbf{u^I}$ and tractions $\mathbf{t^I}$ in the boundary Γ^I of each subdomain must satisfy conditions of displacements continuity and tractions equilibrium.

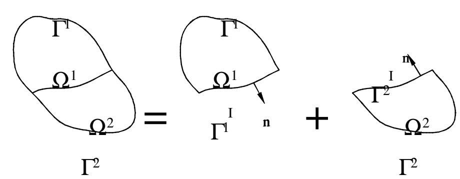

Figure 2. Domain division using sub-region.

After setting up conditions of displacements continuity and tractions equilibrium in the interface and knowing that they are unknown, the Eq. (6) it can be rewritten in the following way

$$\begin{bmatrix} \mathbf{H_1} & \mathbf{H_1^I} & -\mathbf{G_1^I} & 0 \\ 0 & \mathbf{H_2^I} & \mathbf{G_2^I} & \mathbf{H_2^I} \end{bmatrix} \begin{Bmatrix} \mathbf{u_1} \\ \mathbf{u^I} \\ \mathbf{t^I} \\ \mathbf{u_2} \end{Bmatrix} = \begin{bmatrix} \mathbf{G_1} & 0 \\ 0 & \mathbf{G_2} \end{bmatrix} \begin{Bmatrix} \mathbf{t_1} \\ \mathbf{t_2} \end{Bmatrix} \tag{8}$$

Applying the boundary conditions in the system of Eq. (8) the system shown in Eq. (7) is obtained from which unknown boundary and interface values of displacements and tractions are obtained.

4. Numerical results and discussions

Some examples of application of the developed formulation are presented. The aim of these examples is to demonstrate the applicability of the method and to assure that its results are in good agreement with results of the literature. All results are shown for different meshes and a little dependence of the results can be verified in relation with mesh used. In the numerical analyses carried out was investigated the effect of the changes in the relationships among the modules of the materials in the accuracy of the obtained results.

4.1. Interfacial crack in a dissimilar half-planes

Analysis of problems of cracks lying in an interface of two bonded dissimilar half-planes subjected to tension and shear remotely applied are presented. Such problems have analytic expressions (Murakami et al., 1987). Figure 3 shows the physical problem considered. In the numerical treatment, a finite size plate was treated instead, but the width and height of the plate was taken to be 20 times the size of the crack. Plane stress conditions were assumed. Also, to ensure continuity conditions for the strain ε_{xx} along the bimaterial interface, the applied stress $(\sigma_x)_{II}^{\infty}$ was taken to be

$$(\sigma_x)_{II}^{\infty} = \frac{E_2}{E_1}(\sigma_x)_I^{\infty} + \left[\nu_2 - \frac{E_2}{E_1}\nu_1\right](\sigma_y)^{\infty} \tag{9}$$

were E_1 and E_2 are the Young moduli of materials I and II, respectively.

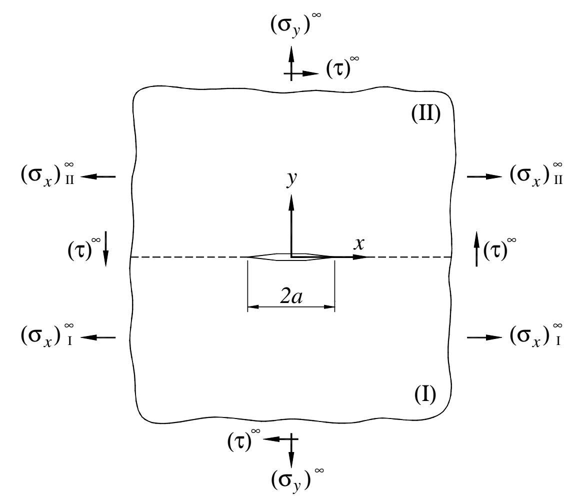

Figure 3. Interfacial crack in a dissimilar half-planes.

Figure 4 shows the mesh employed. Also in this figure it can be seen the original and deformed mesh.

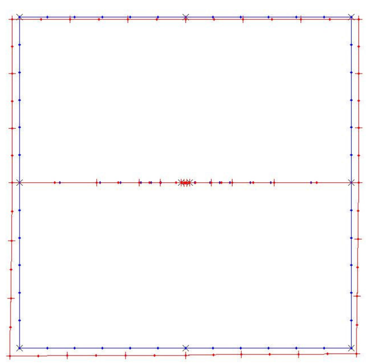
Figure 4. Original and deformed mesh for a problem of an interfacial crack in a dissimilar half-planes.

Five different values of μ_2/μ_1 ratio were considered. Table 1 presents the stress intensity factor K_0 obtained from the boundary element analysis compared with analytic stress intensity factor calculated according expressions presented by Murakami et al. (1987).

4.2. Infinite row of interfacial crack in a dissimilar half-planes

Figure 5 shows the physical problem considered, that is, an infinite bimaterial plate with an infinite row of collinear cracks lying at interface, subjected to remotely tension and shear applied. As in the previous example, in these numerical analyses were modeled a finite plate, but the plate dimensions were taken to be 20 times the size of the crack. Plane stress conditions were assumed and to ensure continuity conditions for the strain ε_{xx} along the bimaterial interface, the applied stress $(\sigma_x)_{II}^{\infty}$ was taken according to Eq. (9).

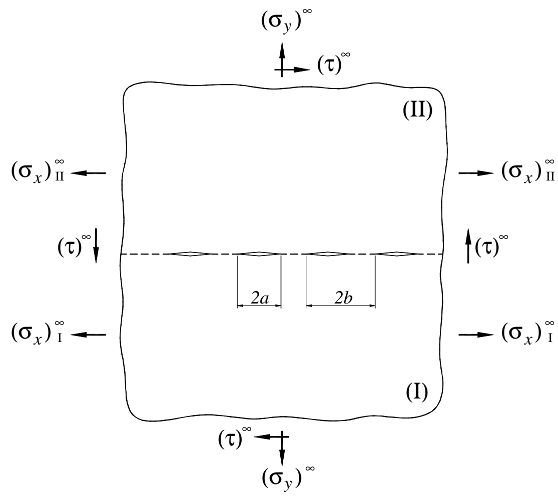

Figure 5. Infinite row of interfacial crack in a dissimilar half-planes.

In this example, five different values of μ_2/μ_1 ratio were considered. Table 2 presents the analytic stress intensity factor $K_{analytic}$ presented by Murakami et al. (1987) and the numerical stress intensity factor K_0 calculated using the implemented program. They are compared and the error found is shown.

Table 1. Stress intensity factors for an interfacial crack in a dissimilar half-plane.

$\frac{\mu_2}{\mu_1}$	$K_{analytic}$ $(Pa\sqrt{m})$	K_0 $(Pa\sqrt{m})$	Error (%)
1	8.8623	8.8958	0.3787
5	8.5040	8.6306	2.1112
10	8.3691	8.4945	2.2624
20	8.2825	8.4005	2.2740
100	8.2013	8.3075	2.2268

Table 2. Stress intensity factors for an infinite row of interfacial crack in a dissimilar half-plane

$\frac{\mu_2}{\mu_1}$	$K_{analytic}$ $(Pa\sqrt{m})$	K_0 $(Pa\sqrt{m})$	Error (%)
1	8.8990	8.8958	0.3787
5	7.8567	8.0498	2.4578
10	7.5714	7.9626	5.1668
20	7.4002	7.8358	5.8863
100	7.2463	7.7535	6.9994

5. Conclusions

In this paper the implementation of a program for stress intensity factors calculation of interfacial cracks in bimaterial structures formed by dissimilar isotropic materials was presented. The program was based on boundary element and subregions methods. The stress intensity factors were calculated from nodal values of tractions obtained through traction singular quarter-point elements.

Numerical analysis which involved varying the ratios of the material properties have been carried out on some well known problems and results show little dependence of the discretization and a good agreement with the analytic results presented in the literature. According with these results it can be assured that the used approach is suitable for treatment of problems of bimaterial structures containing interfacial cracks, dealing with accentuated ratios between elastic constants of the materials.

6. Acknowledgement

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