

CELLULAR COMPOSITES FOR DOUBLE CURVATURE APPLICATIONS

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Abstract. *Conventional materials undergo lateral contraction when stretched and lateral expansion when compressed, a material behavior which is not acceptable for special engineering applications. Bulletproof materials, for instance, require an increase in density, which results in a higher resistance to the damage caused by bullet penetration. In order to achieve it, the ideal material has to undergo a lateral contraction when under compression. A behavior that is only possible with a negative Poisson ratio. Cellular materials, e.g. honeycombs and foams, are suitable materials for this type of application as it is possible to obtain the negative Poisson ratio effect by altering their microstructure. This paper takes into consideration the micro mechanics of honeycombs and how their morphology affects the composite overall mechanical properties. The negative Poisson ratio effect is mainly controlled by the honeycomb morphology, but the wall contribution has to be superposed to the morphological effects. This new composite can be used as core materials for sandwich structures with low impact resistance increase.*

Keywords: Honeycombs, Negative Poisson Ratio, Cellular Composites, Sandwich Composites

1. Introduction

Two high-strength skins bonded to a thick core often compose the typical sandwich panel. The skins carry most of the bending, twisting and in-plane loadings. On the other hand, the core material separates and stabilizes the skins in other to reach the desired panel bending stiffness and loading-bearing capacity (Lee and Tostsis, 2000). As mentioned by Vaidya et all (2001) current generation of sandwich structures primarily use foam, balsa and honeycomb as core materials, as they exhibit lightweight advantages and good damage resistance. According to Kim and Christensen (2000) from the large variety of core materials used the most popular is the hexagonal honeycomb. This core has a two-dimensional hexagonal pattern made of thin-walled structures in one plane. Their good structural efficiency is due to the direct connection between the face panels and the honeycombs. Both, laminate and core, materials must be analyzed considering their behavior under loading and unloading conditions. Laminates, special polymeric matrix composites exhibit a near linear elastic behavior. Guo and Gibson (1999), however, pointed out that most of core materials, also called cellular materials by some authors; under uniaxial compressive stress have an elastic-plastic behavior. The walls suffer an elastic bending followed by buckling or by plastic yielding. As the deformation is further increased, cell collapse progresses at nearly constant loading, resulting in a stress plateau, until all cells have collapsed. After the collapse of the entire number of cells there is a sharp increase on stress due to the contact among the collapsed walls. The same pattern was observed by Papka and Kyriakides (1994) during their experiments with aluminum honeycombs under uniaxial compression. They stated that although the honeycomb performance was highly non-linear the same pattern was noticed for honeycombs with different relative densities. This is an indication that honeycomb morphology, more specifically the cell geometry, plays a major role into the overall mechanical behavior.

Several previous studies have examined the hexagonal honeycomb behavior not only under uniaxial compression (Gibson and Ashby, 1997; Kim and Christensen, 2000; Vaidya et all, 2001) but also shear (Zhang and Ashby, 1992), bending (Daniel and Abot, 2000), and biaxial loading (Gibson and Ashby, 1997). The results are, most of time, acceptable. However, when double curvature is required hexagonal honeycombs do not perform so well due to Poisson effects when the core is forced to curve. According to Zenkert (1997), this problem can be overcome using another cell shape. As mentioned by Gibson and Ashby (1997), Poisson ratios for honeycombs are strongly dependent on cell geometry. A core with negative Poisson ratio can be very useful; as the honeycomb is curved in one direction the secondary curvature in the other direction has the same signal. Hence, a core can be manufactured to fit to a specific double curvature. Moreover, as mentioned by Bitzer (1997), in the case of a positive in-plane Poisson ratio, as for hexagonal shape honeycombs, the bending will be anticlastic making the sheet difficult to fit the shape a doubly curved mould. However, if the Poisson ratio is negative, the reaction to single bending moment will be synclastic (same sign on K_x and K_y) and the core sheet will be easier to handle.

Lakes (1991) and Milton (1992) demonstrate mathematically the possibility of existence of core materials with negative Poisson ratio. According to them, this phenomenon is mainly due to microstructure geometrical configurations. They called these special microstructures as re-entrant arrays. Since then, many other researchers (Gibson and Ashby, 1997; Choi and Lakes, 1995; Lubarda and Mayers, 1999; Yang et all, 2003; Chaves et all, 2003) have been working on materials with negative Poisson ratio. Our objectives are not only to present a new design for honeycombs but also to derive the mathematical formulation for computing their elastic moduli. This special type of honeycomb can be applied for double curvature situations due to their in-plane Poisson ratio close to –1. Furthermore, the two in-plane Poisson ratios, minor and major, are also close to each other and equilibrated. As the result of this feature curvature in two directions is easily obtained.

2. The butterfly honeycomb and Its mathematical formulation

The new honeycomb design is a variation of the model presented by Milton (1991) called re-entrant cell. In the present model, the stress concentration on the corners is minimized by avoiding sharp connections between the walls. Once a curvature radius is introduced on the corners a smooth transition is established. The strategy employed to develop the new set of equations for stiffness estimation is based on the unit cell approach. Once the unit cell is isolated, a set of unitary loadings (uniaxial compression, shear loading, and biaxial compression in and out-of plane) are applied. Each effective/average stiffness modulus is computed by using the equilibrium equations. This is essentially the methodology employed by Gibson and Ashby (1997) in their work with good results. However, changes in some geometrical parameters can result in large variations into the effective elastic moduli (Gibiansky and Torquato, 1995). This phenomenon is common to all honeycomb configurations including the new re-entrant honeycomb.

Figure 1 shows the main dimensions considered into the butterfly honeycomb design. Notice that the vertical walls have thickness twice as much as the others. This is due to the fact that, according to Vinson (1999), it is a common feature into commercial honeycombs. According to Masters and Evans (1996), in all re-entrant honeycombs the angle θ must be negative. Furthermore, a comparison between regular hexagonal and re-entrant honeycombs, it is only possible when the wall length (l) is half of the perpendicular wall length (h), in other words, for the re-entrant honeycomb h=2l. Notice that regular hexagonal honeycombs have equal wall length (h=l) and they are more frequently classified following their cell size (c). Therefore, the comparison only can be made by considering honeycombs with the same or at least close relative densities. Another important issue is the stress concentration factor. Most commercial honeycombs have some smoothing transition on the corners. In our case, this transition is made by employing two curvature radiuses. Following Chaves et all. (2003), the external radius (R_2) is made twice the internal radius (R_1) not only to create a smooth transition but also due to manufacturing limitations.

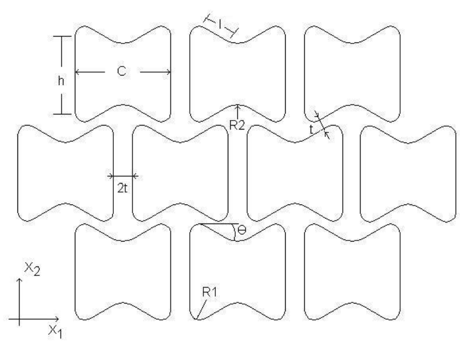

Figure 1: Butterfly honeycomb main geometrical parameters

The new honeycomb design allows an in-plane behavior between a quasi-isotropic condition and a complete dissimilar property on X_1 and X_2 directions. Moreover, the in-plane Poisson ratio will be negative. Notice that the out-of-plane Poisson ratio is still positive as the honeycomb design is essential two dimensional. The mathematical formulation to estimate the elastic moduli can be defined as:

Effective Young's modulus parallel to X_1:

$$E_1^* = \left(\frac{t}{\beta}\right)^3 E_s \frac{\cos\theta}{\sin^2\theta\left[\left(\frac{\alpha}{\beta}\right)+\sin\theta\right]} \tag{1}$$

where:

$$\alpha = h + \frac{4\pi R_1}{3} \qquad \beta = l + \frac{4\pi R_1}{3} \tag{1-b}$$

the E_s is the wall Young's modulus, the wall thickness is represented by t, and θ and R_1 are defined in figure 1.

Effective Young's modulus parallel to X_2:

$$E_2^* = \left(\frac{t}{\beta}\right)^3 E_s \frac{\left[\left(\frac{\alpha}{\beta}\right) + \sin\theta\right]}{\cos^3\theta} \tag{2}$$

From the theory of elasticity (Green and Zerna, 1968) the major Poisson ratio as the negative ratio between the normal stresses on X2 and X1 directions, respectively.

For honeycombs the major Poisson ratio definition leads to:

$$\nu_{12}^* = \frac{\cos^2\theta}{\left[\left(\frac{\alpha}{\beta}\right) + \sin\theta\right]\sin\theta} \tag{3}$$

Silva et all. (1995) pointed out that for honeycombs the reciprocity theorem must be valid. Mathematically this expression is given by:

$$E_1^* \nu_{21}^* = E_2^* \nu_{12}^* \tag{4}$$

This brings out the expression:

$$\nu_{21}^* = \frac{\left[\left(\frac{\alpha}{\beta}\right) + \sin\theta\right]\sin\theta}{\cos^2\theta} \tag{5}$$

The effective in-plane shear modulus is defined as:

$$G_{12}^* = E_s \left(\frac{t}{\beta}\right)^3 \frac{\left(\frac{\alpha}{\beta}\right) + \sin\theta}{\left(\frac{\alpha}{\beta}\right)^2 \cos\theta \left[1 + 16\left(\frac{\alpha}{\beta}\right)\right]} \tag{6}$$

For the out-of-plane mechanical properties, the following equations must hold. The effective out-of-plane Young's modulus is defined as:

$$E_3^* = \left(\frac{t}{\beta}\right) E_s \frac{1 + \left(\frac{\alpha}{\beta}\right)}{\left[\left(\frac{\alpha}{\beta}\right) + \sin\theta\right]\cos\theta} \tag{7}$$

The effective Poisson ratios are defined by Gibson & Ashby (1997) as:

$$\nu_{31}^* = \nu_{32}^* \approx \nu_s \tag{8}$$

and

$$\nu_{13}^* = \nu_{23}^* \approx 0 \tag{9}$$

The expressions for the effective out-of-plane shear are modifications of the ones proposed by Kelsey et all (1958). The effective shear modulus for the 1-3 plane is defined as follows:

$$G_{13}^{*} = G_s\left(\frac{t}{\beta}\right)\frac{\cos\theta}{\left[\left(\frac{\alpha}{\beta}\right)+\sin\theta\right]} \tag{10}$$

Although there is an expression derived by Grediac (1993), for the effective shear modulus for the 2-3 plane is better to use the upper and lower bounds proposed by Kelsey et all (1958) The reason is that Grediac's equation is an approximation of Kelsey's formulation. The modified Kelsey's lower bound derived by Avila et all. (2003) is given by:

$$\left(\frac{t}{\beta}\right)\frac{\left[\left(\frac{\alpha}{\beta}\right)+\sin\theta\right]}{\left[\left(\frac{\alpha}{\beta}\right)+1\right]\cos\theta} \leq \frac{G_{32}^{*}}{G_s} \tag{11}$$

The upper bound is given by:

$$\frac{G_{32}^{*}}{G_s} \leq \frac{\left[\left(\frac{\alpha}{\beta}\right)+\sin^2\theta\right]}{\left[\left(\frac{\alpha}{\beta}\right)+\sin\theta\right]\cos\theta}\left(\frac{t}{\beta}\right) \tag{12}$$

where G_s is the wall shear modulus.

The mathematical formulation presented is a guide to the sandwich composite engineering designers. The main objective is to provide the homogenized or effective stiffness, with these properties on hand the designer can choose the some important geometrical parameters, e.g. transverse area, as a function of bending loads, etc. However, there are a great number of possible configurations. To be able to understand how a specific configuration can affects the overall stiffness a morphological study must be performed.

3. Morphology changes versus stiffness variations

Lakes (1991) stated that for re-entrant honeycombs not only the negative angle θ must be considered but also the ratio between the lengths α and β. As it can be observed on equations 1 through 11 the elastic moduli are mainly dependent on a set of factors, i.e. the wall material properties, the θ angle, the wall thickness (t) and the α/β ratio. For each specific set of parameters there is a unique re-entrant honeycomb. The morphology changes ranges from a near rectangular shape to a close to a butterfly configuration. To give an idea how changes on θ and α/β parameters affects the honeycomb morphology Tab. 1 was created.

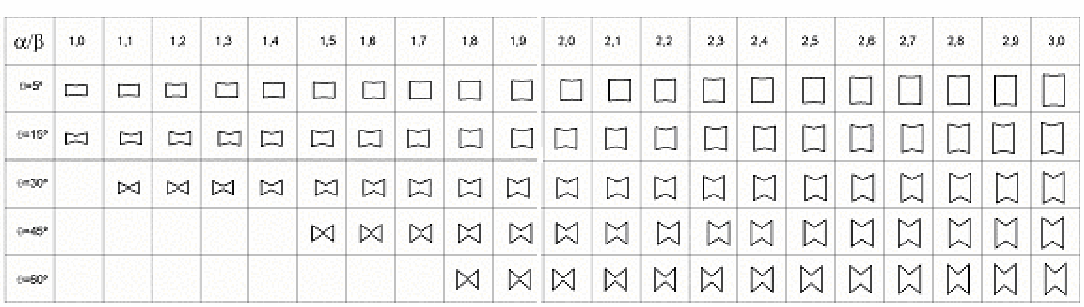

Table 1: Morphological changes on honeycombs

The empty spaces on Table 1 mean that there is not possible to make honeycombs with these angles and correspondent ratio α/β. There is no doubt that changes on these parameters will affect the honeycomb morphology, but how does it affect the elastic moduli? To answer this question a set of numerical simulations were performed. The t/β ratio are kept constant and equals to 0.2. By doing the thin wall condition can be assumed. Even though the honeycomb is under three effects acting at same time, i.e. bending, stretching and hinging, the major contribution is given by the bending effect due to the thin wall condition assumed [8]. Therefore, for this case only bending will be considered. Moreover, the α/β ratio will vary from 1 to 3, and the butterfly honeycomb walls will be made of a quasi-isotropic cross-ply fiber glass/epoxy laminate. The mechanical properties are listed on Table 2.

Material	E (GPa)	G (GPa)	ν
Fiber glass/epoxy	36.0	9.0	0.25

Table 2: fiber glass/epoxy mechanical properties

Figure 2 gives us a hint of how is the Young's modulus (E_1) behavior as a function of the θ angle and the α/β ratio. For a constant θ, the maximum reduction on stiffness as a function of changes on α/β ratio is around 76%, while for the reverse the situation, a fixed α/β and changing on θ, the maximum stiffness reduction is close to two orders of magnitude. For small angles the honeycomb morphology is close to a rectangular shape, which leads a stiffer microstructure, at least for the X_1 direction loadings. Once the θ angle gets negatively larger the micro structural stiffness is reduced due to the rotations on the walls that were parallel or near parallel to the X_1 direction. This phenomenon is potentially worst with increase of α/β ratio. A large difference between α and β lengths makes easy to bend these rotated walls, as they are connected in a geometric configuration, near to butterfly shape. It is possible to conclude that the Young's modulus E_1 is inversely proportional to the α/β ratio. Moreover, as the θ angle gets more negative, there is also a decrease on stiffness (E_1). Besides, the leading parameter for the E_1 modulus seems to be the θ angle. The E_2 modulus behavior is shown in figure 3. Notice that for E_2 the two in-plane Poisson ratios must also be considered. That is why the E_2 maximum value is amount on order of magnitude smaller than E_1.

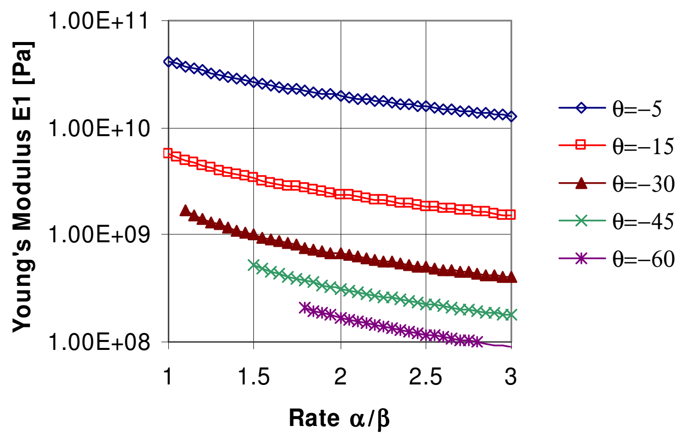

Figure 2: Young's modulus (E_1) variation

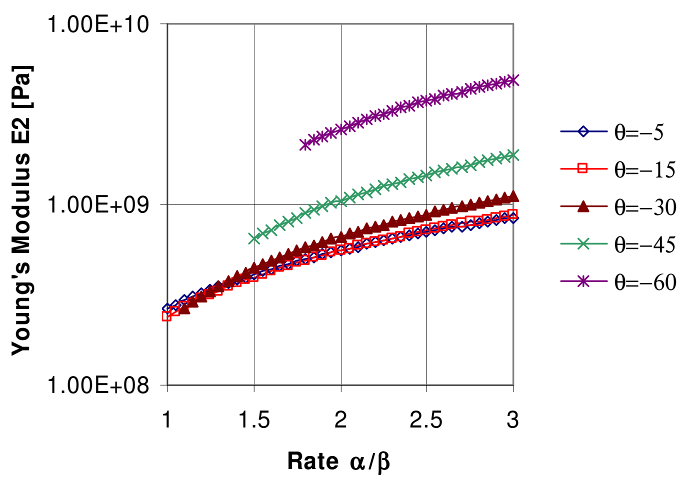

Figure 3: Young's modulus (E_2) variation

When dealing with in-plane Poisson ratio some considerations must be made. For the major Poisson ratio (ν_{12}) as the α/β ratio increases, a near asymptotic behavior is observed, as it can be noticed in figure 4. It is interesting to recognize that smaller negative angles lead to even more negative ν_{12}. An opposite behavior is perceived for the minor Poisson ratio (ν_{21}), as seem in figure 5. The minor Poisson ratio behavior is due to the reciprocity theorem from theory of elasticity. Large variations on ν_{12}, close to 69%, are observed special for small angles when the α/β ratio increases. The reason is that for these configurations the E_1 elastic modulus is high, which leads a small normal deformation on X_1 direction. Notice that stresses are assumed constant. As the major Poisson ratio is defined as the ratio between the normal strain at X_2 direction (ε_2) and the normal strain at X_1 direction (ε_1), a small ε_1 will naturally guides to a large ν_{12}. As the α/β ratio increases the E_1 elastic modulus decreases at same time that E_2 increases, but in different rates. This behavior results to near asymptotic behavior for ν_{12}. For practical applications, however, an equilibrated Poisson ratio, i.e. ν_{12} and ν_{21} close to each other, seems to be more appropriate, as the normal strains on perpendicular directions will be roughly the same. This particular case can be observed when α/β is around two and the θ angle is close to -30 degrees, in other words, considering this configuration the two in-plane Poisson ratios (ν_{12} and ν_{21}) will be roughly -1. The contraction and/or expansion observed can be attributed to the spring effect generated by the morphological changes on honeycomb microstructure.

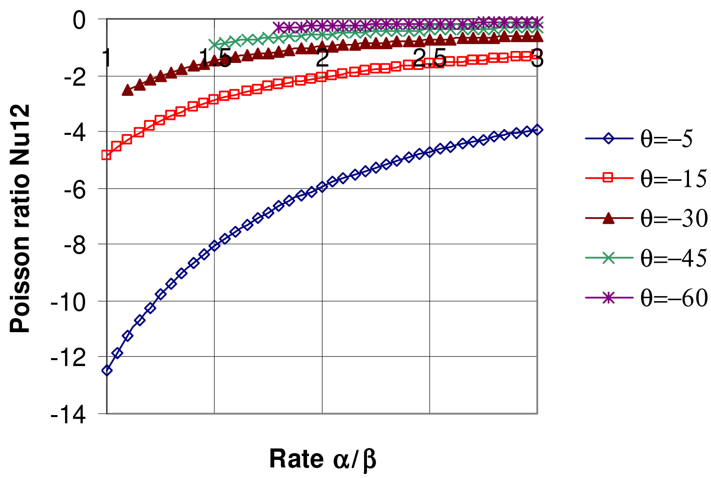

Figure 4: Poisson ratio (ν_{12}) variation

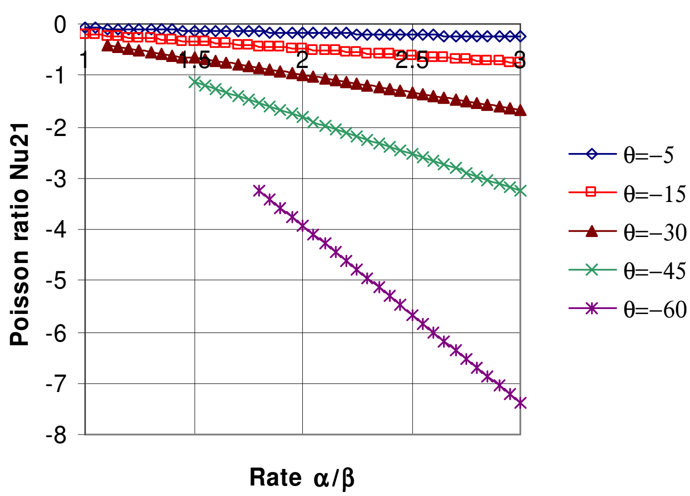

Figure 5: Poisson ratio (ν_{21}) variation

The in-plane shear modulus G_{12} variation is shown in figure 6. It seems that for this case, the angle variation has little effect on G_{12} variation. In fact, the largest variation on G_{12}, keeping the α/β ratio constant, is around 33%. Meanwhile, the in-plane shear modulus variations, considering a constant θ angle, are close to one order of magnitude. This large variation can be due to changes on the transverse section area. A possible conclusion for this behavior is that the most important parameter for G_{12} is the α/β ratio. However, this analysis will not be completed without the out-of-plane analysis. According to Vinson (1999), the out-of-plane mechanical properties, i.e. Young's modulus (E_3), shear moduli (G_{13} and G_{32}) and Poisson ratios, are crucial for sandwich structures design. As it can be observed at figure 7, the out-of-plane Young's modulus (E_3) seems to be strongly affected not only by the θ angle changes, but also by variations on the α/β ratio. When α/β is kept constant and θ varies from -60 to -5 degrees, the maximum stiffness reduction observed is close to 73%. On the other hand, drastic changes on α/β ratio for large angles can lead to a non feasible honeycomb design. For small angles, close to -5 degrees, changes on α/β lead to near constant values, as the angles increase those variations are more significant. This is due to the sine on the equation (6), i.e. for small angles the sine approaches to zero and denominator on equation (6) is close to a constant value.

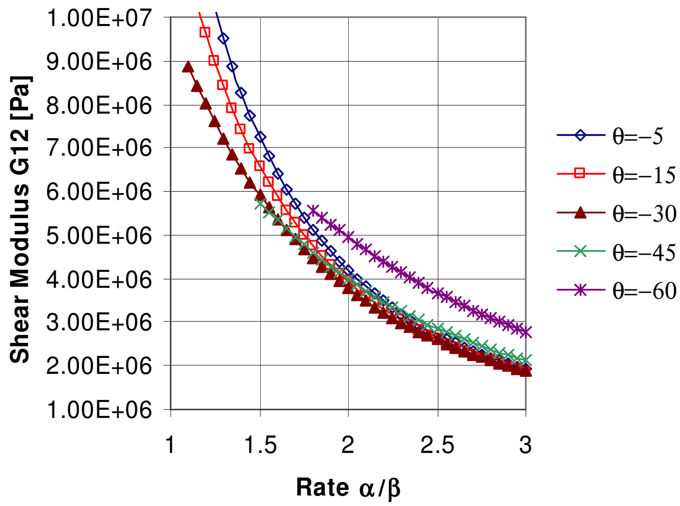

Figure 6: In-plane shear modulus (G_{12}) variation

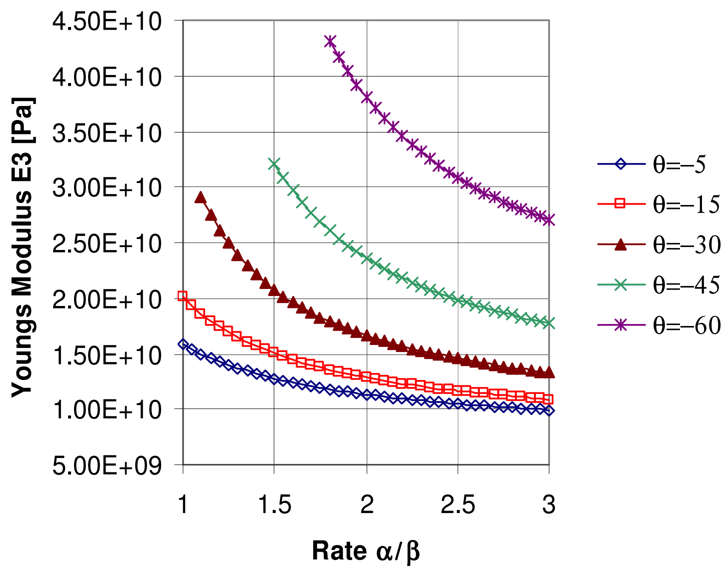

Figure 7: Young's modulus (E_3) variation

The G_{13} variation is shown in figure 8. Considering a constant α/β ratio, changes on θ angle will result in a maximum stiffness alteration close to 34%. Meanwhile, when the α/β ratio changes from 1 to 3 the stiffness decreases to one quarter of its original value, i.e. a 75 % reduction. It seems that for the out-of-plane modulus (G_{13}) changes on θ angles play a secondary role on stiffness definition. The last elastic modulus to be analyzed is the out-of-plane shear modulus. The best way to study the G_{23} modulus is consider its effective value, i.e. mean value between the upper and lower bounds. In figure 9 it can be observed that variations on α/β ratio can lead up to a 25% stiffness changes. However, when the θ angle varies from -60 to -5 degrees, for a constant α/β ratio, there is a decrease on stiffness around 73%. Once the θ angle increases the projected transverse area also increases leading to a higher G_{23} as the applied stresses are constant. It seems that for G_{23} the dominant parameter is the θ angle.

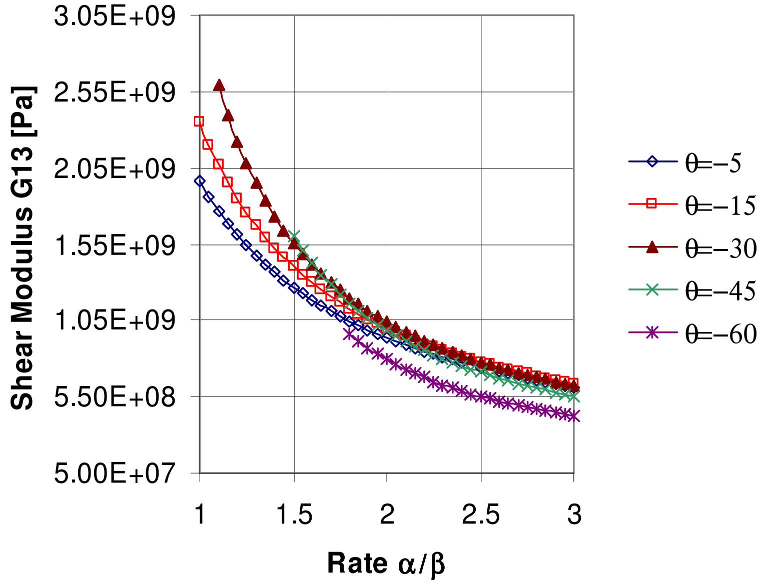

Figure 8: Shear modulus (G_{13}) variation

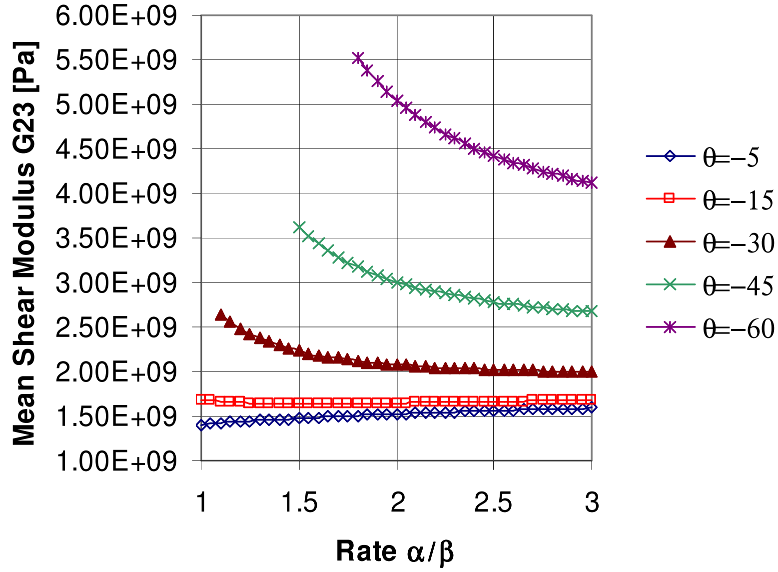

Figure 9: Shear modulus (G_{23}) mean values variation

The Poisson's ratio out-of-plane are not shown because they have a near constant behavior for the first ones (ν_{31} and ν_{32}) the value is close to the wall's Poisson ratio, i.e. ≈ 0.30, and the last ones (ν_{13} and ν_{32}) are close to zero. When all elastic moduli are studied simultaneously some comments could be made. For in-plane and out-plane conditions the best solution is a ratio α/β close to 1. Nevertheless, the in-plane Poisson effect for the α/β ratio close to 1 is complete unlike behavior. In other words, the in-plane contraction or traction in two perpendicular directions will be very different in magnitude. For single curvature applications this is not a problem, but for structures with double curvature this fact could causes much trouble. Milton (1992) suggests a honeycomb with in-plane Poisson's ratio close to -1 to solve the double curvature problem. A prototype of the new butterfly honeycomb design and manufactured. Its cell size is around 10 mm (≈3/8"), and the t/β and α/β ratios are 0.2 and 2.0, respectively. The walls are made of fiber glass/ epoxy cross-ply laminate. The prototype elastic moduli are shown in Table 3.

E_1[MPa]	E_2[MPa]	E_3[MPa]	G_{12}[MPa]	G_{13}[MPa]	G_{23}[MPa]	ν_{12}	ν_{21}	$\nu_{31}=\nu_{32}$	$\nu_{13}=\nu_{23}$
665.108	665.108	16.627	3.779	1.039	2.079	-1.000	-1.000	0.250	≅0.000

Table 3: Butterfly honeycomb effective moduli

According to Kim and Christensen (2000) the relative density is the parameter that allows comparisons among honeycombs. Lets consider a regular hexagonal honeycomb, where θ is equal to 30 degrees, t/β is made equal to 0.2 and the α/β ratio is identical to 1, and the prototype shown in figure 10. These honeycombs have similar relative densities. Moreover, the elastic constants are the same with the exception of in-plane Poisson ratios that are equal to +1 for the hexagonal honeycomb and -1 for the butterfly honeycomb. The -1 in-plane Poisson ratios are the key factors that allow the strains at perpendicular directions have the same sign and equivalent magnitudes, enabling the double curvature construction.

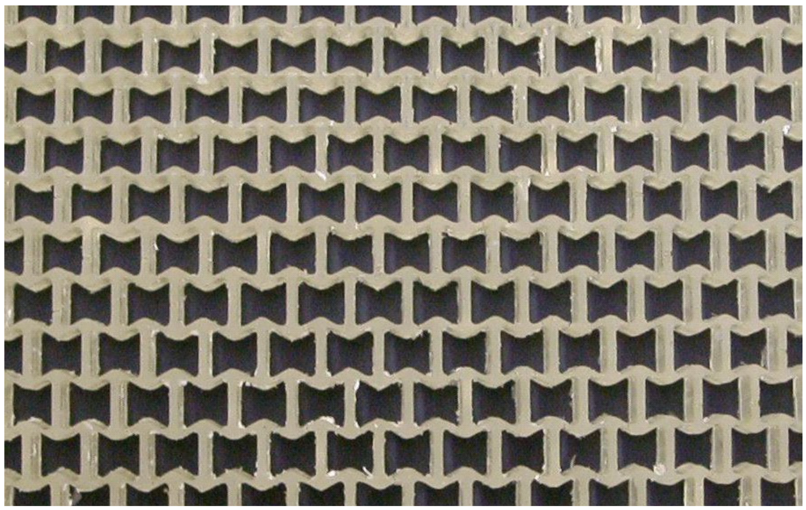

Figure 10: New butterfly honeycomb prototype picture

4. Closing Remarks

For re-entrant honeycombs the morphology and stiffness are related. Four different parameters, three geometrical and one material, can be identified for these honeycombs, i.e. the θ angle, the α/β and t/β ratios, and the mechanical properties of the cells walls. Variations on these parameters have direct effect not only on effective moduli but also on honeycomb morphology itself. The t/β defines if the honeycomb has thin or thick walls. In the first case the bending effect is predominant, while for second case besides bending, stretching and hinging must be considered as acting at same time.

Variations on θ angle, and the α/β ratio do affect the in-plane Young's moduli (E_1 and E_2). As α/β ratio increases there is a decrease on stiffness. Moreover, E_1 behavior is inversely proportional to the increase of the negative angle. The E_2 modulus presents an opposite behavior. The major in-plane Poisson ratio (ν_{12}) seems to have an asymptotic behavior as the α/β ratio gets larger. Furthermore, smaller angles lead to a more negative major Poisson ratio. Due to the reciprocity theorem from the theory of elasticity the minor Poisson ratio (ν_{21}) has the opposite behavior of its analogous (ν_{12}). The in-plane shear modulus (G_{12}) has a near independent behavior of θ angle variation. The major variations seems to be due to the α/β changes, as they are more related to the cell walls transverse area.

The out-of-plain Young's modulus (E_3) seems to be more affected by the θ angle variations than by variations on α/β ratio than angle changes. For the out-of-plane shear modulus, two different behaviors are observed. G_{13} is independent of the θ angle variations, while θ variations can lead to significant variations on G_{23}. The main reason for this behavior is the transverse area on the 1-3 and 2-3 planes. It is important to mention that an optimum stiffness does not mean best performance. In this case, α/β close to 1 will guide to a near maximum stiffness. However, the Poisson effect is unbalanced, which leads to complete different normal strains in the two in-plane perpendicular directions. This configuration is mostly like to direct problems for double curvature applications. The new honeycomb design will probably fit well to double curvature engineering applications as it has the two in-plain Poisson's ratios close to -1 besides good out-of-plane mechanical properties. A series of lab tests on this new honeycomb design is under development.

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