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EDDY CURRENTS TORQUE MODEL FOR SPIN STABILIZED EARTH SPACECRAFT

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Abstract. *An analytical approach for the eddy currents torque acting on the spin-stabilized satellites is presented. It is assumed an inclined dipole model for the Earth´s magnetic field and the method of averaging the eddy currents torque over each orbital period is applied to obtain the components of the torque in the satellite body frame reference system. The developments are presented in terms of the mean anomaly and contain terms of second order in eccentricity. It is observed that the eddy currents torque causes an exponential decay of the angular velocity magnitude. Numerical implementations performed with data of the SCD1 and SCD2 Brazilian satellites show the agreement between the developed analytical solution and the actual satellite behavior.*

Keywords*. Attitude of the artificial satellite, spin-stabilized spacecraft, eddy currents torque, angular velocity magnitude, exponential decay.*

1. Introduction

This work addresses the rotational motion dynamics of spin stabilized Earth satellites (which has the spin axis along the geometric satellite axis), through an analytical approach for attitude prediction. The emphasis is placed on modeling of the eddy currents torque associated with these satellites. A spherical coordinates system fixed in the satellite body is used to located the spin axis of the satellite in relation to the terrestrial equatorial system. The direction of the spin axis is specified by the right ascension (α) and the declination (δ) which are represented in the Fig. 1.

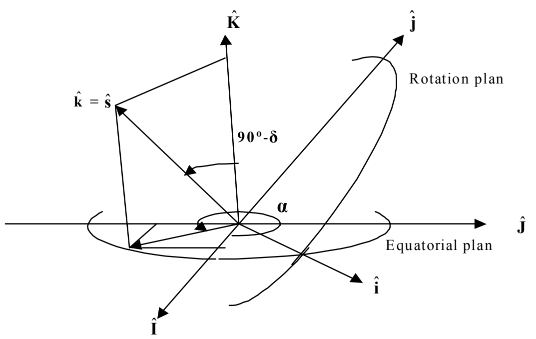

Figure 1. Spin axis orientation ($\hat{s}$): Equatorial System ($\hat{I}, \hat{J}, \hat{K}$), System of the Satellite ($\hat{i}, \hat{j}, \hat{k}$), right ascension (α) and declination (δ) of the spin axis.

The eddy currents torque appears due to the interactions of such currents circulating along the satellite structure chassis and the Earth field. In this paper the torque analysis is performed through the modeling of the inclined Earth magnetic dipole, which orientation depends on the magnetic colatitude and on the ascending node of the magnetic plane. Essentially an analytical averaging method is applied to determine the torque over an orbital period. To compute the average components of eddy currents torque in the satellite body frame reference system (satellite system), an average in time in terms of the mean anomaly is utilized, which involves rotation matrices dependent on the orbit elements, the magnetic colatitude, the ascending node of the magnetic plane and the right ascension and declination of the satellite axis.

The inclusion of this torque on the rotational motion differential equations of spin stabilized satellites yields the conditions to derive an analytical solution. The solution shows clearly that the eddy currents torque causes an exponential decay of the angular velocity magnitude along the time. Numerical implementations performed with data of the SCD1 and SCD2 Brazilian satellites show the agreement between the analytical solution for angular velocity magnitude and the actual satellite behavior.

2. Geomagnetic field

An inclined Earth magnetic dipole model is assumed in this paper. Its orientation depends on the magnetic colatitude (β) and on the ascending node of the magnetic plane (η). The magnetic reference system, which axis z_m is along the dipole vector, β and η are represented in the Fig. 2.

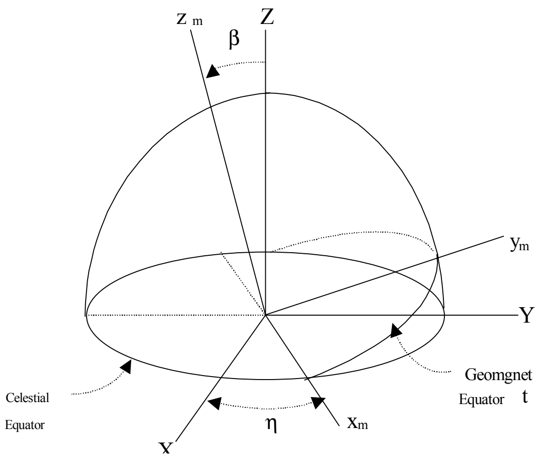

Figure 2 - Magnetic system ($O'x_my_mz_m$) and Equatorial l ($O'XYZ$)

It is well known that the Earth magnetic dipole model (Thomas and Capellari, 1964; Wertz,1978) may be expressed by:

$$\vec{B} = \frac{\ell}{4\pi\mu_o r^3}\left[\hat{k}_m - 3\left(\hat{i}_s \cdot \hat{k}_m\right)\hat{i}_s\right] \tag{1}$$

where ℓ is the magnetic moment of Earth's field magnitude, μ_o the permeability of free space, r the radius vector magnitude of the satellite, $\hat{k}_m$ the unit vector along the dipole vector and $\hat{i}_s$ the unit vector along the radius vector of the satellite ($\vec{r}$). The units vector $\hat{k}_m$ and $\hat{i}_s$ can be expressed in the satellite system through rotation matrices dependent on the orbit elements, right ascension and declination of the satellite spin axis and the angles β and η.

3. Eddy currents torque

The torques induced by eddy currents are caused by the spacecraft spinning motion. It is known (Wertz, 1978) that the eddy currents produce a torque which precesses the spin axis and causes an exponential decay of the spin rate. If $\vec{W}$ is the spacecraft's angular velocity vector and p is a constant coefficient which depends on the spacecraft geometry and conductivity, this torque may be modeled by:

$$\vec{N}_i = p \ \vec{B} \ x \ (\vec{B} \ x \ \vec{W}) \tag{2}$$

Here the magnetic torque is developed for spin stabilized satellites. In this case, the spacecraft's angular velocity vector and the satellite magnetic moment are along the z-axis, and induced eddy currents torque can the expressed by (Kuga et al., 1987):

$$\vec{N}_i = p\,W \ \ \vec{B} \ x \ (\vec{B} \ x \ \hat{k}) \tag{3}$$

where B_x, B_y, B_z are the components of the geomagnetic field in the satellite fixed system. These components are obtained in terms of the geocentric inertial components of the geomagnetic field and the right ascension and declination of the satellite.

4. Mean eddy currents torque

In order to obtain the mean eddy currents torque, it is necessary to integrate the instantaneous torque $\vec{N}_i$, given for (3), over one orbital period (T):

$$\vec{N}_{i_m} = \frac{1}{T} \int_{t_i}^{t_i + T} \vec{N}_i \, dt \tag{4}$$

where: t is the time, t_i the initial time and T the orbital period. Changing the independent variable to the fast varying true anomaly, the mean eddy currents torque can be obtained by (Quirelli, 2002):

$$\vec{N}_{i_m} = \frac{1}{2\pi} \int_{\upsilon_i}^{\upsilon_i + 2\pi} \vec{N}_i \frac{r^2}{h} \, d\upsilon \tag{5}$$

where υ_i is the true anomaly at instant t_i and h is the specific angular moment. Since the instantaneous torque is given by (3) and

$$r = \frac{a\left(1 - e^2\right)}{1 + e \, Cos \, \upsilon}, \qquad h = \frac{2\pi a^2 \left(1 - e^2\right)^{1/2}}{T} \tag{6}$$

where a is the semi-major axis and e the eccentricity of orbit, the mean eddy currents torque (5) becomes:

$$\begin{aligned} \vec{N}_{i_m} = \Re_i \int_{\upsilon_i}^{\upsilon_i + 2\pi} W \left[\hat{k}_m - 3\left(\hat{i}_s \cdot \hat{k}_m\right)\hat{i}_s\right] \times \\ \left\{\left[\hat{k}_m - 3\left(\hat{i}_s \cdot \hat{k}_m\right)\hat{i}_s\right] \times \hat{k}\right\} \left(1 + e \, Cos \, \upsilon\right)^4 \, d\upsilon \end{aligned} \tag{7}$$

with

$$\Re_i = \frac{p \, \ell^2}{32 \pi^3 \mu_o^2 a^6 \left(1 + e^2\right)^{9/2}} \tag{8}$$

To evaluate the integral of (7) we will use the elliptic expansions of the true anomaly in terms of the mean anomaly M (Brouwer & Clemence, 1961), including terms up to first order in the eccentricity (e). Then the present development can be applicable for elliptical orbits without loss of precision. For simplification of the integrals we will consider the initial time for integration equal to the instant that the satellite passes through the perigee. The components of the unit vector $\hat{k}_m$ in the satellite system depend on the magnetic colatitude (β) and ascending node of the magnetic plane (η) and right ascension (α) and declination (δ) of the spin axis (Quirelli, 2002; Thomas & Capellari, 1964). In this paper, we will consider (Thomas and Capellari ,1964):

$$\eta = \eta_o + bM \qquad \text{and} \qquad b = \frac{\omega_e \, T}{2\pi} \tag{9}$$

where η_o is the initial position of the ascending node of the geomagnetic equator at the instant the satellite is at the perigee and ω_e is the angular velocicity of the Earth.

The components of the unit vector $\hat{i}_S$ in the satellite system depend on ascending node orbit (Ω), orbital inclination (i), the true anomaly (ν) and right ascension (α) and declination (δ) of the spin axis (Quirelli, 2002). For one orbital period the angles Ω , i, ν, α, δ and β are constant.

Thus, using trigonometry properties and after exhausting but simple algebraic developments, the mean eddy currents torque can be expressed by (Quirelli, 2002):

$$\vec{N}_{im} = N_{ix} \, \hat{i} + N_{iy} \, \hat{j} + N_{iz} \, \hat{k} \tag{10}$$

with N_{ix}, N_{iy} and N_{iz} terms presented in the Appendix.

5. Applications

The variations of the angular velocity, the declination and the ascension right of the spin axis are given by the Euler equations in spherical coordinates (Kuga et al, 1987):

$$\dot{W} = \frac{1}{I_Z} N_Z \qquad \dot{\delta} = \frac{1}{I_Z\ W} N_y \qquad \dot{\alpha} = \frac{1}{I_Z\ W\ \mathrm{Cos}\,\delta} N_x \tag{11}$$

where I_Z is the moment of inertia along the spin axis, N_x, N_y and N_Z are components of external torques in the satellite body frame reference system (satellite system).

Then it is possible to observe that the eddy currents torque affects the angular velocity magnitude and the spin axis. In this paper it will be analyzed the angular velocity magnitude. By substituting N_{iz} given in (A.3), the variation of the angular velocity magnitude can the expressed as:

$$\frac{d\,W}{d\,t} = k\ d\,t \tag{12}$$

with

$$k = \frac{N_{iz}\ \Re_i}{I_Z} \tag{13}$$

If the parameter k is considered constant for one orbital period, the analytical solution of eq. (12) is:

$$W = W_0\ e^{k\,t} \tag{14}$$

where W_0 is the initial angular velocity. Then when $k < 0$ the angular velocity magnitude decays exponentially.

The results obtained by computer implementation of the developed theory, using data of SCD1 and SCD2 Brazilian satellites (given in table 1) are shown in the Fig. 3 and 4. The Fig. 5 and 6 present the deviation obtained during the period of the tests, compared with the INPE's control center archives. The behaviour of angular velocity magnitudes for the SCD1 and SCD2 satellites show the agreement between the proposed analytical solution and the actual satellite behavior.

Table 1 – Brazilian Satellites Data (SCD 1 and SCD 2).

	SCD 1 Data 24/07/1993	**SCD2 Data** 16/04/2002
a (meters)	7139615.83	7133679.1
E	0.00453	0.00174
i (°)	25.002	24.99
Ω (°)	260.429	353.304
ω (°)	260.32	346.341
I_z (Kg m^2)	13	14.5
W(rpm)	90.76	32.96
α_o (°)	233.94	256.36
δ_o (°)	77.43	58.29
β (°)	11.4	11.4
μ_o (Weber/A m)	$4\,\pi \,.\, 10^{-7}$	$4\,\pi \,.\, 10^{-7}$
ℓ (Weber m)	10^{17}	10^{17}
M_s (A m^2)	-0.63	0.11

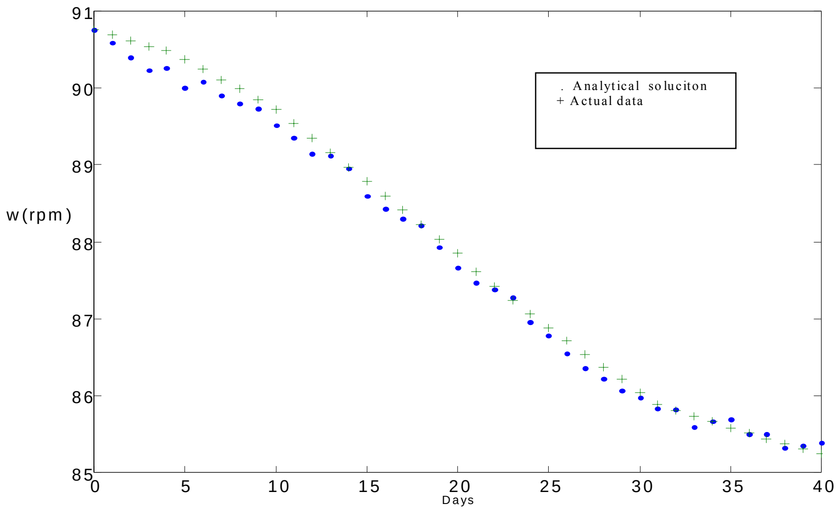

Figure 3 – Evolution of angular velocity of SCD1

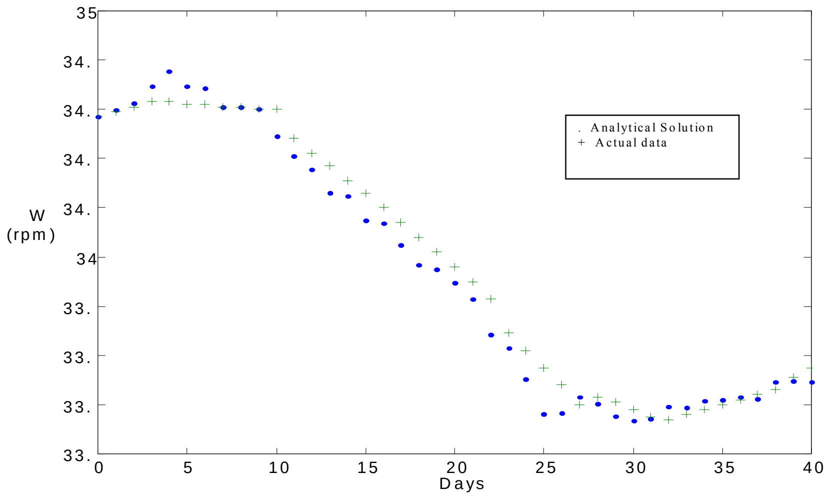

Figure 4 – Evolution of angular velocity for CSCD2

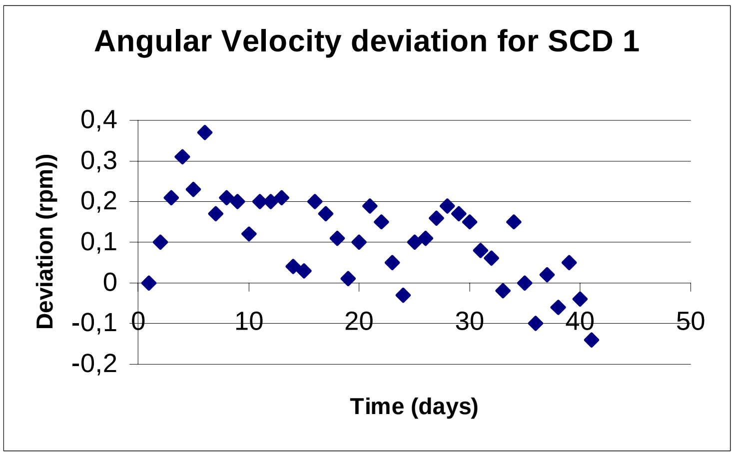

Figure 5 – Angular velocity deviation for SCD1.

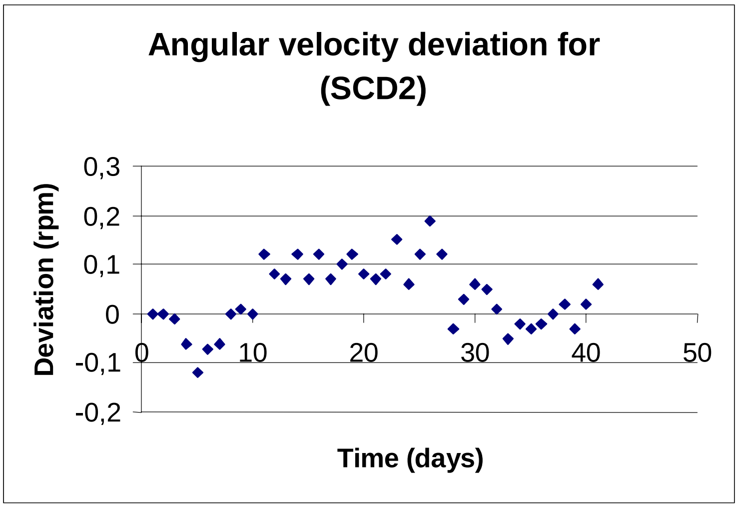

Figure 6 – Angular velocity deviation for SCD2.

6. Summary

The eddy currents torque model was discussed in this paper, considering applications to spin stabilized satellites. The mean components of this torque in the satellite body frame reference system were obtained through the development of an analytical approach. Analysis of the analytical solution shows clearly that the eddy currents torque causes an exponential decay of the angular velocity magnitude along the time. The theory developed accounts also for orbit element time variations, not restricted to circular orbits. Such approach gave rise to some hundreds of trigonometric integrals which were solved analytically.

The theory was coded in a PC micro-computer. Then the program was executed using the data of SCD1 and SCD2 Brazilian satellites. Results have shown the agreement between the analytical solution for the angular velocity and actual satellite behavior. For the period of the tests, the mean difference was in the order of 0.10rpm for SCD1 and 0.04rpm for SCD2, well within the accuracy level in which the attitude determination system of INPE’s control center estimates the angular velocity magnitude.

7. References

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Appendix

$$N_{ix} = \Re_i \; W \{ \{ \{ s\beta \{ - A_{1x} \, s\alpha - B_{1x} \, c\alpha - 3(C_{1xa} - C_{1xd})$$

$$\{ (-s(\alpha - \Omega)(c\alpha \, c^2\delta \, c(\alpha-\Omega) + c\alpha c(\alpha+\Omega) s^2\delta + s\alpha \, s(\alpha - \Omega))) \} -$$

$$3(C_{1xb} - C_{1xe})\{ -c^2 i \, c(\alpha-\Omega) s\Omega \} - 3(C_{1xc} - C_{1xf})\{ (c \, i \, c(\alpha-\Omega)(c\alpha \, c^2\delta \, c(\alpha-\Omega) + c\alpha$$

$$c(\alpha+\Omega) s^2\delta + s\alpha \, s(\alpha - \Omega))) \} - 3(C_{1xc} + C_{1xf}) \{ c i s(\alpha-\Omega) s\Omega \} + 3(D_{1xa} + D_{1xd})$$

$$\{(-\left(2+2\,c2\alpha-c2(\alpha-\delta)+2c2\delta-c2(\alpha+\delta)\right)s(\alpha-\Omega)s\Omega)/4\}+3(D_{1xd}+D_{1xe})\{c^2ic(\alpha-\Omega)$$

$$c\Omega\}+3(D_{1xc}+D_{1xf})\left\{-cic\Omega s(\alpha-\Omega)\right\}+3(D_{1xc}+D_{1xf})\left\{\left(\left(2+2c2\alpha-c2(\alpha-\delta)+2\,c2\delta-\right.\right.\right.$$

$$\left.\left.\left.c\,2(\alpha+\delta)\right)cic(\alpha-\Omega)s\Omega\right)/4\right\}\}\}+\left\{c\beta\left\{-3(F_{1xa}-F_{1xd})\left\{-s\alpha\;s2\delta s(\alpha-\Omega)\,s\Omega\right\}-\right.\right.$$

$$-3\;(F_{1xb}-F_{1xe})\left\{\frac{1}{2}c(\alpha-\Omega)\,s\,2i\right\}-3\left(F_{1xc}-F_{1xf}\right)\left\{-si\;s(\alpha-\Omega)\right\}\}\}+$$

$$+\left\{s^2\beta\left\{-N_{IVa1}\,s\alpha-N_{IVa2}\,s\alpha+[3W[-B_{1a}J-B_{1b}O]+9[C_{1a}Y+(C_{1c}+C_{1d})Y_2-D_{1a}Y_5+D_{1b}Y_6+\right.\right.$$

$$(D_{1c}+D_{1d})Y_7]]\{(-s(\alpha-\Omega)(c\alpha\,c^2\delta c(\alpha-\Omega)+c\alpha\,c\,(\alpha+\Omega)\,s^2\,\delta+s\alpha\,s(\alpha-\Omega)\,)\,)\,)\}+$$

$$+\left[3(-B_{1a}L+B_{1b}N)+9\left(C_{1b}Y_1+C_{1a}Y_1+C_{1b}Y_3-D_{1a}Y_6-\left(D_{1b}Y_8+(D_{1c}+D_{1d})Y_9\right)\right)\right]\left\{-c^2ic(\alpha-\Omega)s\Omega\right\}+$$

$$+\left[3(-B_{1a}N+B_{1b}Q)+9\left((C_{1c}+C_{1d})Y_1+C_{1a}Y_2+C_{1b}Y_4\,(C_{1c}+C_{1d})Y_1-D_{1a}Y_7-D_{1b}Y_9-(D_{1c}+D_{1d})Y_6\right)\right]$$

$$\left\{ci\,c(\alpha-\Omega)\left(c\alpha c^2\delta c(\alpha-\Omega)+c\alpha c(\alpha+\Omega)\,s^2\delta+s\alpha\,s\,(\alpha-\Omega)\right)\right\}+\left[3(-B_{1a}\;N-B_{1b}\;Q)+\right.$$

$$\left.+9\left(C_{1a}\;Y_2+C_{1b}\;Y_4+(C_{1c}+C_{1d})\,Y_1-D_{1a}\,Y_7-D_{1b}\,Y_9-\left(D_{1c}+D_{1d}\right)Y_6\right)\right]\left\{ci\,s(\alpha-\Omega)\,s\,\Omega\right\}+$$

$$+\left[3\,(B_{1a}\;O-B_{1b}\;S)+9\left(-C_{1a}\;Y_5-C_{1b}\;Y_6-(C_{1c}+C_{1d})\,Y_7+D_{1a}\,Y_{10}+D_{1b}\;Y_{11}+\right.\right.$$

$$\left.\left.+(D_{1c}+D_{1d})\,Y_{12}\right)\right]\left\{\left\{\left((2+2\,c\alpha-c\,2(\alpha-\delta)+2\,c\,2\delta-c2(\alpha+\Omega))s(\alpha-\Omega)\,s\,\Omega\right)\right\}/4\right\}+$$

$$+\left[3(B_{1a}\,N-B_{1b}T)+\;9\left(-C_{1a}Y_6-C_{1b}Y_8-(C_{1c}+C_{1d})Y_9+D_{1a}Y_{11}+D_{1b}\;Y_{13}++(D_{1c}+D_{1d})Y_{14}\right)\right]$$

$$\left\{c^2ic(\alpha-\Omega)c\,\Omega\right\}+\left[3(B_{1a}Q-B_{1a}\;U)+9\left(-C_{1a}\;Y_7-C_{1b}\,Y_9-(C_{1c}+C_{1d})\,Y_6+\right.\right.$$

$$\left.\left.+(D_{1c}+D_{1b})Y_{11}\right)\right]\left\{-ci\,c\,\Omega\,s(\alpha-\Omega)\right\}+\left[3\,(B_{1a}\;Q-B_{1b}\;U)+9(-C_{1a}\,Y_7-C_{1b}\;Y_9-(C_{1c}+C_{1d})Y_6+\right.$$

$$\left.+(D_{1c}+D_{1b})Y_{11}\right)\Big]\left\{\left[\left(\left(2+2c2\alpha-c2(\alpha-\delta)+2c2\delta-c2(\alpha+\delta)\right)c\;ic(\alpha-\Omega)s\Omega\right)/4\right]\right\}+$$

$$+\left[3(B_{1a}C_1-B_{1b}D_1)+9\left(C_{1a}Z_7+C_{1b}Z_8+(C_{1c}+C_{1d})Z_9-\;D_{1a}Z_{12}-D_{1b}Z_{13}-(D_{1c}+D_{1d})Z_{14}\right)\right]$$

$$\left\{-s\alpha s2\delta\;\;s(\alpha-\Omega)\,s\Omega\right\}+\left[3(B_{1a}\;C_2+B_{1b}\;D_2)+9\left[C_{1a}\;Z_8+C_{1b}\;Z_{10}+(C_{1c}+C_{1d})\,Z_{11}+\right.\right.$$

$$\left.\left.+D_{1a}\;Z_{13}+D_{1b}\;Z_{15}+\;+(D_{1c}+D_{1d})\,Z_{16}\right]\right]\left\{\tfrac{1}{2}\,c(\alpha-\Omega)\,s2i\right\}+\left[3\,(B_{1a}\;C_3+B_{1b}\;D_3)+\right.$$

$$\left.+9\left[C_{1a}\;Z_9+C_{1b}\;Z_{11}+\;+(C_{1c}+C_{1d})Z_8+D_{1a}Z_{14}-D_{1b}Z_{16}-(D_{1c}+D_{1d})Z_{13}\right]\right]$$

$$\left\{ci\,c(\alpha-\Omega)s\alpha s2\delta\,s\Omega\right\}+\left[3(B_{1a}C_3+B_{1b}D_3)+9\left[C_{1a}\,Z_9+C_{1b}\,Z_{11}+(C_{1c}+C_{1d})\,Z_8+\right.\right.$$

$$\left.\left.+\;D_{1a}\;Z_{14}+D_{1b}\,Z_{16}++(D_{1c}+D_{1d})Z_{13}\right]\right]\left\{-si\,s(\alpha-\Omega)\right\}\}\}\} \tag{A.1}$$

$$N_{iy}=\Re_i\;W\left\{\left\{\left\{s\,\beta\left\{-A_{1x}\;s\delta c\alpha+B_{1x}\;s\delta\,s\alpha-3(C_{1xa}-C_{1xd})\left\{-c\,(\alpha+\Omega)\,s\delta\right.\right.\right.\right.\right.\right.$$

$$\left(c\alpha\, c^2\delta\, c(\alpha-\Omega) + c\alpha\, c(\alpha+\Omega)s^2\delta + s\alpha\, s(\alpha-\Omega) \right) \} - 3(C_{1xb} - C_{1xe}) \{ -c i s\Omega$$

$$(-(c i c\Omega\, s\alpha\, s\delta) + c\delta s i + c\alpha c i s\delta s\Omega) \} - 3(C_{1xc} - C_{1xf}) \{ (c\alpha\, c^2\delta\, c(\alpha-\Omega) +$$

$$c\alpha\, c(\alpha+\Omega)s^2\delta + s\alpha s(\alpha-\Omega))(-(c i c\Omega s\alpha s\delta) + c\,\delta\, s\, i + c\,\alpha\, c i\, s\delta\, s\Omega) \} - 3(C_{1xc} + C_{1xf})$$

$$\{ c\, i\, c(\alpha+\Omega)s\delta s\Omega \} + 3(D_{1xa} + D_{1xd}) \{ c\, i\, c(\alpha+\Omega)\, s\,\delta\, s\,\Omega \} + 3(D_{1xb} + D_{1xe})$$

$$\mathrm{Cos}\,\alpha\,\mathrm{Cos}\,i\,\mathrm{Sin}\,\delta\,\mathrm{Sin}\,\Omega) \} + 3(D_{1xc} + D_{1xf}) \{ - c i\, c\Omega\, c(\alpha+\Omega)s\delta \} + 3(D_{1xc} + D_{1xf})$$

$$\{ [(2 + 2\, s2\alpha - c\,2(\alpha-\delta) + 2\, c\,2\,\delta - c2(\alpha+\delta)) s\Omega (-(c i c\Omega s\alpha s\delta) + c\delta\, s i +$$

$$+ c\alpha\, c i\, s\delta\, s\Omega)] / 4 \} \} \} + \{ c\beta \{ E_{1x}\, c\delta - 3(F_{1xa} - F_{1xd}) \{ c(\alpha+\Omega)s\alpha\, s\delta\, s2\delta\, s\Omega \} -$$

$$-3(F_{1xb} - F_{1xe}) \{ s i (-(c i\, c\Omega\, s\alpha\, s\delta) + c\delta\, s i + c\alpha\, c i\, s\delta\, s\Omega) \} - 3(F_{1xc} - F_{1xf}) \{ s\alpha\, s2\delta s\Omega$$

$$(-(c i c\Omega\, s\alpha\, s\delta) + c\delta\, s i + c\alpha\, c\, i\, s\delta\, s\Omega) \} - 3(F_{1xc} + F_{1xf}) \{ c(\alpha+\Omega)s\delta\, s\, i \} \} \} +$$

$$+ \{ s^2\beta \{ N_{IVa1}\, s\delta\, c\alpha - N_{IVa2}\, s\delta\, c\,\alpha + + [3[-B_{1a} J + B_{1b} O] + 9[C_{1a}Y + (C_{1c} + C_{1d})Y_2 -$$

$$D_{1a}Y_5 + D_{1b}Y_6 + (D_{1c} + D_{1d})Y_7]] \{ (-c(\alpha+\Omega)s\delta(c\alpha c^2\delta\, c(\alpha-\Omega) + c\alpha\, c(\alpha+\Omega)s^2\delta +$$

$$\mathrm{Sin}\,\alpha\,\mathrm{Sin}\,(\alpha-\Omega))) \} + [3(-B_{1a}\, L + B_{1b}\, N) + + 9[C_{1b}Y_1 + C_{1a}Y_1 + C_{1b}Y_3 - D_{1a}Y_6 - D_{1b}Y_8 +$$

$$(D_{1c} + D_{1d})Y_9]] \{ (-c i\, s\Omega(-(c i\, c\Omega\, s\alpha\, s\delta) + + c\delta\, s i + c\alpha\, c i\, s\delta\, s\Omega)) \}$$

$$+ [3[-B_{1a}N + B_{1b}Q] + 9[(C_{1c} + C_{1d})Y_1 + C_{1a}Y_2 + C_{1b}Y_4 + (C_{1c} + C_{1d})Y_1 - - D_{1a}\, Y_7 - D_{1b}Y_9 -$$

$$-(D_{1c} + D_{1d})Y_6]] \{ (c\alpha\, c^2\delta\ \ c(\alpha-\Omega) + c\alpha c(\alpha+\Omega)s^2\delta + s\alpha\, s(\alpha+\Omega)s^2\delta + + s\alpha\, s(\alpha-\Omega))$$

$$(-(c i\, c\Omega\, s\alpha s\delta) + c\delta\, s i + c\alpha\, c i\, s\delta\, s\Omega) \} + [3[-B_{1a}N + B_{1b}Q] + + 9[C_{1a}Y_2 + C_{1b}Y_4 +$$

$$+ (C_{1c} + C_{1d})Y_1 - D_{1b}Y_7 - D_{1b}\, Y_9 - (D_{1c} + D_{1d})Y_6]] \{ (c i\, c(\alpha+\Omega)s\delta\, s\Omega) \} +$$

$$+ [3[B_{1a}O - B_{1b}\, S] + 9[-C_{1a}Y_5 - C_{1b}Y_6 - (C_{1c} + C_{1d})Y_7 + 9[C_{1a}Y_2 + C_{1b}Y_4 + (C_{1c} + C_{1d})Y_1$$

$$\{ (-(2 + 2\, c2\alpha - c2(\alpha-\delta) + 2\, c2\delta - c2(\alpha+\delta)) + 9[C_{1a}Y_2 + C_{1b}Y_4 + (C_{1c} + C_{1d})Y_1 +$$

$$+ [3(B_{1a}\, N - B_{1b}\, T) + + 9[-C_{1a}Y_6 - C_{1b}Y_8 - (C_{1c} + C_{1d})Y_9 + D_{1a}Y_{11} + D_{1b}\, Y_{13} + (D_{1c} + D_{1d})Y_{14}]]$$

$$\{ c\, i\, c\Omega(-(c\, i\, c\Omega\, s\alpha\, s\delta) + c\delta s i + c\alpha\, c i s\delta s\Omega) \} + [3[B_{1a}\, Q - B_{1b}\, U] + 9[-C_{1a}\, Y_7 - C_{1b}\, Y_9 -$$

$$-(C_{1c} + C_{1d})Y_6 + + (D_{1c} + D_{1d})Y_{11}]] \{ -c i\, c\Omega\, c(\alpha+\Omega)s\delta \} + [3[B_{1a}Q - B_{1b}U] +$$

$$9[-C_{1a}Y_7 - C_{1b}Y_9 - (C_{1c} + C_{1d})Y_6 + + (D_{1c} + D_{1d})Y_{11}]] \{ [(2 + 2\, c2\alpha\, c2(\alpha-\delta) +$$

$$+2c2\delta - c2(\alpha+\delta)\Big) s\Omega\Big(-\Big(ci\, c\Omega s\alpha\, s\delta \Big) + + c\delta\, si + c\alpha ci s\delta s\Omega \Big)\Big]/4 \Big\} + \Big[-B_{1a}\ A + B_{1b}\ B +$$

$$3\,(V + V_1)\Big]\{c\delta\} + \Big[\ 3\Big[\ B_{1a}\ C_1 - B_{1b}\ D_1\ \Big] + \ 9\Big[\ C_{1a}\ Z_7 + C_{1b}\ Z_8 + (C_{1c} + C_{1d})\,Z_9 - D_{1a}\ Z_{12} -$$

$$D_{1b}\ Z_{13} - (D_{1c} + D_{1d})Z_{14}\ \Big]\Big]\Big\{\ c(\alpha+\Omega)s\alpha\ s\delta\ \ \mathrm{Sin}\,2\delta \mathrm{Sin}\,\Omega\ \Big\} + \Big[\ 3[B_{1a}C_2 + B_{1b}D_2\ \Big] +$$

$$+9\Big[\ C_{1a}Z_8 + C_{1b}Z_{10} + (C_{1c}+C_{1d})Z_{11} + D_{1a}Z_{13} + D_{1b}Z_{15} + (D_{1c}+D_{1d})Z_{16}\ \Big]\Big]$$

$$\Big\{\ s\,i\Big(-\Big(\ ci\ c\Omega\ s\alpha\ s\delta\ \Big) + c\delta\ si\ +\ \ c\alpha\ ci\ s\delta\ s\Omega\ \Big)\Big\} + \Big[\ 3\Big[\ B_{1a}\ C_3\ + B_{1b}\ D_3\ \Big] +$$

$$+9\Big[\ C_{1a}\ Z_9 + C_{1b}Z_{11} + (C_{1c} + C_{1d})Z_8 - D_{1a}Z_{14} - D_{1b}Z_{16} - \ (D_{1c} + D_{1d})Z_{13}\ \Big]\Big]\Big\{\ s\alpha\ s2\delta\ s\Omega\Big(-\Big(\ ci\,c\Omega\ s\alpha$$

$$s\delta\ \Big) + c\delta\ s\ i\ c\alpha\ ci\ s\delta\ s\Omega\ \Big)\Big\} + \ \Big[\ 3[B_{1a}\ C_3\ -\ B_{1b}\ D_3\,] + 9\Big[\ C_{1a}\ \ Z_9\ +\ C_{1b}\ \ Z_{11}\ +$$

$$+(C_{1c} + C_{1d}\,)\,Z_8 + D_{1a}\ \ z_{14} + D_{1b}\ \ Z_{16} + (D_{1c} + D_{1d}\,)\,Z_{13}\ \Big]\Big]\Big\{\ -c(\alpha+\Omega)\,s\,i\ \Big\}\Big\} \qquad \text{(A.2)}$$

$$N_{iz} = \ \Re_i\ \ W\ \Big\{\Big\{\ s\beta\Big\{\ A_{1x}\ \ c\delta\ c\alpha\ -\ B_{1x}\ \ c\delta\ s\alpha\ -\ 3(C_{1xa} - C_{1xd}\,)\Big\{\ c\delta\ c(\alpha-\Omega)\Big(\ \ c\alpha\ c^2\delta\ c(\alpha-\Omega) +$$

$$+c\alpha\ c(\alpha+\Omega)\,s^2\delta + s\alpha\ s(\alpha-\Omega)\ \Big)\Big\} - 3(C_{1xb} - C_{1xe})\Big\{\Big(\ \ -ci\ s\Omega\ s\alpha + s\delta\ si + c\alpha\ c\delta ci c\Omega\ \Big)\Big)\Big\} -$$

$$3(C_{1xc} - C_{1xf}\,)\Big\{\ -c\delta\ c\ i\ c(\alpha-\Omega)\,s\Omega\ \Big\} + 3(D_{1xa}\ + D_{1xd}\ )$$

$$\Big\{\Big(\ c\delta\Big(\ 2 + 2c2\alpha - c(2(\alpha-\Omega)) + 2c2\delta - c2(\alpha+\delta)\ \Big)c(\alpha-\Omega)s\Omega\ \Big)/4\ \Big\} + 3(D_{1xb} + D_{1xe}\,)\Big\{\ ci\ c\Omega\Big(\ c\delta$$

$$ci\ c\Omega\ c\alpha + s\delta\ si - c\alpha c\delta ci s\Omega\ \Big)\Big\} + \ 3(D_{1xc} + D_{1xf})\Big\{\ c\delta ci\ c(\alpha-\Omega)\,c\Omega\ \Big\} +$$

$$3(D_{1xc} + D_{1xf})\Big\{\Big[\Big(\ \ 2 + 2\ c2\alpha - c2(\alpha-\delta) - \ c2(\alpha+\delta)\ \Big)$$

$$s\Omega\Big(\ c\delta ci c\Omega\ s\alpha + s\delta\ si -\ c\alpha\ c\delta\ ci\ s\Omega\ \Big)\Big]/\ 4\ \Big\}\Big\}\Big\} + \ \ \Big\{\ c\beta\Big\{\ E_{1x}\ \mathrm{Sen}\,\delta - 3(F_{1xa}\ - F_{1xd}\ )$$

$$\Big\{\ c\delta\ c(\alpha-\Omega)\,s\alpha\ s2\delta\ s\Omega\ \Big\} - 3(F_{1xb}\ - F_{1xe}\ )\Big\{\ si\Big(\ \ c\delta\ ci\ c\Omega\ s\alpha + s\delta\ s\ i -$$

$$c\alpha\ c\delta\ ci\ s\Omega\ \Big)\Big\} - 3(F_{1xc} - F_{1xf}\ )\Big\{\ s\alpha\ s2\delta\ s\Omega\ \ \Big(\ \ c\delta\ ci\ c\Omega\ s\alpha + s\delta\ si -\ c\alpha\ c\delta\ ci\ s\Omega\ \Big\}\Big\}\Big\}$$

$$-\{\ \ N_{3a1}\ +\ N_{3a2}\ - 3\big[A_{1e}C_1 + B_{1e}C_2 + \ (C_{1e} + D_{1e}\,)C_3\ \big] + 3(E_{1e}\ D_1 + F_{1e}\ D_2 + \ (G_{1e}\ \ +\ H_{1e}\ )D_3\ ) +$$

$$+9\ (I_{1e}\ \ C_1\ \ +\ J_{1e}\ \ C_2\ \ +\ (L_{1e}\ \ +\ M_{1e}\ )\ C_3\ ) - 9(N_{1e}\,D_1\ \ +\ O_{1e}\ \ D_1\ \ +\ (P_{1e}\ \ +Q_{1e}\ )D_3)\ \ +\ \ 10B\ -$$

$$-2\ (R_{1e}\ D_1 + S_{1e}\ D_2 + (T_{1e} + U_{1e}\,)D_3\ )\ \ \} + \ \Big\{\ s^2\ \beta\Big\{\ -N_{4a1}\ \ c\delta\ c\alpha\ +\ N_{4a2}\ \ c\delta\ c\alpha\ + \Big[\ 3\ \ (-B_{1a}\ \ J\ +$$

$$+B_{1b}O\ \ ) + 9\Big[\ \ C_{1a}Y + (C_{1c} + C_{1d})Y_2 - D_{1a}Y_5 + D_{1b}Y_6 + \ \ (D_{1c} + D_{1d})Y_7\ \ \Big]\Big]\Big\{\ c\delta\ c(\alpha-\Omega)\Big(\ \ c\alpha\ c^2\delta\ c(\alpha-\Omega) +$$

$$+c\alpha\ c(\alpha+\Omega)\ s^2\delta + s\alpha\ s(\alpha-\Omega)\ \Big)\Big\} + \Big[\ \ 3\ (-B_{1a}\ L + \ B_{1b}N\ \ ) + 9\Big(\ \ C_{1b}Y_1 + C_{1a}Y_1 + C_{1b}Y_5 + D_{1a}Y_6 - D_{1b}Y_8 +$$

$$+(D_{1c} + D_{1d})Y_9\ \ \Big]\Big]\Big\{\Big(\ \ -c\ i\ s\Omega\Big(\ \ c\delta\ ci\ c\Omega\ s\alpha + \ + s\delta\ si + c\alpha\ c\delta\ ci\ c\Omega\ \ \Big)\Big)\Big\} + \Big[\ \ 3(-B_{1a}N + B_{1b}Q) +$$

$$+9\Big((C_{1c}+C_{1d})Y_1 + C_{1a}Y_2 + C_{1b}Y_4 + (C_{1c}+C_{1d})Y_1 - C_{1a}Y_2 + C_{1b}Y_4 + (C_{1c}+C_{1d})Y_1 -$$

$$- D_{1a}Y_7 - D_{1b}Y_9 - -(D_{1c}+D_{1d})Y_6 \Big]\Big]\Big\{\Big(c\alpha\, c^2\delta\, c(\alpha-\Omega) + c\alpha\, c(\alpha+\Omega)\, s^2\delta +$$

$$+ s\alpha\, s(\alpha-\Omega)\Big)\Big(s\delta\, ci\, c\Omega s\alpha + s\delta\, si - c\alpha\, c\delta\, ci\, s\Omega \Big)\Big\} + \Big[3(-B_{1a}N + B_{1b}Q) + 9\, W\Big[C_{1a}Y_2 +$$

$$+ C_{1b}Y_4 + (C_{1c}+C_{1d})Y_1 - D_{1a}Y_7 - D_{1b}Y_9 - (D_{1c}+D_{1d})Y_6 \Big]\Big]\Big\{ -c\delta\, ci\, c(\alpha-\Omega)\, s\Omega \Big\} +$$

$$+\Big[3\ (B_{1a}\ O - B_{1b}\ S) + 9\Big[- C_{1a}Y_5 - C_{1b}Y_6 - (C_{1c}+C_{1d})Y_7 + D_{1a}Y_{10} + D_{1b}Y_{11} +$$

$$+ (D_{1c}+D_{1d})Y_{12} \Big]\Big]\Big\{\Big(c\delta\Big(2 + 2\, c2\alpha - c2(\alpha-\Omega) + 2\, c2\delta - c2(\alpha-\delta) \Big)$$

$$c(\alpha-\Omega)\, s\Omega \Big)/4 \Big\} + \Big[3\ (B_{1a}\ N - B_{1b}\ T) + 9\Big[-C_{1a}Y_6 - C_{1b}Y_8 - (C_{1c}+C_{1d})Y_9 +$$

$$+ D_{1a}Y_{11} + D_{1b}Y_{13} + (D_{1c}+D_{1d})Y_{14} \Big]\Big]\Big\{ c\ i\ c\,\Omega\Big(c\delta\, ci\, c\Omega\, c\alpha + s\delta\, si - c\alpha\, c\delta\, ci\, c\Omega \Big)\Big\} +$$

$$+\Big[3\ (B_{1a}\ Q - B_{1b}\ U) + 9\Big[-C_{1a}Y_7 - C_{1b}Y_9 - (C_{1c}+C_{1d})Y_6 ++ (D_{1c}+D_{1b})Y_{11} \Big]\Big]$$

$$\Big\{ c\delta\, ci\, c(\alpha-\Omega)\, c\Omega \Big\} + \Big[3\,(B_{1a}\ Q - B_{1b}\ U) + 9\Big(-C_{1a}Y_7 - C_{1b}Y_9 - (C_{1c}+C_{1d})Y_6 +$$

$$+ (D_{1c}+D_{1b})Y_{11} \Big]\Big]\Big\{\Big(\Big(2 + 2\, c2\alpha - c2(\alpha-\delta) + +2\, c\, 2\delta - c\, 2(\alpha+\delta) \Big) s\Omega\Big(c\delta\, ci\, c\Omega\, s\alpha +$$

$$s\delta\, si - c\alpha\, c\delta\, ci\, s\Omega \Big)\Big)/4 \Big\} + \Big[\Big(B_{1a}\ A + B_{1b}\ B + 3\big(V + V_1\big)\Big)\Big]\Big\{ s\,\delta \Big\} +$$

$$+\Big[3(B_{1a}C_1 - B_{1b}D_1) + 9\Big(C_{1a}Z_7 + C_{1b}Z_8 + (C_{1c}+C_{1d})Z_9 - D_{1a}Z_{12} - D_{1b}Z_{13} - (D_{1c}+D_{1D})Z_{14} \Big]\Big]$$

$$\Big\{ c\delta\, c(\alpha-\Omega)\, s\alpha\, s2\delta s\Omega \Big\} + \Big[3(B_{1a}C_2 + B_{1a}D_2) + 9\Big[C_{1a}Z_8 + C_{1b}Z_{10} + (C_{1c}+C_{1d})Z_{11} + D_{1a}Z_{13} +$$

$$+ D_{1b}Z_{15} + (D_{1c}+D_{1d})Z_{16} \Big]\Big]\Big\{ si\Big(c\delta\, ci\, c\Omega\, s\alpha + + s\delta\, si - c\alpha c\delta ci\, s\Omega \Big)\Big\} + \Big[3\Big(B_{1a}C_2 +$$

$$B_{1b}\ D_3 \Big] + 9\Big[C_{1a}Z_9 + C_{1b}Z_{11} + (C_{1c}+C_{1d})Z_8 - D_{1a}Z_{14} - D_{1b}Z_{16} -$$

$$- (D_{1c}+D_{1d})Z_{13} \Big]\Big]\Big\{ s\alpha\, s2\delta\, s\Omega(c\delta\, si\, c\Omega\, s\alpha + s\delta\, si - c\alpha\, c\delta\, ci\, s\Omega \Big)\Big\} + \Big[3\big(B_{1a}C_3 - B_{1b}D_3\big) +$$

$$9\Big[C_{1a}Z_9 + C_{1b}Z_{11} + (C_{1c}+C_{1d})Z_8 + D_{1a}Z_{16} + (D_{1c}+D_{1d})Z_{13} \Big]\Big]\Big\{ c\delta\, c(\alpha-\Omega)\ si \Big\}\Big\} -$$

$$\Big\{ N_{4b1}s^2\beta - (N_{4b2} + N_{4b3} + N_{4b4} + N_{4b5})s\beta\, c\beta \Big\}\Big\} \qquad (A.3)$$

where s = Sin and c = Cos and all coefficients are explicitly described in Quirelli (2002).