PARAMETER SETTING AND PRELIMINARY SIZING OF A LRE

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Abstract. *It is described a bipropellant, liquid rocket engine parametric design and analysis method, which underlies a computer program for preliminary screening and selection of the propulsion type and cycle, as well as for engine parametric optimization. The method could be applied to the analysis of pressure-fed systems and three classical pump-fed systems: gas generator, staged combustion, and expander cycles. The basic inputs for the method are the mission (initial and final altitudes and velocities) and design (take-off mass, propellant nature and mass, etc.) requirements and the chosen configuration for a stage of the vehicle. Emphasis is given in the used mass-estimating relations, most of which are based on historical data from engines developed between 1960 and 1980. Results of the methodology were compared to the second stage of Zenit launch vehicle, showing reasonable agreement.*

Keywords: *Conceptual design, Preliminary design, Liquid rocket engine, Satellite launch vehicle.*

1 Introduction

The conceptual and preliminary design phases of a launch vehicle are complex, iterative procedures requiring synthesis of a wide variety of engineering disciplines. To obtain an initial reference vehicle that fulfill a given mission, after a multidisciplinary study one must assume certain initial values for various vehicle parameters in the process (such as the vehicle thrust-to-weight ratio and the mass split between stages). Once this baseline vehicle had been defined, these parameters can be varied in an attempt to better understand the role that each one plays in the design process and to determine the optimal system to perform the required mission.

The vehicle's propulsion systems determine largely the complexity and cost of the whole vehicle. Therefore, judicious study should be done before the proper choice of a propulsion system. Liquid rocket engines are either pressure-fed or pump-fed, depending on the mission requirements. Once mission, engine type and cycle are determined, the engine's mass and performance can be estimated for some range of values of independent parameters. Hence parametric propulsion data are a basic need for conceptual/preliminary design.

Different methods and tools had been used worldwide for analysis of liquid rocket engine (LRE) cycles. They could be relatively simple as, for example, that one due to Kozlov (1993), or more detailed, as the modular method of Goertz (Goertz, 1995; Manski et al., 1998). However, such tools are not available to general public.

This paper describes a bipropellant, liquid rocket engine parametric design and analysis method, which underlies a computer program for preliminary screening and selection of the propulsion type and cycle, as well as for engine parametric optimization. In addition to pressure-fed systems, the method permits the analysis of three classical and more often used pump-fed engine systems: the gas generator cycle, the staged combustion cycle, and the expander cycle.

The basic inputs for the method are the mission requirements (initial and final altitudes and velocities), the design requirements (take-off mass, propellant nature and mass, etc.), and the chosen configuration for a stage of the vehicle.

The current mass/sizing module utilizes various empirical mass-estimating relations (MER's) based on historical data where possible. The flight-performance module uses a 2-D trajectory model, in which the aerodynamic forces are calculated using very simple empirical formulae. Because the launcher is not designed for high maneuverability as are air-to-air and surface-to-air missiles, a 2-DOF model is all that is necessary to capture the overall performance of the vehicle. In addition, a more complete model would be difficult to construct because details of the dynamic performance of the launcher are not readily available during conceptual design. For these reasons, a 2-DOF model was deemed sufficient.

Since for a mass-optimized vehicle the system mass is a main concern, this article will give special emphasis on the MER's used in the calculations.

2 Disciplinary Analysis

The synthesis of a satellite launch vehicle includes analysis for many disciplines: aerodynamics, propulsion, architecture/sizing, mass estimation, trajectory, and optimization. These disciplines include the main disciplines required for a complete conceptual design process, which is illustrated in the Fig. 1.

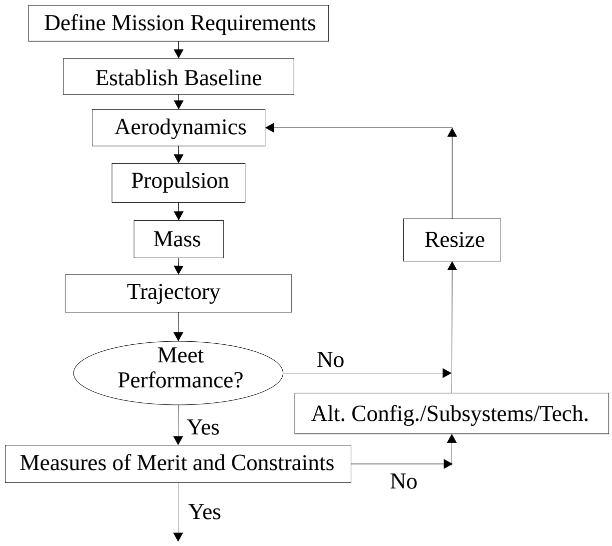

Figure 1: Process for conceptual launcher sizing and synthesis.

The program is constructed so that each discipline is handled on an individual module. This breakdown allows the user to focus on each discipline individually. Linking of the individual disciplines/modules is done through one master module, which is designed to handle user-inputs, to coordinate the stage synthesis and optimization (from the disciplinary modules), and to save properly selected output results.

2.1 Trajectory

The trajectory simulation "flies" the sized vehicle through a user-specified sequence of trajectory events. It uses the vehicle model and the equations of motion to determine the vehicle position, velocity, acceleration, and other performance quantities as a function of time.

In the present version of the computer code, the trajectory simulation portion has been simplified to the simulation of a point mass planar motion over a spherical nonrotating earth. This is normally detailed enough to determine the proper vehicle performance within small errors when vehicle model inputs of sufficient accuracy are provided.

The equations of powered, planar motion over a spherical, nonrotating earth are

$$\frac{dv}{dt} = \frac{F - D}{m} - g \sin\theta \,, \tag{1}$$

$$\frac{d\theta}{dt} = \left(\frac{v}{r} - \frac{g}{v}\right) cos\theta \,, \tag{2}$$

$$\dot{x} = \frac{r_o}{r}\, v \cos\theta \,, \tag{3}$$

$$\dot{h} = v \sin\theta \,, \tag{4}$$

where: t = time; v = velocity magnitude; θ = flight path angle relative to the local horizontal (angle between the direction of the velocity vector and the local horizontal); x = surface range, i.e., curvilinear distance along the surface from the launch point to a point under the vehicle; h = altitude above the earth surface; m = vehicle mass; $g = g_o\,(r_o/r)^2$ = acceleration due to gravity; g_o = acceleration of gravity at earth's surface ($r = r_o$); $r = r_o + h$ = radius from the earth center to the point mass; r_o = radius of the earth; D = drag force; $F = F(t,\, h)$ = thrust.

For a complete launcher, the whole ascent trajectory is broken into three phases: vertical ascent (from liftoff instant up to approximately 10 s), kick-over manoeuver, and gravity turn. Coast periods could be freely defined by the user.

As already pointed out, the effect of the earth's rotation has been neglected. For most launchers during their burning phase, this effect can be introduced as a differential correction added on at the end of burning of the second stage .

For a given vehicle stage, the following initial conditions ($t = t_0$) must be prescribed: $h(t_0) = h_0$, $x(t_0) = x_0$, $v(t_0) = v_0$, $\theta(t_0) = \theta_0$, and $m(t_0) = m_0$. Additionally, depending on the objective of the calculations, the terminal states (at $t = t_F$) of the following parameters could be specified: $\theta(t_F) = \theta_F$ and $h(t_F) = h_F$.

2.2 Propulsion

The propulsion discipline is designed to handle pressure-fed or pump-fed systems, as previously described. The user inputs the propellant-type, combustion chamber pressure, burn time, and expansion ratio. From these data, simple rocket relations are used to calculate the throat area, exit area, thrust, and specific impulse. The program warns the user whenever the exit area is greater than the maximum diameter of the stage.

2.3 Aerodynamics

Normally, the aerodynamics discipline requires inputs that define the vehicle geometry, including nose fineness, length, diameter, etc. However, the present method uses a very simple model, which consists only in the calculation of the drag, according to the equation:

$$D = C_D\, A_{\mathrm{ref}}\, \frac{\rho}{2}\, v^2\ . \tag{5}$$

Here, A_{ref} denotes a reference area for drag estimation, $\rho = \rho(h)$ is the density of atmosphere (1976 USA Standard Atmosphere used), and $C_D = C_D(\alpha,\, M,\, \mathrm{stage})$ is the drag coefficient.

The drag coefficient was calculated by using a simple analytical function of the Mach number (M), holding for null angle-of-attack ($\alpha = 0$):

$$C_D = \begin{cases} 0.29 & 0 \le M \le 0.80\ , \\ M - 0.519 & 0.80 < M \le 1.068\ , \\ 0.09 + 0.5/M & M > 1.068\ . \end{cases} \tag{6}$$

2.4 Thermochemistry Data

2.4.1 Theoretical Values

The CETPC (1994) computer program was used to compute the theoretical values of the engine performance parameters, assuming chemical equilibrium and infinite area combustor. The input data for the CETPC program are: the composition of the propellants (fuel and oxidizer), their assigned initial temperature and enthalpy, the propellant mass mixture ratio (k_{m}), the combustion chamber pressure (p_{c}), and the gas expansion pressure ratio (ε_p) or the nozzle expansion area ratio (ε_A).

Output data from CETPC, for a given propellant combination and a chosen range of the input parameters, are reduced and saved in tabular form. In addition to the values of k_{m}, p_{c}, ε_p and ε_A, values of the following parameters are included in such tables: theoretical specific impulse $I_{\mathrm{sp,t}} = I_{\mathrm{sp,t}}(k_{\mathrm{m}},\, p_{\mathrm{c}},\, \varepsilon_p)$, theoretical characteristic exhaust velocity $c^*_{\mathrm{t}} = c^*_{\mathrm{t}}(k_{\mathrm{m}},\, p_{\mathrm{c}})$, temperature of the combustion products T_{c}, and average specific heat ratio n.

During a code run, the values of ε_A, c^*_{t}, $I_{\mathrm{sp,t}}$ and n, corresponding to given values of k_{m}, p_{c} and ε_p, are calculated from the tabled data by using a spline interpolation routine. The sequence is the following:

1. Calculation of ε_A by using ε_p;
2. Interpolation of c^*_{t};
3. Interpolation of $I_{\mathrm{sp,t}}$;
4. Interpolation of n (if required).

2.4.2 Effective Values

The real specific impulse of the thrust force produced by a LRE thrust chamber is determined according to the following relation:

$$I_{\mathrm{sp,e}} = \varphi_{\mathrm{c}}\, \varphi_{\mathrm{n}}\, I_{\mathrm{sp,t}}\ , \tag{7}$$

where $I_{\mathrm{sp,t}} = I_{\mathrm{sp,t}}(k_{\mathrm{m}},\, p_{\mathrm{c}},\, \varepsilon_p)$ is the theoretical value of the specific impulse, φ_{c} is the c^*-efficiency (in-chamber specific impulse loss factor), and φ_{n} is the nozzle-efficiency (in-nozzle specific impulse loss factor).

Accordingly, the real characteristic velocity of the engine is given by

$$c^*_{\mathrm{e}} = \varphi_{\mathrm{c}}\, c^*_{\mathrm{t}}\ . \tag{8}$$

where c^*_{t} is the theoretical value of the characteristic velocity.

3 Stage Sizing and Mass

Mass characteristics have been obtained as sums over all subsystems depending on the main operational parameters of the vehicle. The mass of a separate unit has been represented in analytical form as a function of operational parameters. Scaling factors have been determined by processing statistical data of real structures.

Most formulae implemented in the computer program — and described in the following sections — were based in references (Kozlov et al., 1988; Kozlov, 1993; Kozlov et al., 1999; Garanin and Samokhin, 1993; Garanin, 1996). However, it must be pointed out that these references contain several and important misprints and errors, which, it is hopped, are now corrected!

3.1 Mass of Propulsion System and Accessories

In this study, are supposed known the initial mass gross of the stage, m_{init}, and the initial propellant mass, m_{p}. The potential payload mass, m_{pl}, should be determined.

The mass of propulsion system and accessories, m_{psa}, could be expressed in the following form:

$$m_{\text{psa}} = m_{\text{init}} - m_{\text{p}} - m_{\text{pl}} \, . \tag{9}$$

In this study, m_{psa} is estimated as a function of the main parameters of the propulsion system of the stage, which could be used as variables in an optimization procedure. A further analysis shows that m_{psa} could also be written as

$$m_{\text{psa}} = m_{\text{eng}} + (m_{\text{tf}} + m_{\text{to}}) + (m_{\text{pc}} + m_{\text{pg}}) + (m_{\text{rcs}} + m_{\text{p,rcs}}) + m_{\text{misc}} \, , \tag{10}$$

where: m_{eng} is the mass of the engine; m_{tf} and m_{to} are masses of the propellant tanks; m_{pc} and m_{pg} are the masses of the pressurant container and gas, respectively; m_{rcs} is the mass of the reaction control system; $m_{\text{p,rcs}}$ is the mass of propellant of the reaction control system; and m_{misc} accounts for all the mass not included in the other subsystems.

Given m_{init} and m_{p}, after the calculation of m_{psa} one can estimate the payload mass by using the Eq. (9), which could be rewritten in the form:

$$m_{\text{pl}} = m_{\text{init}} - m_{\text{p}} - m_{\text{psa}} \, . \tag{11}$$

3.2 Engine Mass

All the mass properties of the LRE parts are based on analytical functions of the operation parameters, considering statistical factors reflecting properties of construction materials and the present state of production engineering.

In the following relations, n_{cham} denotes the number of thrust chambers, F is the thrust of engine in newtons, m_{tc} is the mass of a thrust chamber, and m_{tpus} is the total mass of the turbo-pumps[1].

The mass of a **staged combustion cycle** engine is computed by the correlation given in (Kozlov et al., 1988)

$$m_{\text{eng}} = \begin{cases} n_{\text{cham}}\, m_{\text{tc}} + m_{\text{tpus}} + 4.305 \cdot 10^{-4}\, n_{\text{cham}}\, F + 57.0\,, & \text{if } (n_{\text{cham}} F < 1 \cdot 10^5) \\ n_{\text{cham}}\, m_{\text{tc}} + m_{\text{tpus}} + 3.881 \cdot 10^{-4}\, n_{\text{cham}}\, F - 73.1\,, & \text{otherwise} \end{cases} \, . \tag{12}$$

For a engine with **gas generator cycle**, the mass is estimated by

$$m_{\text{eng}} = \begin{cases} m_{\text{tc}}\, n_{\text{cham}} + m_{\text{tpus}} + 2.125 \cdot 10^{-4}\, n_{\text{cham}}\, F + 57.5\,, & \text{if } F n_{\text{cham}} \le 9 \cdot 10^4 \\ m_{\text{tc}}\, n_{\text{cham}} + m_{\text{tpus}} + 3.70 \cdot 10^{-4}\, n_{\text{cham}}\, F - 93.1\,, & \text{otherwise} \end{cases} \, . \tag{13}$$

For a engine with **expander cycle**, the following expression is used for calculation of the engine mass:

$$m_{\text{eng}} = m_{\text{tc}}\, n_{\text{cham}} + m_{\text{tpus}} + 1.895 \cdot 10^{-4}\, n_{\text{cham}}\, F + 54.0 \, . \tag{14}$$

In case of a **pressure-fed** engine, the mass is given by

$$m_{\text{eng}} = m_{\text{tc}}\, n_{\text{cham}} + 1.895 \cdot 10^{-3}\, n_{\text{cham}}\, F + 74.5 \, . \tag{15}$$

In order to estimate the mass of each thrust chamber, $m_{\text{tc}} = m_{\text{tc}}(\dot{m}_{\text{c}},\, p_{\text{c}},\, \varepsilon_p,\, n,\, \beta_{\text{e}},\, c^*_{\text{e}},\, q)$, the thrust chamber is further subdivided into: cylindrical part of the chamber, convergent (subsonic) and divergent (supersonic)

[1] Unless otherwise explicitly cited, all the parameters used here are expressed in the normal SI units. Therefore, mass is in kg; distance and length in m; force in N; pressure in Pa; temperature in K; speed in m/s; specific impulse in N·s/kg; area in m^2; volume in m^3; and mass-density in kg/m^3.

parts of the nozzle, injector head, and inlet manifold. This way, defining $w \equiv p_c\, c_e^*\, \dot{m}_c = p_c^2\, A_t$, the following expressions could be used to estimate the thrust chamber mass for the different feed systems. For a **staged combustion cycle** or a **expander cycle**:

$$m_{tc} = A_t \left[\gamma_{cc}(\bar{S}_{cyl} + \bar{S}_{conv}) + \gamma_{div}\, \bar{S}_{div} + 396.2/w^{1/4} + 17.58\, w^{1/8}\right] - 13.3\,. \tag{16}$$

For a **gas generator cycle**:

$$m_{tc} = A_t \left[\gamma_{cc}(\bar{S}_{cyl} + \bar{S}_{conv}) + \gamma_{div}\, \bar{S}_{div} + 163.0/w^{1/4}\right] - 8.5\,. \tag{17}$$

For a **pressure-fed engine**:

$$m_{tc} = A_t \left[\gamma_{cc}(\bar{S}_{cyl} + \bar{S}_{conv}) + \gamma_{div}\, \bar{S}_{div} + 16.3/w^{1/4}\right] - 8.5\,. \tag{18}$$

In the above formulae: A_t is the throat section area; γ_{cc} is the mass per unit surface area of the combustion chamber; $\bar{S}_{cyl}$ and $\bar{S}_{conv}$ are the ratios of the areas of cylindrical and convergent parts, respectively, by the throat area; $\bar{S}_{div}$ is the nozzle divergent part surface area divided by the throat section area; γ_{div} is the mass per unit surface area of the nozzle divergent part; and the other terms are empirical correlating coefficients.

$$\gamma_{cc} = \begin{cases} 3.03 \cdot 10^{-6}\, p_c/\sqrt{2R_t} - 17\,, & \text{if } p_c/\sqrt{2R_t} > 12.21 \cdot 10^6 \\ 20\,, & \text{otherwise} \end{cases}, \tag{19}$$

$$\bar{S}_{cyl} = 2\,\frac{L^*}{R_t}\,\sqrt{q\, c_e^*} - 2\,\frac{1}{\sqrt{q\, c_e^*}} + \sqrt{q\, c_e^*} + 1\,, \tag{20}$$

$$\bar{S}_{conv} = \frac{2}{q\, c_e^*} + \frac{0.8187}{\sqrt{q\, c_e^*}} - 0.9736\,, \tag{21}$$

$$\gamma_{div} = \begin{cases} 10.0\,, & \text{if } p_c/\sqrt{2R_t\, \varepsilon_p} < 0.632 \cdot 10^6 \\ 0.05894 \left(\dfrac{p_c}{\sqrt{2R_t\, \varepsilon_p}}\right)^{0.475} - 23.58\,, & \text{otherwise} \end{cases}, \tag{22}$$

$$\bar{S}_{div} = \bar{S}_o \left[1 - \left(1.415 - \frac{0.274}{\sqrt{r}}\right) f(z)\right], \tag{23}$$

$$\bar{S}_o = (32 - 10\, n)(r-1) + (2.1 + 1.6\, n^4)(r-1)^{2.25}\,, \tag{24}$$

$$f(z) = 1 - \exp\left[-(1-z)^{1/3}\right]. \tag{25}$$

The relative radius at the nozzle exit section, defined by

$$r \equiv \frac{R_e}{R_t} = \sqrt{\frac{A_e}{A_t}}\,, \tag{26}$$

is a function of the pressure expansion ratio ($\varepsilon_p = p_c/p_e$) and the isentropic expansion coefficient n, i.e., $r = r(\varepsilon_p, n)$. Here, R_e and R_t denote the radii at the exit and throat cross sections, respectively.

The parameter z is an auxiliary parameter for calculation of m_{tc}. It is defined by

$$z(n,\, \varepsilon_p,\, \beta_e) \equiv \frac{R_e - R_t}{R_{max} - R_t} = \frac{r-1}{\bar{R}_{max} - 1}\,, \tag{27}$$

where R_{max} is the nozzle section radius for which $\beta_e = 0$. It will be estimated by

$$z = 1 - \left[\frac{\sin\beta_e}{0.6 - (0.018\, n - 0.0175)(r+24)}\right]^{4/3}. \tag{28}$$

Here, β_e is the angle of nozzle contour at the exit section.

In the above formulae, $\dot{m}_c$ stands for the mass flow rate through the thrust chamber of the main engine; L^* is the characteristic length of the combustion chamber; and q is the "flow rate stress level", as defined in (Kozlov et al., 1988).

3.3 Mass of Turbopumps

The total mass of the Turbopump Units (TPU's) accounts for all the main and auxiliary pumps and turbines:

$$m_{\rm tpus} = m_{\rm tpu,o} + m_{\rm tpu,f} + m_{\rm bp,o} + m_{\rm bp,f}\,. \tag{29}$$

In the following description, analytical formulae and empirical correlations are presented for single-shaft and independent turbopumps.

3.3.1 Single-Shaft Turbopump

For a single-shaft TPU arrangement, one turbine drives both the fuel pump and oxidizer pump that are mounted on the same shaft. Therefore, both pumps rotate at the same rotational speed, i.e., $\omega_{\rm f} = \omega_{\rm o}$.
For **staged combustion** or **expander cycles**:

$$m_{\rm tpu,f} + m_{\rm tpu,o} = 19.0 + 0.232 \cdot 10^{-3}\mathcal{D}\,, \quad \text{for } 2.93 \cdot 10^4 \le \mathcal{D} \le 1.82 \cdot 10^6\,. \tag{30}$$

For g**as generator cycle**:

$$m_{\rm tpu,f} + m_{\rm tpu,o} = \begin{cases} 6.29 + 0.981 \cdot 10^{-3}\,\mathcal{D}\,, & \text{if} \quad 0.117 \cdot 10^4 \le \mathcal{D} \le 3.22 \cdot 10^4, \\ 21.0 + 0.54 \cdot 10^{-3}\,\mathcal{D}\,, & \text{if} \quad 2.93 \cdot 10^4 \le \mathcal{D} \le 7.52 \cdot 10^5. \end{cases} \tag{31}$$

In the preceding formulae, $\mathcal{D}$ is a hydromechanical parameter given by

$$\mathcal{D} = \dot m_{\rm o}\, h_{\rm o}^{3/2}/\omega_{\rm o} + \dot m_{\rm f}\, h_{\rm f}^{3/2}/\omega_{\rm f}\,. \tag{32}$$

The rotational speed of the oxidizer pump, $\omega_{\rm o}$, is obtained from the following expression (note again that here $\omega_{\rm f} = \omega_{\rm o}$):

$$\omega_{\rm o} = C_{\rm cav,o}\, \frac{[\Delta p_{\rm suc,o}/\rho_{\rm o}]^{3/4}}{298\sqrt{\dot V_{\rm o}/n_{\rm ent,p,o}}}\,. \tag{33}$$

In the above formula, $n_{\rm ent,p,o}$ denotes the number of entrances into the oxidizer pump.
The parameters $h_{\rm f}$ and $h_{\rm o}$ are defined by

$$h_{\rm f} = \frac{p_{\rm ex,f} - p_{\rm in,f}}{\rho_{\rm f}}\,, \tag{34}$$

$$h_{\rm o} = \frac{p_{\rm ex,o} - p_{\rm in,o}}{\rho_{\rm o}}\,. \tag{35}$$

The differences $(p_{\rm ex,f} - p_{\rm in,f})$ and $(p_{\rm ex,o} - p_{\rm in,o})$ are the pressure gains across the pumps. The parameters $C_{\rm cav,f}$ and $C_{\rm cav,o}$ are the cavitating specific speed coefficients, which usually assume values of approximately 4200.

The values of the entrance pressures at the fuel and oxidizer pumps are set as input parameters in the present method. The following expressions were used to calculate the fuel and oxidizer pump discharge pressures. For **staged combustion** or **expander cycles** ($p_{\rm ex,f,2}$ is for a second stage of a fuel pump) the discharge pressures are calculated by:

$$p_{\rm ex,f} = 130.0 \cdot 10^5 + 1.75\, p_{\rm c}\,, \tag{36}$$

$$p_{\rm ex,o} = 80.0 \cdot 10^5 + 1.667\, p_{\rm c}\,, \tag{37}$$

$$p_{\rm ex,f,2} = 270.0 \cdot 10^5 + 1.5\, p_{\rm c}\,. \tag{38}$$

For a **gas generator cycle**:

$$p_{\rm ex,f} = 1.714\, p_{\rm c}\,, \tag{39}$$

$$p_{\rm ex,o} = 1.428\, p_{\rm c}\,. \tag{40}$$

The head available for suction is obtained by subtracting the propellant saturation pressure, $p_{\rm v}$, and a reserve pressure drop, $\Delta p_{\rm cav}$, from the entrance pressure, $p_{\rm in}$:

$$\Delta p_{\rm suc,f} = p_{\rm in,f} - p_{\rm v,f} - \Delta p_{\rm cav,f}\,, \tag{41}$$

$$\Delta p_{\rm suc,o} = p_{\rm in,o} - p_{\rm v,o} - \Delta p_{\rm cav,o}\,. \tag{42}$$

Total fuel mass and volume flow rates are:

$$\dot{m}_\mathrm{f} = n_\mathrm{cham}\,\dot{m}\,\frac{1}{k_\mathrm{m}+1}\,, \tag{43}$$

$$\dot{V}_\mathrm{f} = \frac{\dot{m}_\mathrm{f}}{\rho_\mathrm{f}}\,. \tag{44}$$

Total oxidizer mass and volume flow rates are:

$$\dot{m}_\mathrm{o} = n_\mathrm{cham}\,\dot{m}\,\frac{k_\mathrm{m}}{k_\mathrm{m}+1}\,, \tag{45}$$

$$\dot{V}_\mathrm{o} = \frac{\dot{m}_\mathrm{o}}{\rho_\mathrm{o}}\,. \tag{46}$$

3.3.2 Separated Turbopumps

For this arrangement, there are two distinct turbopump units mounted on different shafts: one turbine drives the fuel pump, and other turbine drives the oxidizer pump. This way, the oxidizer and fuel pumps rotate at different speeds (i.e., $\omega_\mathrm{f} \neq \omega_\mathrm{o}$), which are given by:

$$\omega_\mathrm{f} = C_\mathrm{cav,f}\,\frac{\left[\Delta p_\mathrm{suc,f}/\rho_\mathrm{f}\right]^{3/4}}{298\,\sqrt{\dot{V}_\mathrm{f}/n_\mathrm{ent,p,f}}}\,, \tag{47}$$

$$\omega_\mathrm{o} = C_\mathrm{cav,o}\,\frac{\left[\Delta p_\mathrm{suc,o}/\rho_\mathrm{o}\right]^{3/4}}{298\,\sqrt{\dot{V}_\mathrm{o}/n_\mathrm{ent,p,o}}}\,. \tag{48}$$

The masses of fuel and oxidizer TPU's, for a **staged combustion cycle**, could be estimated by the following expressions:

$$m_\mathrm{tpu,f} = 19.0 + 6.911\cdot 10^{-3}\,\mathcal{D}\,, \qquad \mathcal{D} = \dot{m}_\mathrm{f}\,h_\mathrm{f}^{3/2}/\omega_\mathrm{f}\,, \tag{49}$$

$$m_\mathrm{tpu,o} = 19.0 + 6.911\cdot 10^{-3}\,\mathcal{D}\,, \qquad \mathcal{D} = \dot{m}_\mathrm{o}\,h_\mathrm{o}^{3/2}/\omega_\mathrm{o}\,. \tag{50}$$

3.4 Propellant Tanks

3.4.1 Propellant Component Masses and Volumes

Knowing the total propellant mass m_p and the mass mixture ratio k_m, the masses and volumes of the propellant components are given by:

$$m_\mathrm{f} = m_\mathrm{p}/(k_\mathrm{m}+1)\,, \tag{51}$$

$$m_\mathrm{o} = m_\mathrm{p}\,k_\mathrm{m}/(k_\mathrm{m}+1)\,, \tag{52}$$

$$V_\mathrm{f} = m_\mathrm{f}/\rho_\mathrm{f}\,, \tag{53}$$

$$V_\mathrm{o} = m_\mathrm{o}/\rho_\mathrm{o}\,. \tag{54}$$

3.4.2 Masses of Fuel and Oxidizer Tanks

The masses of load-carrying thermo-insulated cylindrical tanks of fuel and oxidizer are determined by (see (Kozlov et al., 1988)):

$$m_\mathrm{tf} = K_\mathrm{dmt,f}\left\{2.1\,V_\mathrm{f}\left[\frac{f_\mathrm{t,f}\,\rho_\mathrm{t,f}}{\sigma_\mathrm{u,t,f}}\,p_\mathrm{tb,f} + 2\,\frac{\delta_\mathrm{ins,f}\,\gamma_\mathrm{ins,f}}{D_\mathrm{t,f}}\right] + 0.649\,D_\mathrm{t,f}^3\,\frac{f_\mathrm{t,f}\,\rho_\mathrm{t,f}}{\sigma_\mathrm{u,t,f}}\,p_\mathrm{tb,f}\right\}\,, \tag{55}$$

$$m_\mathrm{to} = K_\mathrm{dmt,o}\left\{2.1\,V_\mathrm{o}\left[\frac{f_\mathrm{t,o}\,\rho_\mathrm{t,o}}{\sigma_\mathrm{u,t,o}}\,p_\mathrm{tb,o} + 2\,\frac{\delta_\mathrm{ins,o}\,\gamma_\mathrm{ins,o}}{D_\mathrm{t,o}}\right] + 0.649\,D_\mathrm{t,o}^3\,\frac{f_\mathrm{t,o}\,\rho_\mathrm{t,o}}{\sigma_\mathrm{u,t,o}}\,p_\mathrm{tb,o}\right\}\,. \tag{56}$$

Here: $K_\mathrm{dmt,f}$ and $K_\mathrm{dmt,o}$ are the factors for the mass of details of the tanks; V_f and V_o are the fuel and oxidizer volumes; $p_\mathrm{tb,f}$ and $p_\mathrm{tb,o}$ are the fuel and oxidizer feed pressures; $\rho_\mathrm{t,f}$ and $\rho_\mathrm{t,o}$ are the mass-densities of the tank materials; $\sigma_\mathrm{u,t,f}$ and $\sigma_\mathrm{u,t,f}$ are the strength limits of the tank materials; $f_\mathrm{t,f}$ and $f_\mathrm{t,o}$ are the tank safety factors; $\delta_\mathrm{ins,f}$ and $\delta_\mathrm{ins,o}$ are the thicknesses of tank insulations; $\gamma_\mathrm{ins,f}$ and $\gamma_\mathrm{ins,o}$ are the mass per unit area of the tank insulations; $D_\mathrm{t,f}$ and $D_\mathrm{t,o}$ are the diameters of the tanks.

It should be noted that an estimation based on the above formulae could give a worse value for an insulated tank when decreases the tank length to diameter ratio.

The feed pressure is estimated by the pressure at the tank bottom, which, in turn, can be estimated by:

$$p_{\mathrm{tb,f}} = p_{\mathrm{tu,f}} + N_{\mathrm{x}}\, g_{\mathrm{o}}\, m_{\mathrm{f}}/(\pi\, D_{\mathrm{t,f}}^2/4)\ , \tag{57}$$

$$p_{\mathrm{tb,o}} = p_{\mathrm{tu,o}} + N_{\mathrm{x}}\, g_{\mathrm{o}}\, m_{\mathrm{o}}/(\pi\, D_{\mathrm{t,o}}^2/4)\ , \tag{58}$$

where $p_{\mathrm{tu,f}}$ and $p_{\mathrm{tu,o}}$ are the ullage pressures at the oxidizer and fuel tanks, respectively; N_{x} is the maximum axial g-load, and g_{o} is the sea-level gravity acceleration.

The ullage pressures at the propellant tanks for a **pressure-fed engine** are computed by formulae obtained by curve fitting of statistical data:

$$p_{\mathrm{tu,f}} = \begin{cases} 5.0 \cdot 10^5 + 1.42\, p_{\mathrm{c}}\ , & \text{for regenerative cooling;} \\ 3.0 \cdot 10^5 + 1.32\, p_{\mathrm{c}}\ , & \text{for radiation/ablative cooling;} \end{cases} \tag{59}$$

$$p_{\mathrm{tu,o}} = 1.2\, p_{\mathrm{c}}\ . \tag{60}$$

In the other hand, for a **pump-fed engine** the ullage pressures must be prescribed in the input data for the computer program.

3.5 Pressurization System Mass

The mass of the pressurization system is defined by the pressurant container mass (m_{pc}) plus the pressurant gas mass (m_{pg}):

$$m_{\mathrm{ps}} = m_{\mathrm{pc}} + m_{\mathrm{pg}}\ . \tag{61}$$

3.5.1 Pressurant Volumes

The pressurant volumes for pressurization of fuel and oxidizer tanks and valve control are given by:

$$V_{\mathrm{pc,f}} = K_{\mathrm{pev}}\, \frac{V_{\mathrm{f}}}{1 - K_{\mathrm{u,i,f}}}\, \frac{p_{\mathrm{tu,f}}}{T_{\mathrm{ftu,f}}} \left[\frac{p_{\mathrm{i,pc,f}}}{T_{\mathrm{ipc,f}}\, Z_{\mathrm{ipc,f}}} - \frac{p_{\mathrm{min,f}}}{T_{\mathrm{fpc,f}}\, Z_{\mathrm{fpc,f}}} \right]^{-1}\ , \tag{62}$$

$$V_{\mathrm{pc,o}} = K_{\mathrm{pev}}\, \frac{V_{\mathrm{o}}}{1 - K_{\mathrm{u,i,o}}}\, \frac{p_{\mathrm{tu,o}}}{T_{\mathrm{ftu,o}}} \left[\frac{p_{\mathrm{i,pc,o}}}{T_{\mathrm{ipc,o}}\, Z_{\mathrm{ipc,o}}} - \frac{p_{\mathrm{min,o}}}{T_{\mathrm{fpc,o}}\, Z_{\mathrm{fpc,o}}} \right]^{-1}\ , \tag{63}$$

where: $K_{\mathrm{pev}} = 1.8$ is a gas margin factor for electro-pneumo valve (EPV) control, $p_{\mathrm{i,pc,f}}$ and $p_{\mathrm{i,pc,o}}$ are the initial pressures of gas in the vessels, $T_{\mathrm{ipc,f}}$ and $T_{\mathrm{ipc,o}}$ are the initial temperatures of gas in the vessels, $Z_{\mathrm{ipc,f}}$ and $Z_{\mathrm{ipc,o}}$ are the initial compressibility factors of gas, $Z_{\mathrm{fpc,f}}$ and $Z_{\mathrm{fpc,o}}$ are the final compressibility factors of gas, $K_{\mathrm{u,i,f}}$ and $K_{\mathrm{u,i,o}}$ are the ullage factors, $T_{\mathrm{fpc,f}}$ and $T_{\mathrm{fpc,o}}$ are the final temperature of gas in the vessel, $T_{\mathrm{ftu,f}}$ and $T_{\mathrm{ftu,o}}$ are the final temperature of gas in the tank ullage, $p_{\mathrm{min,f}}$ and $p_{\mathrm{min,o}}$ are the initial pressures of gas in the vessels (which are given by ullage pressure plus the pressure drop on the regulator, i.e., $p_{\mathrm{min,f}} = p_{\mathrm{tu,f}} + \Delta p_{\mathrm{reg,f}}$ and $p_{\mathrm{min,o}} = p_{\mathrm{tu,o}} + \Delta p_{\mathrm{reg,o}}$), $\Delta p_{\mathrm{reg,f}}$ and $\Delta p_{\mathrm{reg,o}}$ are the pressure drops on the pressure regulators.

3.5.2 Mass of Pressurant Gas

The mass of gas used for propellant expulsion and for EPV control is determined by the following formula:

$$m_{\mathrm{pg}} = V_{\mathrm{pc,f}}\, \frac{p_{\mathrm{i,pc,f}}}{Z_{\mathrm{ipc,f}}\, \mathcal{R}_{\mathrm{pg,f}}\, T_{\mathrm{ipc,f}}} + V_{\mathrm{pc,o}}\, \frac{p_{\mathrm{i,pc,o}}}{Z_{\mathrm{ipc,o}}\, \mathcal{R}_{\mathrm{pg,o}}\, T_{\mathrm{ipc,o}}}\ , \tag{64}$$

where $\mathcal{R}_{\mathrm{pg,f}}$ and $\mathcal{R}_{\mathrm{pg,o}}$ are the gas constants of the pressurants used for pressurization of the fuel and oxidizer tanks, respectively.

3.5.3 Mass of Pressurant Container

The total mass of the pressurant containers, considering a reserve for control devices, is determined according to relation:

$$m_{\mathrm{pc}} = 3\, V_{\mathrm{pc,f}}\, \rho_{\mathrm{pc,f}}\, K_{\mathrm{dmpc,f}}\, f_{\mathrm{pc,f}}\, \frac{p_{\mathrm{i,pc,f}}}{\sigma_{\mathrm{u,pc,f}}} + 3\, V_{\mathrm{pc,o}}\, \rho_{\mathrm{pc,o}}\, K_{\mathrm{dmpc,o}}\, f_{\mathrm{pc,o}}\, \frac{p_{\mathrm{i,pc,o}}}{\sigma_{\mathrm{u,pc,o}}}\ , \tag{65}$$

where: $\rho_{\mathrm{pc,f}}$ and $\rho_{\mathrm{pc,o}}$ are the mass-densities of the container materials, $K_{\mathrm{dmpc,f}}$ and $K_{\mathrm{dmpc,f}}$ are the factors for the mass of details of the pressurant containers, $f_{\mathrm{pc,f}}$ and $f_{\mathrm{pc,o}}$ are the safety factors of the container materials, $\sigma_{\mathrm{u,pc,f}}$ and $\sigma_{\mathrm{u,pc,o}}$ are the strengths of the container materials.

3.6 Engine Fairing Mass

The mass of the engine fairing (cylindrical compartment that houses the engine) is

$$m_{\mathrm{fair}} = \pi\, D_{\mathrm{fair}}\, L_{\mathrm{fair}}\, \gamma_{\mathrm{fair}}\, n_{\mathrm{fair}} \,, \tag{66}$$

where D_{fair}, L_{fair}, and γ_{fair} denote, respectively, the diameter, the length, and the mass per unit area of the engine fairing. n_{fair} is the number of fairings.

The total length of the engine fairing is essentially the length of the thrust chamber. Therefore, it can be estimated by the sum of the lengths of combustion chamber and nozzle (Garanin, 1996):

$$L_{\mathrm{fair}} = \bar{L}_{\mathrm{cc}}\, R_{\mathrm{t}} + \bar{L}_{\mathrm{n}}\, R_{\mathrm{t}} \,. \tag{67}$$

The length of the combustion chamber is estimated by (Garanin, 1996):

$$\bar{L}_{\mathrm{cc}} = 2\sqrt{\bar{R}_{\mathrm{c}}^2 - 1} + \frac{L^*}{R_{\mathrm{t}}\, \bar{R}_{\mathrm{c}}^2} - \frac{6.8\,(\bar{R}_{\mathrm{c}} - 1) + 2.1\,(\bar{R}_{\mathrm{c}} - 1)^3}{\bar{R}_{\mathrm{c}}^2} \,, \tag{68}$$

where $\bar{R}_{\mathrm{c}} \equiv R_{\mathrm{c}}/R_{\mathrm{t}}$, which, in a first estimate could be taken as $\bar{R}_{\mathrm{c}} = \sqrt{6}$ (Garanin, 1996) or estimated from the following expression for the chamber contraction ratio (Kozlov et al., 1988):

$$\frac{A_{\mathrm{c}}}{A_{\mathrm{t}}} = \frac{5000}{\sqrt{2 R_{\mathrm{t}} p_{\mathrm{c}}}} \,. \tag{69}$$

The nozzle length is given by

$$\bar{L}_{\mathrm{n}} = \bar{L}_{\mathrm{no}} \left\{ 1 - \left[1.15 + \frac{(20\, n - 17)(r - 1)}{1000\, z} \right] f_1(z) \right\} \,, \tag{70}$$

where

$$\bar{L}_{\mathrm{no}} = 3.2\, r^{(0.829 + 0.298 n^2)} \,, \tag{71}$$

$$f_1(z) = 1 - \exp\left[-(1 - z)^{0.4} \right] \,. \tag{72}$$

For using of above formulae, n, R_{t}, $r = R_{\mathrm{e}}/R_{\mathrm{t}} = \sqrt{A_{\mathrm{e}}/A_{\mathrm{t}}}$ and z are supposed known (they were previously calculated).

4 Application Results

The second stage of the Zenit launch vehicle (Ukraine/Russia) will be used in order to illustrate the applicability of the discussed method to the conceptual design.

The Zenit medium capacity launcher is a two-stage liquid-fueled vehicle. First stage has one RD-170 booster engine (one turbopump with four separate combustion chambers) burning LOX/kerosene fed from first stage tanks, generating 7,262 kN of thrust. Second stage has one NPO Energomash RD-120 fixed chamber sustainer engine plus four gimballed NPO Yuzhnoye RD-8 vernier (one turbopump with four separate combustion chambers) for steering, burning LOX/kerosene fed from second stage tanks, generating a total of 912 kN of vacuum thrust (FAS, 2003; Braeuning, 2003).

The main specifications of the Zenit second stage (SL-16/J-1) and of the RD-120 engine are given in tables 1 and 2, respectively, together with the calculated/assumed data, for comparison.

In general, as shown in Tab. 1 and 2, the estimated parameters are in very good agreement with the real values. The only sensible discrepancy was the rotational speed of the main turbopump, since the estimated value was 24,564 rpm, while the real (or, at least, the reference) value is 19,000 rpm. The true reasons for this miss-estimation are being analyzed, and will not be presented here, due to lack of space.

The estimated engine length (3.5 m) was less than the real value (3.9 m). This was expected, since the method takes as the engine length only the sum of the combustion chamber length with the nozzle divergent part length, without including the injector head and the main propellant valves.

The calculated total dry mass of the stage (8,403 kg) was slightly larger than the real value (8,300 kg). One possible reason for this overestimation could be the fact that it was used a higher propellant mass in the calculations (86,400 kg versus 80,600 kg). A larger propellant mass increases the sizes of propellant tanks and pressurization system, resulting in a increased dry mass of the overall propulsion system. This larger propellant mass also explains, at least in part, the larger overall stage length.

Table 1: Comparisons for the second stage of the Zenit launcher (SL-16/J-1).

Stage Parameter	**Real Value**	**Estimated Value**
Overall length	11.50 m	13.14 m
Diameter	3.9 m	3.9 m
Dry mass	8,300 kg	8,403 kg
Oxidizer	Liquid Oxygen (LOX)	Liquid Oxygen (LOX)
Fuel	Kerosene (T-1)	Kerosene (RP-1)
Propellant mass	80,600 kg	86,400 kg
Thrust main engine (vac)	833.5 kN	853.2 kN
Thrust verniers (vac)	78.4 kN	78.5 kN
Thrust total (vac)	911.9 kN	931.7 kN
Total Mass Flow Rate	270.8 kg/s	271.6 kg/s
Burn time (main)	about 300 s	about 318 s
Burn time verniers	up to 1,100 s	

Table 2: Comparisons for the RD-120 engine.

Engine Parameter	**Real Value**	**Estimated Value**
Oxidizer	Liquid Oxygen (LOX)	Liquid Oxygen (LOX)
Fuel	Kerosene (T-1)	Kerosene (RP-1)
Engine Cycle	Staged Combustion	Staged Combustion
Average Thrust	830.28 kN vac	853.2 kN vac
Chamber Pressure (p_c)	162.72 bar	162.72 bar
Mixture Ratio	2.6	2.6
Specific Impulse vac	3433.5 N.s/kg	3430.8 N.s/kg
Nozzle Area Ratio	106	106
Propellant Mass Flow Rate	241.8 kg/s	243.6 kg/s
Engine Mass	1,124 kg (dry)	1,135 kg (dry)
Engine Thrust to Weight	75	76.6
Throttle Range (% p_c)	85 – 100%	85 – 100 %
Restart Capability	No	No
Engine Length	3.9 m	3.5 m
Engine Diameter	2.0 m	2.0 m
Main turbopump speed	19,000 rpm	24,564 rpm
Gas Generator Temperature	735 K	735 K

5 Final Remarks

It was briefly described a method useful for parametric design and analysis of rocket stages based on bipropellant, liquid rocket engines. The computational tool developed based on this method could be used for preliminary screening and selection of the propulsion type and cycle, as well as for engine parametric optimization. In addition, the tool permits to the user select, among the available engines, one that meets the design requirements.

Although the program is in initial phase of development, the presented results indicated that, in general, good estimates for the staged combustion systems could be obtained with the method implemented here. The next step is to investigate the behavior of the program when it is applied to other types of engines.

Some interesting improvements could be easily done in the program. One is the introduction of better configuration module, which would permit improved estimates of tank mass, stage length, etc. Another amelioration could result from the introduction of a library of curves for engine performance parameters for propellant pairs such LOX/LH_2, LOX/CH_4, LOX/Propane, HTP/Kerosene, and HTP/Ethanol.

The final goal of this work is the creation of a integrated environment for sizing, synthesis and optimization of a complete launcher at the conceptual level. Among the next improvement steps, one could mention the inclusion of performance optimization, i.e., optimization of the flight-path for a given vehicle configuration. Note that the current version of the given method only treats of a single stage at a time. So, it is still necessary to include modifications that could propitiate the synthesis of a complete vehicle in a unique run of the program that implements such method.

6 References

Braeuning, Robert A., 2003, *Rocket and Space Technology. Space Launchers – Zenit*. URL: http://users.commkey.net/Braeunig/space/specs/zenit.htm. (Site last accessed in 04/10/2003.)

Program CETPC, Version 2.0, 1994, (See *CET93 and CETPC: An Interim Updated Version of the NASA Lewis Computer Program for Calculating Complex Chemical Equilibria With Applications*. NASA TM-4557, 1994.)

Federation of American Scientists, 2003, *World Space Guide – Zenit*. URL: http://www.fas.org/spp/guide/ukraine/launch/zenit.htm. (Site last accessed in 04/10/2003.)

Garanin, I.; Samokhin, V., 1993, Engine's Parameters Complex Optimization in Aerospace Systems. *44th Congress of the International Astronautical Federation*, Graz, Austria.

Garanin, I., 1996, *Preliminary Analysis of Launcher Propulsion System*. Moscow Aviation Institute, Moscow, October.

Goertz, C., 1995, A Modular Method for the Analysis of Liquid Rocket Engine Cycles. AIAA 95-2966, *31st AIAA/ASME/SAE/ASEEE Joint Propulsion Conference and Exhibit*, July 10-12, San Diego, CA.

Kozlov, A. A.; Novikov, V. N., Soloviov, E. V., 1988, *Sistemy Pitania i Upravlenia JRD Ustanovok* (Feed and Control Systems of LRE Propulsion Units), Mashinostroenie, Moscow. (In Russian.)

Kozlov, A. A., 1993, The Chosing Scheme, Oxidizer and Fuel Components and Fundamental Parameters of LRE Devices on Early Stages of Design Work by Means of Interactive System FORDU. *Third China-Russian Scientific Conference on Aeroengines*, Beijing, China.

Kozlov, A. A.; Hinckel, J. N.; Koreeda, J.; Comiran, A., 1999, The Application of Tripropellant Engines on the First Stage of a Carrier Rocket. *Journal of Propulsion and Power*, Vol. 15, No. 2, pag. 304–309.

Manski, D.; Goertz, C.; Sassnick, H-D.; Hulka, J. R.; Goracke, B. D.; Levack, D. J. H., 1998, Cycles for Earth-to-Orbit Propulsion. *Journal of Propulsion and Power*, Vol. 14, No. 5, pag. 588–604, Sep.-Oct.

Space and Tech Database, 2003, *Rocket Engines: RD-120 Summary and Specifications*. URL: http://www.spaceandtech.com. (Site last accessed in 04/10/2003.)

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