FATIGUE ENDURANCE CRITERION FOR HARD METALS: IMPROVEMENTE UPON A MODEL FOR MULTIAXIAL LOADING CONDITIONS

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Abstract: *In this paper, a new multiaxial high-cycle fatigue endurance criterion is proposed. It considers, as measures of fatigue solicitation: (i) a new function of the shear stress amplitude, which is capable to account for the nonproportional character of the loading history in a very simple manner and (ii) the maximum principal stress along the stress history, rather than the maximum hydrostatic stress usually considered as a measure of solicitation upon embryocracks. Assessment of the resulting criterion shows that it compares very well with experimental data published in the literature.*

Keywords: *multiaxial high-cyle fatigue, fatigue endurance, nonproportional loading.*

1. Introduction

In many practical situations, mechanical components are subjected to complex cycling multiaxial nonproportional loading histories under service conditions. Although experimental testing is of fundamental importance for the product development, it should be kept to a minimum due since it is costly and lengthy. The development of new, more accurate and if possible computationally cheaper multiaxial fatigue models capable to predict the durability of these structures is therefore extremely desirable. Most of the criteria for multiaxial fatigue can be classified into one the following methodologies: a) Empirical, b) Stress Invariants, c) Critical Plane, d) Strain Energy, e) Combined Energy/Critical Plane and f) Mesoscopic. Extensive experimental studies conducted by Gough et al. (1951) and Nishihara and Kawamoto (1945) provided the basis for the development of empirical criteria, which aimed to predict the fatigue strength of metals under multiaxial cyclic loading. Sines (1959) identified the effect of static superimposed stresses on the permissible amplitude of alternating stresses and proposed a multiaxial fatigue criterion based on stress invariants. Crossland (1956) developed a model that considers the maximum value of the hydrostatic pressure rather than its mean value as initially suggested by Sines. More recently, Deperrois (1991) and Bin Li et al. (2000) presented stress invariant based criteria which provided better results than those obtained by Sines and Crossland. Considering that fatigue models could be explored from a different scale, Dang Van (1973) (see also Dang Van & Papadopoulos (1987) and Papadopoulos et al. (1997)) proposed the so called mesoscopic criteria. This approach assumes that a localized plastic deformation of an unfavourably oriented grain precedes fatigue damage. Thus, local microscopic stresses (which can be written as a function of macroscopic stresses) should be used to define a crack initiation criterion. More recently, Zouain & Cruz (2002) proposed a stress based shakedown model for High Cycle Fatigue (HCF), which yielded even better predictions of fatigue strength.

In this paper, a new multiaxial high-cycle fatigue endurance criterion is proposed. It considers, as measures of fatigue solicitation: (i) a new function of the shear stress amplitude, which is capable to account for the nonproportional character of the loading history in a very simple manner and (ii) the maximum principal stress along the stress history, rather than the maximum hydrostatic stress usually considered as a measure of solicitation upon embryocracks. We claim that geometric quantities associated with the elliptic hull containing the stress path (after projection onto the deviatoric stress subspace) gives the correct measure for the shear solicitation to fatigue. A simple theoretical result presented in this paper allows us to compute such quantities in a very simple way. Further, the hydrostatic stress is basically the quantity obtained by averaging the normal stress over all the planes passing through a givem material point. We claim that the worst situation — corresponding to an eventual embryocrack oriented orthogonally to the maximum principal stress — instead of the average one should be taken into account when describing the contribution of the normal stress to the fatigue damage. Thus, the maximum principal stress is considered instead of the maximum hydrostatic stress. Assessment of the resulting criterion shows that it compares very well with experimental data published in the literature. Further, it is very simple to implement, making it very competitive with respect to those proposed, for instance, by Papadopoulos or by Zouain & Cruz.

The paper is organized as follows: the fatigue model is presented in section 2. Measures for shear and normal solicitations to fatigue are proposed in sections 2.1 and 2.2, respectively. The resulting criterion is assessed in section 3. Some concluding remarks are presented in section 4.

2. The fatigue model

Mechanical degradation due to fatigue under high number of loading cycles takes place at stress levels well below the yield limit. According to pioneering studies conducted by Ewin & Rosenhain (1900), high cycle fatigue damage can be associated with cyclic plastic deformations at the grain level, leading to the formation of persistent slip bands and later to the nucleation of microcracks, even if the material shows an essentially elastic behaviour at macroscopic level. On the other hand, if the material point manages to attain cyclic elastic behaviour at grain level, eventually after a number of initially plastic cyclic deformations, then fatigue failure is not expected to occur. Thus, since plasticity plays an important role on crack initiation, shear stresses must be considered as one of the driving forces of the fatigue process. Another varible that must be considered is the normal stress acting upon embryo-cracks, which has been shown by Sines (1959) to affect the fatigue resistance. Its influence has been taken into account by many authors through an average of the normal stress acting upon all the planes passing through the material point. As remarked by Papadopoulos (1997) such average is equal to the hydrostatic stress. Under such assumptions, many fatigue limit criteria can be written as:

$$f(\tau) + g(\sigma) \leq 0, \tag{1}$$

where f and g are functions of the shear stress τ and the normal stress σ, respectively. For instance, the criterion proposed by Crossland (1956) can be written as:

$$\tau_{eq} + a\, p_{max} \leq b, \tag{2}$$

where $\tau_{eq} := \sqrt{J_{2,a}}$ is the J_2 measure of the amplitude of the deviatoric stress $\mathbf{S} := \boldsymbol{\sigma} - \frac{1}{3}\,(tr\,\boldsymbol{\sigma})\,\mathbf{I}$ and p_{max} is the maximum value of the hydrostatic stress observed along the stress history, while a and b are material parameters. The Dang Van (1973) criterion is based on a mesoscopic scale approach and can be written as:

$$\tau(t) + a\, p(t) \leq b, \tag{3}$$

where $\tau(t) := \frac{1}{2}\,|s_{p\,max}(t) - s_{p\,min}(t)|$ is half the difference between the maximum and the minimum principal deviatoric stresses (measured at the grain level) at time instant t. The criterion proposed by Papadopoulos relies on the argument that the accumulated plastic deformations at mesoscopic level, at each slip plane, are proportional to the resolved shear stress amplitude T_a. An average of this quantity within an elementary volume is given by:

$$\sqrt{< T_a^2 >} = \sqrt{5}\,\sqrt{\frac{1}{8\pi^2}\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{\psi=0}^{2\pi}(T_a(\varphi,\theta,\psi)^2\,d\psi\,\sin(\theta)\,d\theta\,d\varphi}. \tag{4}$$

The angle ψ covers all the gliding directions, while φ and θ define the orientation of the material plane inside the elementary volume. The resulting criterion can be expressed as:

$$\sqrt{< T_a^2 >} + a\, p_{max} \leq b, \tag{5}$$

where, once again, p_{max} is the maximum value of the hydrostatic stress observed along the stress history, while a and b are material parameters.

The criterion proposed by Papadopoulos provides very good results when compared with experimental results for a wide range of materials and loading conditions. On the other hand, an disadvantage associated with these criteria is the fact that they require quite lengthy and complicated calculations. Here we present an alternative fatigue endurance criterion based on new definitions for functions $f(\tau)$ and $g(\sigma)$ in expression (1).

2.1. The equivalent shear stress amplitude

The Crossland criterion considers, as a measure of the shear solicitation to fatigue along the loading history, the $\sqrt{J_2}$ radius of the sphere circumscribing the stress path (after projection onto the deviatoric space). As illustrated in Fig. (1), proportional and nonproportional paths can be circumscribed by the same sphere although a more severe solicitation is expected when the nonproportional stress history is considered.

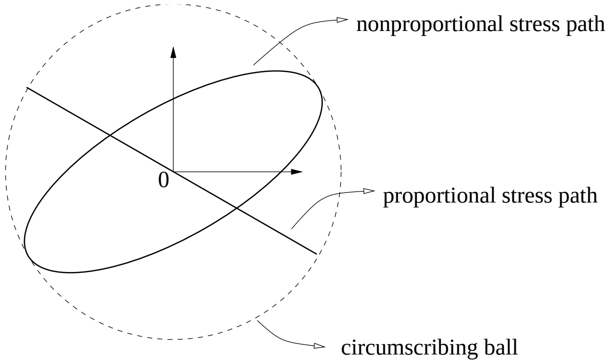

Figure 1. Proportional and nonproportional stress paths associated with the same amplitude $\sqrt{J_{2,a}}$.

As an alternative, one could consider a quantity associated with the minimum ellipsoid circumscribing the stress path, as previously suggested by Deperrois (1991), and later by Bin Li et al. (2000): the basic idea is to consider shear stress amplitudes in several ortogonal directions, summing up their effects to provide a measure of the shear fatigue solicitation. Deperrois proposes, as the equivalent shear stress, the expression:

$$f(\tau) := \frac{1}{2\sqrt{2}}\sqrt{\sum_{i=1}^{5} D_i^2}, \tag{6}$$

where $D_i,\ i = 1, \ldots, 5$ are computed as follows: first, the longest chord D_5 between two distinct points of the stress path in the deviatoric space is determined; next, the stress path is projected into a subspace orthogonal to such chord; a new longest chord D_4 is computed in this subspace, and the process is repeated successively for the remaining dimensions. As remarked by Papadopoulos (1997), in some situations the Deperrois criterion shows a lack of uniqueness of the longest chord, making the definition of the orthogonal subspace an ill posed problem. Another approach is given by Freitas and his collaborators (Bin Li et al. (2000)): the equivalent shear stress is defined as the square root of the sum of the squared semi-axes of the minimum cricumscribing ellipsoid:

$$f(\tau) := \sqrt{R_a^2 + R_b^2}. \tag{7}$$

As described by Bin Li et al., the proposed approach requires two steps to define the minimum circumscribing ellipsoid. First, the minimum circumscribing circle with radius R_a is constructed, according to the minimum circumscribing hypersphere approach. Then, the second semi-axis R_b of the minimum circumscribing ellipsoid is obtained, with the radius R_a as the major ellipsoid semi-axis. Although this measure of shear solicitation is appealing and, for many cases, provides very good results, it contains a drawback. For the sake of illustration, consider the two triangular stress paths A and B shown in Fig. (2.a).

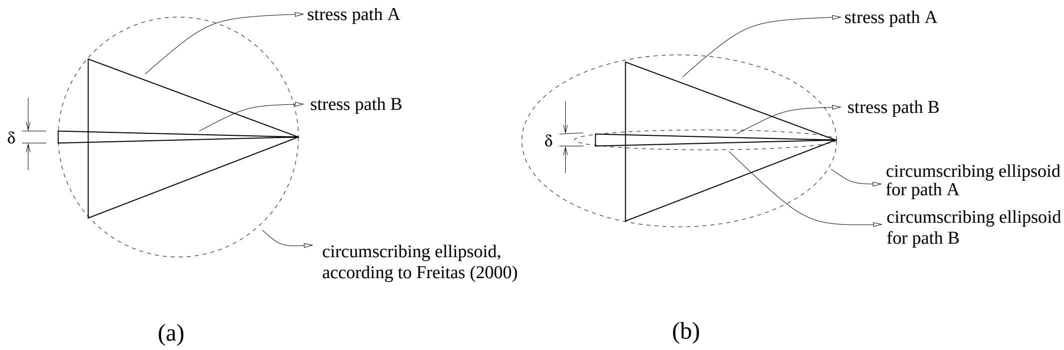

Figure 2. (a) Drawback in the criterion proposed by Freitas and (b) correct circumscribing ellipsoids.

In both cases, due to the fact that R_a is computed as the radius of the minimum circumscribing hypersphere, the methodology proposed by Bin Li et al. defines the same *"minimum ellipsoid"* for both stress paths A and B. This is true even when the length δ of the stress path B tends to zero. This would lead to the same measure of shear solicitation even for the limiting affine case. Figure (2.b) sketches what would be the minima circumscribing ellipsoids for the stress paths A and B. Notice that the semi-axis R_b decreases with the length δ, but this is not taken into account by the methodology proposed by Bin Li et al. Furthermore, quite ellaborate algorithms are required in order to compute the semi-axes $R_i,\ i = 1, \ldots, 5$ of the ellipsoid.

We acknowledge that the concept of minimum circumscribing ellipsoid is an appropriate measure of the equivalent shear stress. The concern here is restricted to the definition of the elliptic hull and to the methodology proposed in order to obtain the geometrical characteristics of the ellipsoid. In this setting, we embrace the idea that the minimum circumscribing ellipsoid contains the information required to characterize the shear solicitation. Thus, we propose as the measure $f(\tau)$ of shear solicitation to fatigue the expression:

$$f(\tau) := \sqrt{\sum_{i=1}^{5} \lambda_i^2}, \tag{8}$$

where $\lambda_i,\ i = 1, \ldots, 5$ are the semi-axes of the ellipsoid circumscribing the stress path (in the deviatoric space). In general, however, such ellipsoid and hence its semi-axes are difficult to determine. The result presented in what follows enable us to derive a new expression for $f(\tau)$ which can be almost trivially computed.

2.1.1. Invariance of the prismatic hull

Let Dev^3 denote the space of symmetric deviatoric tensors from $\mathbb{R}^3$ to $\mathbb{R}^3$ and let $\{\mathbf{N}_i,\ i = 1, \ldots, 5\}$ be an arbitrarily chosen orthonormal basis for such space. Any deviatoric stress state $\mathbf{S}(t)$ can be written as:

$$\mathbf{S}(t) = \sum_{i-1}^{5} s_i(t)\, \mathbf{N}_i. \tag{9}$$

If the basis of Dev^3 is given, for instance, by:

$$\mathbf{N}_1 = \begin{pmatrix} \frac{2}{\sqrt{6}} & 0 & 0 \\ 0 & \frac{-1}{\sqrt{6}} & 0 \\ 0 & 0 & \frac{-1}{\sqrt{6}} \end{pmatrix}, \quad \mathbf{N}_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & \frac{-1}{\sqrt{2}} \end{pmatrix},$$
$$\mathbf{N}_3 = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{N}_4 = \begin{pmatrix} 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix}, \quad \mathbf{N}_5 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} \\ 0 & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}, \tag{10}$$

then the components $s_i(t)$ of $\mathbf{S}(t)$ in this basis can be expressed as:

$$s_1(t) = \sqrt{\frac{3}{2}}\, S_{xx}(t), \quad s_2(t) = \frac{1}{\sqrt{2}}\, (S_{yy}(t) - S_{zz}(t)),$$
$$s_3(t) = \sqrt{2}\, S_{xy}(t), \quad s_4(t) = \sqrt{2}\, S_{xz}(t), \quad s_5(t) = \sqrt{2}\, S_{yz}(t); \tag{11}$$

From (8), it is possible to describe the stress path in terms of a curve in $\mathbb{R}^5$, where each point $\mathbf{s}(t) \in \mathbb{R}^5$ can be expressed as:

$$\mathbf{s}(t) := (s_1(t)\ s_2(t)\ \ldots s_5(t))^T. \tag{12}$$

Let the set of all points $\mathbf{s}(t)$ describing the path of deviatoric stresses in $\mathbb{R}^5$ be represented by the symbol Δ. The result presented below allows us to compute the equivalent shear solicitation to fatigue in an almost trivial way:

Proposition 1 *Given an ellipsoid $\mathcal{E}$ in $\mathbb{R}^m$ with centre located at the origin and an arbitrary orthonormal basis $\{\mathbf{n}_i,\ i = 1, \ldots, m\}$ of $\mathbb{R}^m$, let $\mathcal{P}$ be a rectangular prism circumscribing $\mathcal{E}$ such that its faces are orthogonal to each one of the basis elements. If $\lambda_i,\ i = 1, \ldots, m$ are the magnitudes of the principal semi-axes of $\mathcal{E}$ and $a_i,\ i = 1, \ldots, m$ denote the distances of the centre of the ellipsoid to the faces of the rectangular prism, then:*

$$\sum_{i=1}^{5} \lambda_i^2 = \sum_{i=1}^{5} a_i^2. \tag{13}$$

Proof: Let $\mathbb{S}_1^m$ be the unit sphere in $\mathbb{R}^m$:

$$\mathbb{S}_1^m := \{\mathbf{y} \in \mathbb{R}^m;\ ||\mathbf{y}|| = 1\}, \tag{14}$$

where $||\mathbf{y}|| := \left(y_1^2 + y_2^2 + \ldots + y_m^2\right)^{1/2}$ is the classical Euclidean norm in $\mathbb{R}^m$. The ellipsoid $\mathcal{E}$ can be characterized as the set of points:

$$\mathcal{E} := \{\mathbf{x} \in \mathbb{R}^m;\ \ \mathbf{x} = \mathbf{L}\,\mathbf{y},\ \mathbf{y} \in \mathbb{S}_1^m,\},\tag{15}$$

where $\mathbf{L} : \mathbb{R}^m \to \mathbb{R}^m$ is a symmetric, positive semi-definite matrix with eigenvalues given by the magnitudes λ_i, $i = 1, \ldots, m$ of the semi-axes of $\mathcal{E}$. On the other hand, the distance a_i, from the faces of the rectangular prism orthogonal to a basis element $\mathbf{n}_i$ to the centre of the ellipsoid, can be expressed as:

$$a_i = \sup_{\mathbf{x} \in \mathcal{E}} (\mathbf{x}, \mathbf{n}_i), \quad , i = 1, \ldots, m. \tag{16}$$

where $(\mathbf{x}, \mathbf{y}) = \sum_{i=1}^m x_i \, y_i$ denotes the classical Euclidean inner product in $\mathbb{R}^m$. By considering the fact that the points $\mathbf{x}$ from the ellipsoid $\mathcal{E}$ satisfy (14), we can develop (15) as:

$$a_i = \sup_{\mathbf{x} \in \mathcal{E}} (\mathbf{x}, \mathbf{n}_i) = \sup_{\mathbf{y} \in \mathbb{S}_1^m} (\mathbf{L}\,\mathbf{y}, \mathbf{n}_i) = \sup_{\mathbf{y} \in \mathbb{S}_1^m} (\mathbf{y}, \mathbf{L}\,\mathbf{n}_i) = ||\mathbf{L}\,\mathbf{n}_i||, \tag{17}$$

since the supremum of $(\mathbf{y}, \mathbf{L}\,\mathbf{n}_i)$ among the points $\mathbf{y}$ from $\mathbb{S}_1^m$ is attained for $\mathbf{y}$ parallel to $\mathbf{L}\,\mathbf{n}_i$. Now, let us represent the identity operator on $\mathbb{R}^m$ as:

$$\mathbf{I} = \sum_{i-1}^{m} \mathbf{n}_i \otimes \mathbf{n}_i, \tag{18}$$

where $\otimes$ denotes the tensor product operator such that $(\mathbf{a} \otimes \mathbf{b})\,\mathbf{u} = (\mathbf{a}, \mathbf{u})\,\mathbf{b}$. It follows that:

$$\mathbf{L}^2 = \mathbf{L} \left(\sum_{i-1}^{m} \mathbf{n}_i \otimes \mathbf{n}_i \right) \mathbf{L} = \sum_{i-1}^{m} \mathbf{L}\,\mathbf{n}_i \otimes \mathbf{L}\,\mathbf{n}_i \tag{19}$$

Finally, since the Frobenius norm of the linear operator $\mathbf{L}$ is given by $||\mathbf{L}||_F = (\sum_{i=1}^m \lambda_i^2)^{1/2}$, from (16) and (18) we obtain:

$$\sum_{i=1}^{m} \lambda_i^2 = ||\mathbf{L}||_F^2 = tr\,(\mathbf{L}^2) = tr \left(\sum_{i-1}^{m} \mathbf{L}\,\mathbf{n}_i \otimes \mathbf{L}\,\mathbf{n}_i \right) = \sum_{i-1}^{m} (\mathbf{L}\,\mathbf{n}_i, \mathbf{L}\,\mathbf{n}_i) = \sum_{i-1}^{m} ||\mathbf{L}\,\mathbf{n}_i||^2 = \sum_{i-1}^{m} a_i^2. \tag{20}$$

The aforementioned statement is of fundamental importance for the computation of the measure $f(\tau)$ of shear solicitation to fatigue since it precludes the need to determine the principal semi-axes of the ellipsoid. More specifically, whenever the ellipsoid is a good approximation for the convex hull of the stress path $\boldsymbol{\Delta}$, the shear stress amplitude $f(\tau)$ can be simply computed as:

$$f(\tau) := \sqrt{\sum_{i=1}^{5} a_i^2}, \tag{21}$$

where, in the context of the present study, a_i, $i = 1, \ldots, 5$ are the *amplitudes of the components* $s_i(t)$ *of the deviatoric stresses* defined as:

$$a_i := \max_t |s_i(t)|, \; i = 1, \ldots, 5. \tag{22}$$

The procedure for computation of $f(\tau)$ can be summarized as follows:

- For each time instant t, given the Cauchy stress tensor $\boldsymbol{\sigma}(t)$, compute the corresponding deviatoric stress state:

$$\mathbf{S}(t) = \boldsymbol{\sigma}(t) - \frac{1}{3}(tr\,\boldsymbol{\sigma}(t))\,\mathbf{I}; \tag{23}$$

- For each time instant t, determine the components of the deviatoric stress tensor $\mathbf{S}(t)$ in terms of an arbitrarily chosen orthonormal basis $\mathbf{N}_i$, $i = 1, \ldots, 5$:

$$s_i(t) = (\mathbf{S}(t), \mathbf{N}_i)\,; \tag{24}$$

- Compute the amplitudes of the deviatoric stresses a_i, $i = 1, \ldots, 5$ as:

$$a_i := \frac{1}{2} \left(\max_t s_i(t) - \min_t s_i(t) \right), \; i = 1, \ldots, 5. \tag{25}$$

- Compute the shear solicitation to fatigue $f(\tau)$ as:

$$f(\tau) := \sqrt{\sum_{i=1}^{5} a_i^2}. \tag{26}$$

2.2. The normal stress

Tensile normal stresses contribute to the fatigue degradation by acting (essentially in mode 1) upon eventually existing embryocracks in the material.

Many fatigue endurance criteria consider the hydrostatic stress as the measure of the solicitation to fatigue produced by the normal stresses since, as remarked by Papadopoulos (1997), the hydrostatic stress is basically the quantity obtained by averaging the normal stress over all the planes passing through a givem material point.

In this paper, we claim that the worst situation — which corresponds to considering the existence of an embryocrack oriented orthogonally to the maximum principal stress (among the three eigenvalues of the stress tensor and along all the stress path) — should be considered rather than the average solicitation given by the maximum hydrostatic stress.

2.3. The resulting endurance criterion

Based on the considerations developed along sections 2.2 and 2.3, we propose the following multiaxial hych cycle fatigue endurance criterion:

$$\sqrt{\sum_{i=1}^{5} a_i^2} + \kappa\,\sigma_{p\,max} \leq \lambda, \tag{27}$$

where $a_i,\ i = 1, \ldots, 5$ are defined as in (22) and $\sigma_{p\,max}$ is the maximum principal stress among acting upon the material point along the loading history, while κ and λ are material parameters. If f_1 and t_1 are the fatigue endurance limits under alternate bending and alternate torsion solicitations, respectively, then the parameters κ and λ can be computed as:

$$\kappa = \frac{\sqrt{2}}{f_{-1} - t_{-1}} \left(t_{-1} - \frac{f_{-1}}{\sqrt{3}} \right) \qquad \text{and} \qquad \lambda = \sqrt{2}\,\frac{t_{-1} f_{-1}}{f_{-1} - t_{-1}} \left(1 - \frac{1}{\sqrt{3}} \right). \tag{28}$$

3. Assessment of the criterion

Proportional and out-of-phase multiaxial fatigue experiments for a number of different materials were considered to assess the proposed criterion in predicting fatigue strength under a high number of cycles. The data collected are reported in Tables 1 to 4 and correspond to experiments on hard metals ($1,3 \leq f_{-1}/t_{-1} < \sqrt{3}$) involving biaxial stress states, where f_{-1} and t_{-1} are the fatigue limits under fully reversed bending and torsion, respectively. Data came from publications by Nishihara and Kawamoto (1945) (Table 1), Heidenreich et al. (1983) (Table 2), Lempp (1977) (Table 3) and Froustey and Lassere (1989) (Table 4). The following nomenclature was adopted in these Tables: the subscript a stands for the amplitude of stresses while m represents the mean value. As usual, σ and τ are normal and shear stresses while β contains information concerning phase difference. The stress values reported in each table correspond to the maximum combination of stresses that the specimen can stand without failing, up to a limit of 10^6 cycles.

To assess the quality of the results provided by our model, an error index I is defined as:

$$I = \frac{1}{\lambda} \left(\sqrt{\sum_{i=1}^{5} a_i^2} + \kappa\,\sigma_{p\,max} - \lambda \right) \times 100 \quad (\%), \tag{29}$$

which gives a measure of how close the prediction of the criterion is with respect to the experimental data. A negative I yields a non-conservative fatigue strength prediction since it indicates that the stress solicitation has not attained a critical value while the experimental data are representative of limiting situations. On the other hand, a positive I provides a conservative estimate while $I = 0$ means a perfect prediction for the observed fatigue strength.

Table 1 reports experimental data under in-phase and out-of-phase alternated bending and torsion conditions. Analysis of these data revealed that all the criteria considered show, in general, satisfactory predictions of fatigue strength, regardless of the phase angle. Exception was observed for the Crossland criterion in experiments 1-7 and 1-8, where the calculated error index were respectively -8.35% and -17.81%.

Results 2-1 to 2-6 from Table 2 are also associated with in-phase and out-of-phase alternated loadings producing normal and shear stresses. In this set of data, the Crossland criterion yielded quite poor predictions under out-of-phase loadings, while the other criteria rendered excellent results. Experiments 2-7 to 2-9 were carried out under alternated bending and repeated torsion, while experiments 2-10 to 2-12 considered repeated bending and alternated torsion. Under the presence of mean stresses, the proposed model produced more conservative results when compared with the remaining criteria. The same trend can be observed in the results reported in Tables 3 and 4. This fact is in agreement with the hypothesis that the worst situation — which corresponds to considering the existence of an embryocrack oriented orthogonally to the maximum principal stress (among the three eigenvalues of the stress tensor and along all the stress path) — should be considered rather than the average solicitation given by the maximum hydrostatic stress. In summary, application of our model to the experimental data provided an error index which varied in the worst cases between −8.74% and 15.34% for all materials and loading coditions analysed. The results provided by both Papadopoulos (1997) and by Mamiya & Araújo (2002) varied between −15.3% and 7.3% while the Crossland criterion provided significantly poorer predictions. In our model, a shift of the error index towards the conservative region can be clearly observed whenever a mean stress is present in the loading history.

Table 1 – Fatigue strength of hard steel (t_{-1}=196.2 MPa,f_{-1}=313.9 MPa): experimental data (Nishihara & Kawamoto (1945)) and predictions.

	σ_a(MPa)	σ_m(MPa)	τ_a(MPa)	τ_m(MPa)	β(°)	I^a(%)	I^b(%)	I^c(%)	I^d(%)
1-1	138.1	0	167.1	0	0	-2.27	-2.3	-2.28	-1.91
1-2	140.4	0	169.9	0	30	-2.60	-0.6	-0.64	-0.27
1-3	145.7	0	176.3	0	60	-3.61	3.1	3.10	3.49
1-4	150.2	0	181.7	0	90	-3.74	6.3	6.27	6.66
1-5	245.3	0	122.6	0	0	1.44	1.5	1.44	1.73
1-6	249.7	0	124.8	0	30	0.01	3.3	3.26	3.55
1-7	252.4	0	126.2	0	60	-8.35	4.4	4.39	4.69
1-8	258.0	0	129.0	0	90	-17.81	6.5	6.70	7.01
1-9	299.1	0	62.8	0	0	0.92	0.9	0.92	1.02
1-10	304.5	0	63.9	0	90	-2,99	2.7	2.74	2.83

[a] Crossland, [b] Papadopoulos, [c] Mamiya & Araújo, [d] Current model

Table 2 – Fatigue strength of 34Cr4 (t_{-1}=256 MPa,f_{-1}=410 MPa) experimental data (Heidenreich et al. (1983)) and predictions.

	σ_a(MPa)	σ_m(MPa)	τ_a(MPa)	τ_m(MPa)	β(°)	I^a(%)	I^b(%)	I^c(%)	I^d(%)
2-1	314.0	0	157.0	0	0	-0.55	-0.6	-0.55	-0.27
2-2	315.0	0	158.0	0	60	-12.33	-0.1	-0.11	0.18
2-3	316.0	0	158.0	0	90	-22.93	0.1	0.08	0.37
2-4	315.0	0	158.0	0	120	-12.33	-0.1	-0.11	0.18
2-5	224.0	0	224.0	0	90	-8.38	5.2	5.15	5.55
2-6	380.0	0	95.0	0	90	-7.32	0.4	0.37	0.49
2-7	316.0	0	158.0	158.0	0	0.08	0.1	0.08	6.01
2-8	314.0	0	157.0	157.0	60	-12.69	-0.6	-0.54	5.34
2-9	315.0	0	158.0	158.0	90	-23.17	-0.1	-0.11	5.83
2-10	279.0	279.0	140.0	0	0	-6.38	-6.4	-6.38	-0.21
2-11	284.0	284.0	142.0	0	90	-25.5	-4.8	-4.83	1.45
2-12	212.0	212.0	212.0	0	90	-9.39	3.4	3.41	7.23

[a] Crossland, [b] Papadopoulos, [c] Mamiya & Araújo, [d] Current model

Table 3 – Fatigue strength of 42CrMo4 (t_1=260 MPa,f_1=398 MPa): experimental data (Lempp (1977)) and predictions.

	σ_a(MPa)	σ_m(MPa)	τ_a(MPa)	τ_m(MPa)	$\beta(°)$	I^a(%)	I^b(%)	I^c(%)	I^d(%)
3-1	328.0	0	157.0	0	0	4.19	4.2	4.19	4.63
3-2	286.0	0	137.0	0	90	-28.14	-8.8	-9.13	-8.74
3-3	233 0	0	224.0	0	0	7.30	7.3	7.3	7.94
3-4	213.0	0	205.0	0	90	-14.94	-1.8	-1.84	-1.25
3-5	266.0	0	128.0	128.0	0	-15.34	-15.0	-15.3	-7.80
3-6	283.0	0	1360	136.0	90	-28.89	-9.6	-9.97	-1.97
3-7	333.0	0	160.0	160.0	180	5.92	5.8	5.92	15.34
3-8	280.0	280.0	134.0	0	0	-2.89	-2.7	-2.89	7.04
3-9	271.0	271.0	130.0	0	90	-23.99	-5.8	-5.93	3.67

[a] Crossland, [b] Papdopoulos, [c] Mamiya & Araújo, [d] Current model

Table 4 – Fatigue strength of 30NCD16 (t_{-1}=410 MPa,f_{-1}=660 MPa): experimental data (Froustey & Lasserre (1989)) and predictions.

	σ_a(MPa)	σ_m(MPa)	τ_a(MPa)	τ_m(MPa)	$\beta(°)$	I^a(%)	I^b(%)	I^c(%)	I^d(%)
4-1	485.0	0	280.0	0	0	1.77	1.8	1.77	2.07
4-2	480.0	0	277.0	0	90	-27.27	0.7	0.70	1.00
4-3	480.0	300.0	277.0	0	0	3.91	3.9	3.91	7.63
4-4	480.0	300.0	277.0	0	45	-3.36	3.9	3.91	7.63
4-5	470.0	300.0	270.0	0	60	-10.93	1.6	1.60	5.32
4-6	473.0	300.0	273.0	0	90	-25.12	2.5	2.45	6.17
4-7	590.0	300.0	148.0	0	0	0.11	0.1	0.11	4.32
4-8	565.0	300.0	141.0	0	45	-7.23	-4.1	-4.07	0.14
4-9	540.0	300.0	135.0	0	90	-14.97	-8.1	-8.15	-3.94
4-10	211.0	300.0	365.0	0	0	-0.68	-0.7	-0.68	1.86

[a] Crossland, [b] Papadopoulos, [c] Mamiya & Araújo, [d] Current model

4. Conclusions

A new multiaxial fatigue criterion which is very simple to implement has been proposed. Application of this criterion to a broad range of in-phase and out-of-phase loading conditions involving four different materials under multiaxial, in-phase and out-of-phase states of stress yielded very good predictions of fatigue endurance. The proposed criterion always provided more conservative endurance estimates than all the other criteria considered in the present study, whenever shear or normal mean stresses were present in the loading history. On the other hand, when such mean stresses were absent, the predictions were essentially the same for all criteria with exception of Crossland. A very interesting feature of the proposed model which should be stressed is the great simplicity of implementation of our criterion.

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6. References

Bin Li, Santos, J. L. & Freitas M., 2000, "A unified numerical approach for multiaxial fatigue limit evaluation", Mech. Struct. & Mach., Vol. 28, pp. 85–103.

Crossland, B., 1956, "Effect of large hydrostatic pressures on the torsional fatigue strength of an alloy steel", Proc. Int. Conf. on Fatigue of Metals, IMechE, London, pp. 138–149.

Dang Van, K., 1973, "Sur la résistance à la fatigue des métaux", Sciences et techniques de l armement, Vol. 47, pp. 429–453.

Dang Van, K. & Papadopoulos, I. V., 1987, "Multiaxial fatigue failure criterion: a new approach", Proc. Third Int. Conf. on Fatigue and Fatigue Thresholds, Fatigue 87, Ritchie, R. O. & Starke Jr., E. A., eds., EMAS Warley, U.K., pp.997–1008.

Deperrois, A., 1991, “Sur le calcul des limites d’éndurance des aciers”, Thèse de Doctorat, Ecole Polytechnique, Paris.

Ewing, J. A., & Rosenhain, W., 1900, “Experiments in micro-metallurgy: effects of strain. Preliminary notice”, Philosophical Transactions of the Royal Society, London, Vol. A199, pp. 85–90.

Gough, H. J., Pollard, H. V. & Cleushaw, W. J., 1951, “Some experiments on the resistance of metals to fatigue under combined stress”, Memo 2522, Aeronautical Research Council, HMSO, London.

“Froustey, C. & Lasserre, S., 1989, Multiaxial fatigue endurance of 30NCD16 steel, Int. J. Fatigue, Vol. 11, pp. 169–175.

Heidenreich, R., Zenner, H. & Richter, I., 1983, “Dauerschwingfestigkeit bei mehrachsiger Beanspruchungi. Forschungshefte FKM, Heft 105.

Lempp, W., 1977, “FestigKeitsverhalten von Stählen bei mehrchsiger Dauerschwingbeanspruchung durch Normalspannungen mit überlagerten phasengleichen und phasenverschobenen Schubspannungen. Diss. Uni. Stuttgart.

Nishihara, T. & awamoto, M., 1945, “The strength of metals under combined alternating bending and torsion with phase difference”, Memoirs of the College of Engineering, Kyoto Imperial University, Vol. 11, pp. 85–112.

Papadopoulos, I. V., Davoli, P., Gorla, C., Filippini, M. & Bernasconi, A., 1997, “A comparative study of multiaxial high-cycle fatigue criteria for metals”, Int. J. Fatigue, Vol. 19, pp. 219–235.

Sines, G., 1959, “Behavior of metals under complex static and alternating stresses” in: Sines, G. & Waisman, eds.: Metal Fatigue, McGraw-Hill, New York, pp. 145–169.

Zouain, N. & Cruz, I., 2002, “A high-cycle fatigue criterion with internal variables”, European Journal of Mechanics A/Solids, Vol. 21, pp. 597–614.