AEROELASTIC FUNCTIONAL APPROXIMATION VIA AN ARTIFICIAL NEURAL NETWORK REDUCED-ORDER MODEL

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Abstract. Identification and prediction of aeroelastic behavior presents significant challenge for the analysis and control of adverse aeroelastic phenomena. Aeroelastic modeling requires information from both structural dynamics and unsteady aerodynamic behavior. The majority of methodologies available today are based on the decoupling of structural model from the unsteady aerodynamic model. Therefore, alternative methods are welcome in the research field of aeroelasticity. The multi-layer functional (MLF) allows a rigorous mathematical framework appropriate for aeroelastic modeling and can be realized by means of artificial neural networks. In principle, neural networks provide a systematic procedure to model experimentally acquired motion-induced aeroelastic responses, without suffering from limitations of decoupling the structural dynamics from unsteady aerodynamics. This work presents an identification procedure based on artificial neural networks to represent the motion-induced aeroelastic response functional. A wind tunnel aeroelastic wing model has been used to provide motion-induced aeroelastic responses. The wing has been fixed to a turntable, and an electrical motor provides the incidence motion to the wing. The neural network model has been trained using the Levenberg-Marquardt algorithm to emulate the aeroelastic response from experimental data. This aeroelastic functional representation is then tested for a range of the wind tunnel model operational boundaries. The results showed that the use of neural networks in the aeroelastic response identification is a promising alternative method, which allows fast evaluation of aeroelastic response model.

Keywords. Aeroelasticity, multi-layer functionals, neural networks, Levenberg-Marquardt training, semi-empirical methods.

1. Introduction

The methodologies for aeroelastic modeling are divided into two categories, classical and integrated. The classical methods combine the fluid and structural equations in an uncoupled fashion. They use some restrictions such as inviscid, incompressible, irrotational flow. The integrated methods attempt to solve the aeroelastic modeling problems gathering the structural and aerodynamic model by either exchanging information at each time step or by formulating the aeroelasticity problem in a single unified fluid-structure medium (Marshall and Imregun, 1996). In both cases, the final set of aeroelastic equations is generally complex, in especial for modeling unsteady aerodynamics. Methodologies to deal with unsteady aerodynamic modeling lies from the computationally demanding CFD methods to the semi-empirical ones. The last one comprises studies using ordinary differential equations approximation (Tran and Petot, 1981), functions fitting (Leishman and Beddoes,1986), and neural networks (Marques, 1997).

Unsteady aerodynamic models can be obtained from the principles of dynamic systems theory with mathematical laws so that the generalized aerodynamic force response can be represented as a non-linear functional of a generalized displacement history. The relevance of the functional concept to unsteady aerodynamics is evident in the context of modeling non-linear time-invariant hereditary systems. A coherent modeling approach towards a general non-linear unsteady aerodynamic response functional has been followed by Tobak and coworkers (Tobak and Pearson, 1964; Tobak and Schiff, 1981; Tobak and Chapman, 1985). Other functional forms have been successfully applied, for instance, the case of Volterra series functionals that have been the applied to unsteady aerodynamic and aeroelastic modeling mainly by Silva and co-workers (Silva, 1993; Silva and Perry, 1999; Silva, 1999; Silva et al., 2001; Silva and Raveh, 2001). All the methodologies from semi-empirical to functional-based models have been recently referred as reduced-order models (ROM). A ROM (Silva, 1999) is a mathematical model developed to be a simplified representation of a dynamic system. The reduced size and complexity of ROM allow more efficient computational algorithms at lower cost.

An alternative approach to the approximation of non-linear functionals has been proposed by Modha and Hecht-Nielsen (1993), which provides a unified framework for the input-output description of a large class of non-linear dynamic systems. The so-called multi-layer functionals (MLF) (Modha and Hecht-Nielsen, 1993) are a new parametric family of real-valued mappings defined by a non-linear combination of linear affine functionals on arbitrary normed linear spaces. MLF forms can be conveniently realized by artificial neural networks. Depending on the neural network architecture, classes of MLF can be obtained. Marques and Anderson (2001) have shown that the use of functional representations provides a rigorous mathematical framework that can account for the complex nonlinearities and time-history effects of the unsteady aerodynamic response. Here the authors have presented a multi-layer functional framework that is attainable by a Finite Impulse Response (FIR) neural network.

The simplest neural network architecture that realizes a MLF is the conventional multi-layer feedforward network (with sigmoidal activation functions) (Marques, 2002). By assembling an appropriate sequence of input/output values at a time window, a time-delay neural network (TDNN) can be achieved and trained using typical algorithms for static networks. Faller and Schreck (1996) have shown that artificial neural networks technology have been explored in aeronautics mainly for real-time fault diagnostics and control reconfiguration, for real-time reference models and simulations of the vehicle dynamics, and for adaptive, nonlinear control systems. Applications of neural network to aeroelastic analysis have been not presented in the technical literature.

This work presents an application multi-layer functional approach to the aeroelastic response modeling of a flexible wing. The MLF has been attained by a TDNN and trained with the Levenberg-Marquardt algorithm (Hagan and Menhaj, 1994). The training has been based on experimental data using a scaled flexible wing in wind tunnel. The wind tunnel model has been built using a main structure of fiberglass and non-structural foam and wooden skin. The wing model is fixed to a turntable that allows variations to the incidence angle of the whole model, while strain gages are used to capture local aeroelastic displacements. Motion-induced aeroelastic responses have been collected to a variety of flow speeds. Primarily, the neural network model has been conceived to identify linear motion-induced aeroelastic responses to variations of the wing model incidence angle in a range of flow speeds. Further studies will comprise non-linear aeroelastic responses at high angles of attack. After training the neural network model has been verified for generalization properties. The results have shown adequate features of the identified neural network model.

2. Functional Approximation and Neural Networks

Functionals are functions of functions (Tobak and Pearson, 1964) that provide a suitable mathematical framework to model the relationship between aeroelastic responses and motion history effects. A variety of linear and non-linear functional forms are available, such as, the Volterra series and block-oriented representations (Marques, 1997).

Developments based on the universal approximation theorem have lead to the so-called multi-layer functional (MLF) (Modha and Hecht-Nielsen, 1993). The MLF uses the premise that any time-invariant non-linear system, characterized by continuous functionals, can be approximated by non-linear superposition of linear affine functionals defined in arbitrary normed spaces, that is:

$$F(t) \approx \mathrm{MF}[\mathbf{u}_t] = \sum_{i=1}^{k} \zeta_i \varphi\left(\theta_i + L_i[\mathbf{u}_t]\right), \tag{1}$$

where, $F(t)$ is the system response, k is the number of process units, ζ_i and θ_i are real constants, φ is non-constant, bounded, monotone-increasing continuous function, and $L_i[\mathbf{u}_t]$ denotes linear functionals of generalized motion vector $\mathbf{u}_t$.

Classes of MLF can be obtained from specifying different linear functionals L_i in Eq. (1). MLF can be realized by artificial neural networks. Neural networks are mathematical models that process information by means of a distributed arrangement of processing units (Haykin, 1994). They have been conceived to work analogously to the animal brains, thereby enhancing the capabilities in dealing with complex input-to-output mappings.

Multi-layer feedforward neural networks are a class of MLF, where the basic linear functional L_i is represented as weighted superposition of affine functionals on p-dimensional real valued spaces, modified by a sigmoid non-linear function, that is:

$$F(t) \approx \mathrm{MF}[\mathbf{u}] = \sum_{i=1}^{k} \zeta_i \varphi\left(\theta_i + \sum_{j=1}^{p} w_{ji} u_j\right), \tag{2}$$

where, $F(t)$, k, ζ_i, θ_i and φ are as defined in Eq. (1), $\mathbf{u}$ is the input vector and w denotes a weight value.

The resulting neural network for the MLF in Eq. (2) is basically a static one. To represent a dynamic system with such model the architecture must be specific.

A discrete-time, finite memory temporal neural network, in the form of a finite impulse response neural network (FIRNN) has been used by Marques (Marques and Anderson, 1996; Marques, 1997; Marques and Anderson, 2001) as a realization of MLF. In this case the basic linear functional L_i is in the form of the convolution integral, therefore:

$$F(t) \approx \mathrm{MF}[\mathbf{u}_t] = \sum_{i=1}^{k} \zeta_i \varphi\left(\theta_i + \int_0^l \mathbf{h}_i(\tau)\mathbf{u}(t-\tau)d\tau \right), \tag{3}$$

where, $F(t)$, k, ζ_i, θ_i and φ are as defined in Eq. (1), $\mathbf{h}_i$ is the vector of the unit impulse responses.

By assuming discrete-time the MLF in Eq. (3) results in a temporal or dynamic neural network, the FIRNN.

To achieve a desirable set of synaptic weights to a pre-defined network architecture, a training process is needed. A training process is generally based on an optimization scheme to adjust the network parameters (mainly, weights) in relation to a set of input-to-output to be matched by the neural network model (supervised learning scheme). The backpropagation algorithm based on gradient descent (Haykin, 1994) has been widely applied for general neural network training. More efficient training scheme can be achieved by using the Levenberg-Marquardt algorithm (LMA).

Temporal neural networks can also be trained by means of a modified backpropagation algorithm (Haykin, 1994), that is the so-called temporal backpropagation. The algorithm, however, does not present a satisfactory performance. Unfortunately, alternatives to the training of temporal networks are still very few ones. Marques (1997) has proposed the use of genetic algorithm to train a temporal network in the form of FIRNN. It has revealed to require a substantial amount of computational time.

3. Levenberg-Marquardt Algorithm (LMA)

This algorithm is a variation of the Newton's method for minimizing functions that are sums of squares of other non-linear functions (Hagan et al., 1996). The LMA provides better performance when compared with typical backpropagation algorithms.

From Newton's method the network update rule is:

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \mathbf{H}_n^{-1}\mathbf{g}_n , \tag{4}$$

where, $\mathbf{w}$ is the network weight matrix, n is a step of iteration, $\mathbf{H}$ is the Hessian matrix and $\mathbf{g}$ is the gradient matrix.

For the performance index as a sum of squared functions, the Hessian matrix can be approximated in terms of the Jacobian matrix, $\mathbf{J}$, that contains first derivatives of the network errors with respect to the weights and biases. Thus,

$$\mathbf{H} \cong \mathbf{J}^T\mathbf{J} . \tag{5}$$

When Eq. (5) is substituted into Eq. (4), the Gauss-Newton method is obtained, that is:

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \left[\mathbf{J}_n^T\mathbf{J}_n\right]^{-1}\mathbf{g}_n . \tag{6}$$

A trouble with the Gauss-Newton method is that the matrix $\left[\mathbf{J}^T\mathbf{J}\right]$ may not have inverse. This can be overcome by assuming a modification to the matrix $\left[\mathbf{J}^T\mathbf{J}\right]$ that leads to the LMA:

$$\mathbf{w}_{n+1} = \mathbf{w}_n - \left[\mathbf{J}_n^T\mathbf{J} + \mu_n\mathbf{I}\right]^{-1}\mathbf{g}_n , \tag{7}$$

where, $\mathbf{I}$ is the identity matrix and μ is a scalar.

The scalar μ plays an important role to the LMA. When μ_n is zero, the weight update is basically the Gauss-Newton method. When μ_n is sufficiently large, the Eq. (7) becomes gradient descent with small step size. By choosing the proper value of μ the LMA provides an efficient compromise between the great performance of the Newton's method and the guaranteed convergence of gradient descent approach.

4. Time –Delay Neural Networks

Typical neural networks can only deal with input-to-output mappings that are static. To represent dynamic mappings, for instance, to model a dynamic system behavior, the network must be constructed in a different way. A solution to this case has been given by using the idea of regressive models, in other words, models based on past values of the system input, $u(t)$, and output, $y(t)$. This kind of network has been usually called *time-delay neural network* (TDNN) (Haykin, 1994), and a schematic representation is given in Fig. (1).

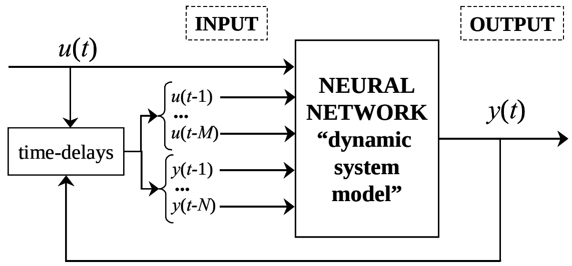

Figure 1. Time-delay neural network

5. Experimental Apparatus

The aeroelastic wing comprises a wind tunnel model of an arbitrary straight rectangular semi-span wing with NACA0012 airfoil from wing's root to tip. The wing model has been fixed to a turntable that allows incidence motion to the wing. The wing semi-span is 800 *mm* and the chord is 290 *mm*.

The model main structure has been constructed using fiberglass and epoxy resin. A tapered plate has been laminated with 20 plies of fiberglass fabric (0º, 90º) using the hand lay-up process. The taper ratio is 1:1.67, where the width at the wing root is 250 *mm*. To provide aerodynamic shape high-density foam has been used. To the wing skin a mahogany shell with thickness of 2 *mm* has been bounded externally to the foam. In order to minimize as much as possible the effects of the skin to the wing structure stiffness, both foam and wooden shell have been segmented at each 100 *mm* spanwise. Figure (2) presents an illustration of the experimental apparatus with indications of the strain gages positions inside the wing model.

Incidence motion is achieved with an electrical motor (THOMSON brushless servo motor TMC-1000 and TD-50 brushless motor amplifier) mounted beneath the turntable. The motor actions are controlled by software integrated to the acquisition system. Strain gages have been fixed to the plate surface to furnish proper measurement of the dynamic response of the wing main structure. The strain gages have been distributed along three lines spanwise. The first and last lines present three strain gages each (enumerated form 1 to 3 and 7 to 9, respectively), all to capture bending motions. The intermediate line presents three strain gages for torsional motion (enumerated from 4 to 6).

The closed circuit wind tunnel presents a testing chamber with 1300×1400 *mm*. The maximum flow speed in the testing chamber is 50 *m/s* with turbulence level of 0.3 %.

Data acquisition and the motion control of the servo motor have been achieved by using a dSPACE® DS1103 PPC controller board and real-time interface for SIMULINK®. The HBM KWS 3073 amplifier for strain gage bridge energizing has been used to acquire and amplify the strain gage signals. The resulting signals are directly acquired by the dSPACE controller board, allowing subsequent data storage into a PC compatible computer.

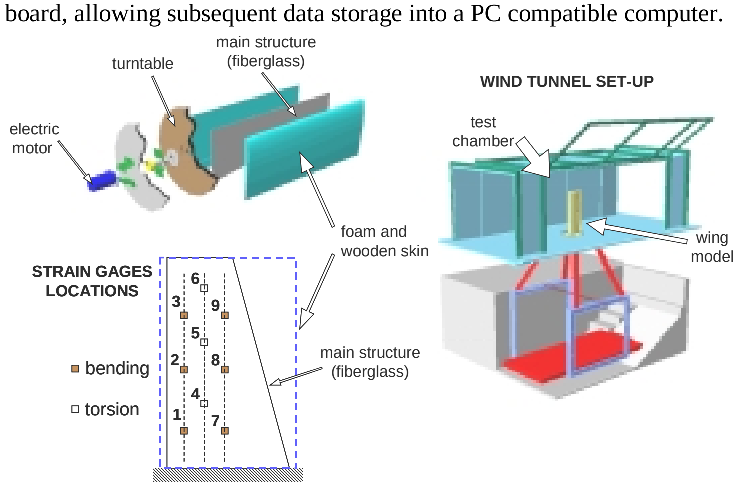

Figure 2. Wind tunnel model and strain gages locations.

6. Aeroelastic Functional Approximation

Experimental data have been used for neural network training to form an input-output database. Here the aeroelastic response time histories have been assumed from the bending displacements acquired by the strain gage at point 3 cf. Fig. (2). Strain gage measurements have been normalized with relation to their maximum absolute values. The incidence angle time history and past values of aeroelastic response comprise the input to the neural network (see Fig. (1)). The aeroelastic response at each measured time composes the output vector. Furthermore, the flow speed in the wind tunnel has been included as a static input to the network. The number of time-delays in the inputs have been assumed 4 for the incidence angle and 4 for the aeroelastic response past values. Therefore a total number of 10 inputs have been used. This network input set can be referred as the time window. A total of 1000 randomly chosen time windows of the respective incidence time histories and system outputs have been taken as examples to the training procedure.

A good practice is to randomize the order of presentation of training examples from one epoch to the next. This randomization tends to promote the stochastic search in weight space over the learning cycles, thus avoiding the possibility of limit cycles in the evolution of the synaptic weight vectors (Haykin, 1994).

A two hidden layers TDNN composed of 12 and 8 neurons with hyperbolic tangent, as activation functions comprises the architecture.

The TDNN has been trained accordingly to the LMA. The training procedure required 100 epochs to achieve a mean squared error of magnitude 10^{-7}, as shown in Fig. (3). After training, it has been presented unknown inputs to the network, that is, generalization procedure has been done.

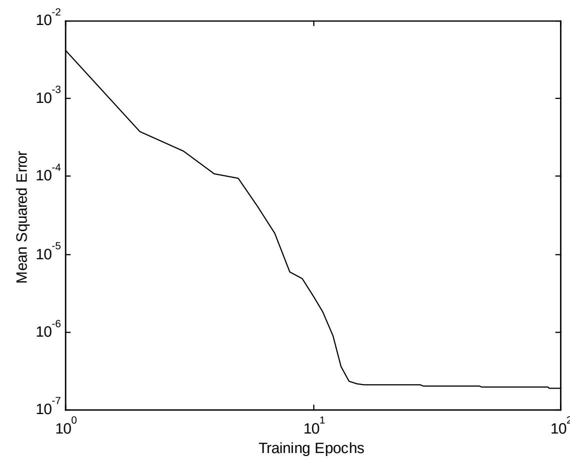

Figure 3. Error decay during the neural network training.

Figures (4) to (6) show the neural network training results to random incidence time-histories at flow speeds of 11.27 *m/s*, 13.64 *m/s* and 16.96 *m/s,* respectively. The random signals have been obtained using white noise and a second order filter to attenuate the signals intensity. The reason for that lies in the difficulties in turning the servo-motor to a pure random signal.

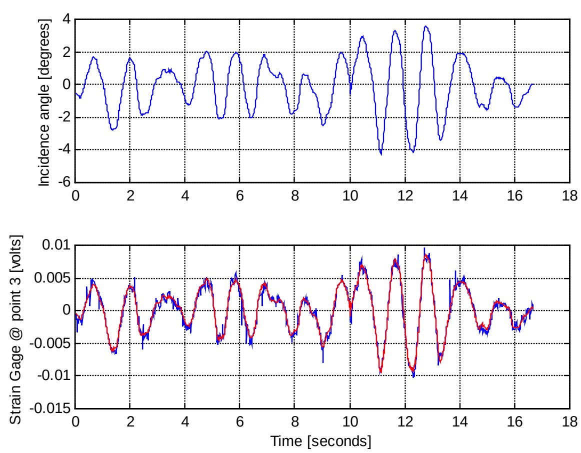

Figure 4. Neural network training results to random motion history and flow speed of 11.27 *m/s*. Blue line - System output; red line - Neural network output.

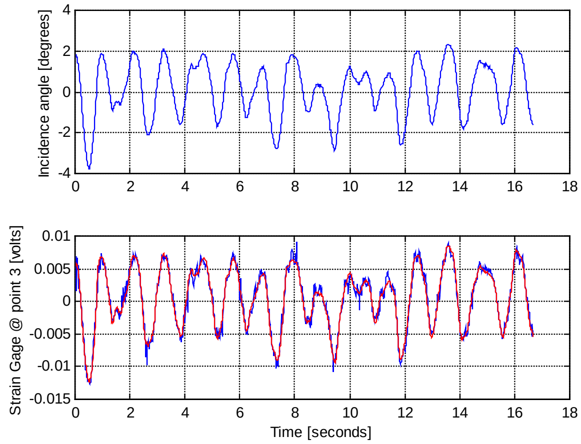

Figure 5. Neural network training results to random motion history and flow speed of 13.64 *m/s*. Blue line - System output; red line - Neural network output.

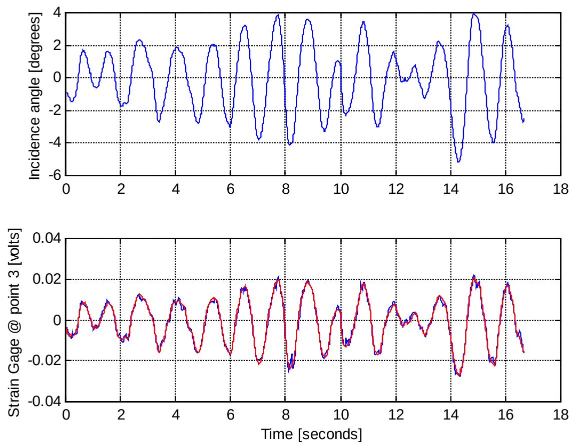

Figure 6. Neural network training results to random motion history and flow speed of 16.96 *m/s*. Blue line - System output; red line - Neural network output.

Analyzing the training results it is possible to notice that the curves regarding the system and neural network outputs are overlapped. This indicates that the training procedure has been efficient. However, making a careful observation it is possible to see that the curve regarding the neural network output is smoother than the experimental one, indicating that the neural network also operates as a filter.

Figures (7) to (9) show generalization tests comprising neural network responses to a random incidence time histories different from those applied during training. The flow speeds for each case are of 11.10 *m/s*, 13.63 *m/s* and 16.89 *m/s*, respectively. The random signals have been obtained by the same way of those used to train the network. For each case it has been also presented a frequency analysis. The idea is to show that the neural network model has also captured the frequency content of the aeroelastic functional.

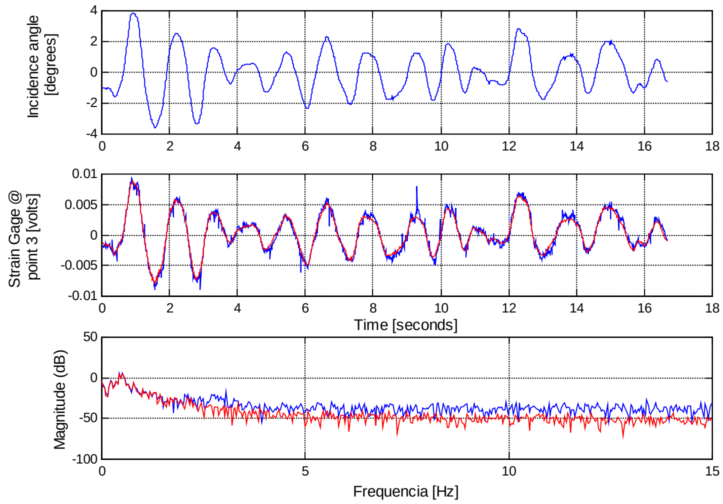

Figure 7. Generalization test of the neural network to random motion history and flow speed of 11.10 *m/s*. Blue line - System output; red line - Neural network output.

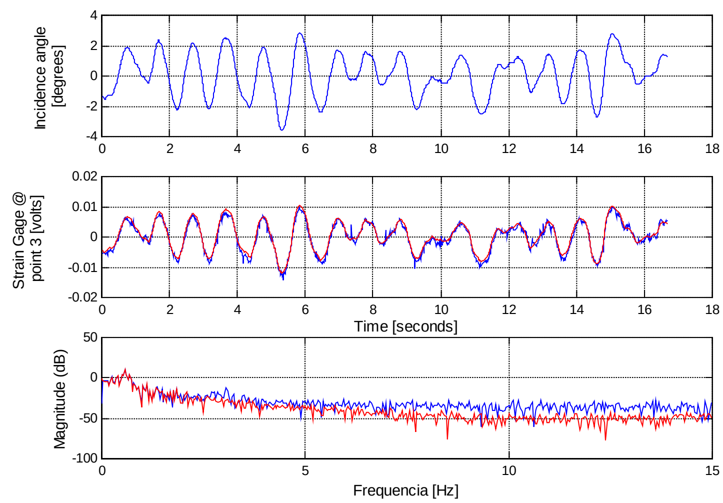

Figure 8. Generalization test of the neural network to random motion history and flow speed of 13.63 *m/s*. Blue line - System output; red line - Neural network output.

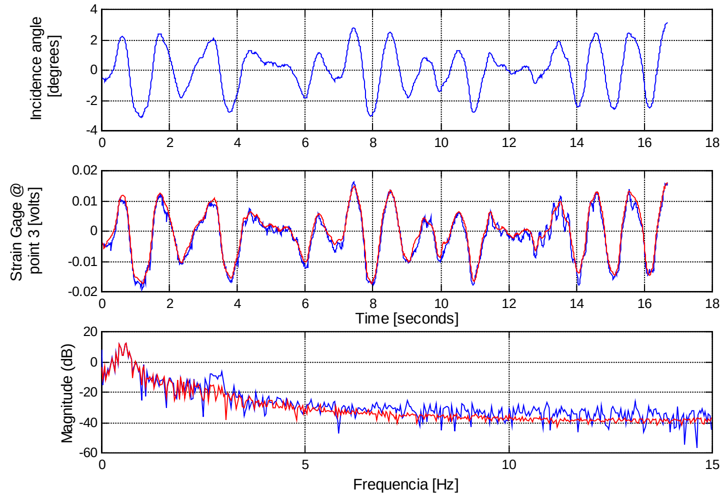

Figure 9. Generalization test of the neural network to random motion history and flow speed of 16.89 *m/s*. Blue line - System output; red line - Neural network output.

Observing Figs. (7) to (9) it is possible to infer that the neural network model has been able to succesfully identify the aeroelastic response functional associated to the structure response at a range of wind tunnel flow speeds.

The frequency spectra show that the neural network has been able to capture the main system frequencies.

Figures (10) to (12) show neural network responses to oscillatory incidence angle time histories for different reduced frequencies, that is, k equals to 0.10338, 0.08497, and 0.06852, respectively.

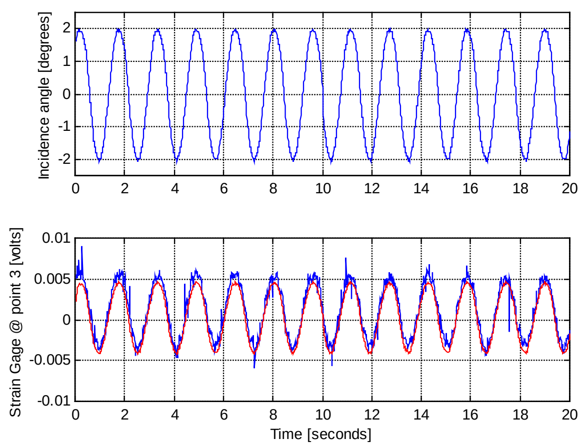

Figure 10. Generalization test of the neural network to oscillatory motion at flow speed of 11.22 *m/s* (k = 0.10338). Blue line - System output; red line - Neural network output.

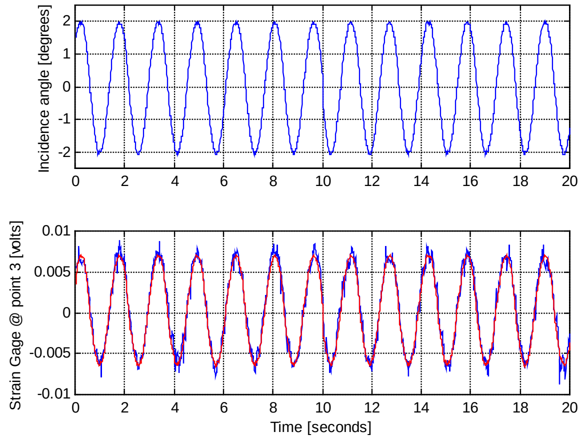

Figure 11. Generalization test of the neural network to oscillatory motion at flow speed of 13.65 *m/s* (k = 0.08497).
Blue line - System output; red line - Neural network output

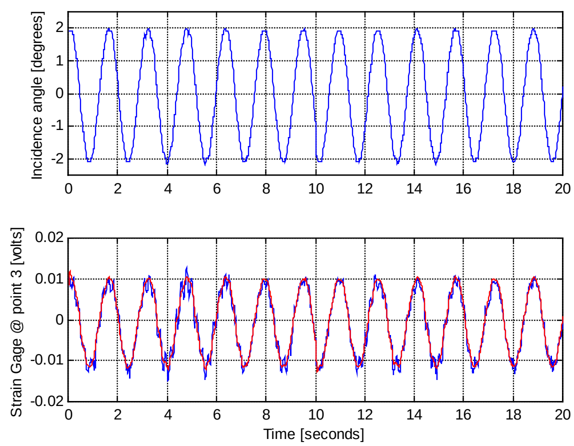

Figure 12. Generalization test of the neural network to oscillatory motion at flow speed of 16.93 *m/s* (k = 0.06852).
Blue line - System output; red line - Neural network output

The curves presented in the Figs. (10), (11) and (12) are overlapped indicating that the neural network model has been able to generalize even when different kind of input has been presented. This fact shows that the neural network is sensitive to changes in the kind of excitation.

7. Conclusions

TDNN models furnish appropriate approximation of the aeroelastic responses. They belong to a class of MLF, therefore allowing rigorous functional representation of dynamic systems. The major advantages associated with this approach are: *(i)* the ability to account for non-linearities and time-history dependencies encountered in unsteady transonic flow, *(ii)* the use of parametric multiple-input/multiple-output models for aeroelastic applications, allowing fast evaluation of the aerodynamic response; *(iii)* static parameters (e.g. Mach number) can be used as inputs to the TDNN model, thereby increasing the range of flow conditions for which the model is applicable; *(iv)* the difficulties associated with standard non-linear system identification approaches are diminished. The training procedure based on the LMA is shown to be an efficient methodology to achieve reliable TDNN models.

Generalization tests demonstrate that the neural network model efficiently predicts the aeroelastic response functional, even when the flow speed in the wind tunnel has been changed and when a different kind of excitation has been applied. These facts show that the use of neural networks is a promising method for aeroelastic modeling.

8. Acknowledgements

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