NONLINEAR SLOSHING OF GRAVITY WAVES IN A RESERVOIR

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Abstract.*.The present paper is concerned with gravity waves which appear on the free surface of a container of liquid, shaped as a two dimensional box undergoing forced horizontal or vertical oscillations. Oscillations induced by a stationary initial condition where the liquid surface is instantly at a non-equilibrium configuration are also considered.*

Keywords*. Free surface flows, potential flows, wave impact, boundary integral method.*

1. Introduction

Free surface waves can be generated by means of an imposed movement on a partially filled reservoir. Violent wave impacts inside confined spaces have been observed on vehicles transporting liquid substances, subject to large accelerations/decelerations. In such cases wave reflection on the walls induces hydro-dynamical forces, which may cause hazardous destabilizing effects. This fluid/structure interaction may cause the container's rupture, instability and loss of maneuverability on vehicles loaded with liquids. That is the case of fuel tanks in trucks, aircraft, spacecraft and ship, which in certain circumstances are subject to harmonic loads of high amplitude, and may reach resonant modes. Bredmose et al. (2002) observed that two very different types of response may coexist in a confined tank: a violent brief impact of the liquid on the container wall and long lasting/large amplitude sloshing motions.

Ockendon & Ockendon (1973) find for a 2D box of length πL and depth hL, undergoing harmonic horizontal oscillations, a discrete spectrum of frequencies:

$$\omega = [gn \tanh(nh / L)]^{1/2} , \tag{1}$$

where g is the acceleration of gravity and n is the wave-number. Resonance occurs, when the fluid is forced to oscillate at these frequencies; for resonant oscillations, are meant forced oscillations, of a small magnitude, which produce an out of proportion large response, when compared with the input harmonic oscillations. A search about the word 'sloshing', at the 'search engines' of the Internet, reveals stories which are both mysterious and scary; some of these tell about serious car accidents, caused by the sloshing of the petrol inside the reservoir; this one has been told by 'witnesses' at ′yahoo.com.′

The generation of patterns of steep standing waves is an important feature observed in sloshing motions. If certain driving frequencies are imposed, vertical accelerations in a container induces the growth of standing waves, also known as Faraday waves. Bredmose et al. (2002) report the formation of "table-top" breaking waves numerically and experimentally when studying steep, breaking, Faraday waves. Longuet-Higgins (2001) shows that vertical jets may result from high amplitude standing waves; that is known as "the bazooka effect". The free surface flows mentioned above have gravity as its main restoring mechanism. On a smaller scale, however, surface tension may significantly affect wave properties. Small scale ripples, or capillary waves, occur due to external accelerations imposed on the reservoir. Billingham (2001) finds that, under zero gravity conditions, periodic and chaotic solutions and solutions where the topology of the fluid changes, either through self-intersection or pinch off, are possible.

The present work is concerned with free surface waves produced, on the surface of a container, shaped as a 2D box, which is partially filled with liquid. The box is made of two vertical walls, set at a distance of L apart; there may be an impermeable horizontal bottom set at a mean.distance H from the free surface; or the water may be deep; it is also possible to consider a horizontal flat lid closing the box. In such cases we are interested in the study of violent horizontal or vertical oscillations. Energetic vertical shaking, is bound to produce very large oscillations; with amplitudes in excess of 2.5 mean depths; in various applications, in different fields, that is of importance for the stability of vehicles transporting liquid loads in confined spaces.

We start with a regular train of periodic waves propagating over a flat impermeable bed; in fact there is no physical bed; as the bed is imposed through a Method of Images. If the wave period, with no loss of generality, fits the length of 2π and then at π, a vertical line of symmetry is imposed on the flow, as a result a vertical line non percolation is created; as the waves are periodic, we have to have an infinite array of such impermeable vertical lines, or walls, with a spacing of π from each other; and then we may consider a horizontal impermeable lid, closing the box; in order to evaluate wave impacts coming from underneath the lid. According to H. Bredmose et al. (2002), an important characteristic of sloshing motions in a tank, is the generation of patterns of steep standing waves. That is of importance because

Some results of numerical experiments with the wave box have been obtained, in our calculations. These are for:

i) Taking a box at rest, with liquid inside, and a free surface instantly tilted to one side, with a configuration of non equilibrium; as the water is released, the liquid sloshes inside the box; the energy which keeps the oscillations is conserved.

ii) The box may undergo forced horizontal oscillations; at the present stage these oscillations have to be gentle, either slow or of a little amplitude in order to preclude premature breaking.

iii) Vertical oscillations of the box, may produce very energetic standing waves, of a high amplitude, producing spectacular splashes; with a sort of mushroom like waves; if there is an impermeable horizontal flat lid, closing the box energetic splashes may hit the lid from the underneath.

There are some striking differences, between traveling gravity waves and standing waves; traveling waves of a certain amplitude are prone to super-harmonic instability; which causes steepening at the front face of the wave leading to wave breaking. Standing waves, due to its intrinsic symmetry, and the vertical motion of the crest, are likely to attain exceptionally high amplitudes without breaking, and starting an experiment with one period

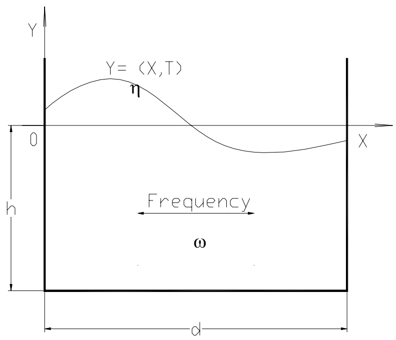

Figure 1. Fluid domain geometry.

Bredmose et al. (2002) investigated experimentally and numerically the generation of steep forced waves in a moving tank filled with liquid, finding a good agreement between experiment and computation.

The present work is related to the study of sloshing in a reservoir modelled as a 2-D impermeable box with a flat horizontal bottom and two vertical walls spaced by a distance d (see figure 1). We suppose an initially flat free surface of liquid with depth h. The aspect ratio h/d and the frequency of oscillations ω are the parameters of the problem. A lateral and/or vertical harmonic motion is imposed at the boundaries of the reservoir.

The nonlinear unsteady free surface flow is numerically simulated by means of a boundary-integral method developed by Dold & Peregrine (1986). For small length scale reservoirs and zero gravity situations, surface tension effects become important and may produce especial features at the free surface (Jervis 1996, Billingham 2002). These cases are of interest and are also included in the modelling.

2. The boundary value problem

We assume a non-viscous and incompressible free surface flow with an arbitrary constant vorticity ζ. Then Laplace´s equation,

$$\nabla^2 \Phi = 0 \ , \tag{2}$$

is satisfied on the fluid domain for a full velocity potential Φ. The oscillations imposed at the box are introduced in the model by decomposing Φ into a regular part ϕ (due to surface waves) and a perturbed part $\bar{\phi}$ (due to a harmonic external forcing). The kinematic boundary condition states that fluid particles move with their own velocity $\vec{u}$,

$$\frac{D\vec{r}}{Dt} = \vec{u} \ , \tag{3}$$

where $\vec{r} = (x, y)$. At the free surface Bernoulli´s equation is satisfied,

$$\frac{D\Phi}{Dt} = \frac{1}{2}|\nabla\Phi|^2 - \frac{p}{\rho} - gy - \tau\kappa + \zeta(y\Phi_x - \Psi), \tag{4}$$

where p is the atmospheric pressure, ρ is the density, τ is the surface tension, κ is the free surface curvature and Ψ is the stream function. At the vertical walls and at the bottom, the boundary condition is of the Neumann type i.e. both walls and bottom are supposed to be rigid and impermeable. In all the computed cases presented in this paper surface tension effects are neglected.

3. Fully nonlinear boundary integral solver

The problem described in section 2 is solved through the use of a boundary integral equation, related to Cauchy´s integral theorem (see Dold & Peregrine 1986). The flow is determined by means of a point discretisation of the free surface contour, which significantly reduces the computational demand for the calculation of the fluid motion, since only surface properties are evaluated. The method of solution consists of the following stages; initially the full potential Φ is known on the surface; the disturbed potential $\bar{\phi}$ is also defined and subtracted from the surface value of Φ, such that the remaining surface wave potential ϕ can be used with Cauchy´s integral theorem, to calculate the velocity $\nabla\phi$ on the free surface. Then the potential $\bar{\phi}$ is added back in and the corresponding total velocities are evaluated. The free surface is stepped in time using a truncated Taylor series. Such stages are repeated until the final time is reached, or the algorithm breaks down.

The complex velocity is given as,

$$q(z) = \frac{df}{dz} = u - iv, \tag{5}$$

where $f = \phi + i\Psi$. The impermeability condition on the bed is satisfied by considering an image region (see figure 2).

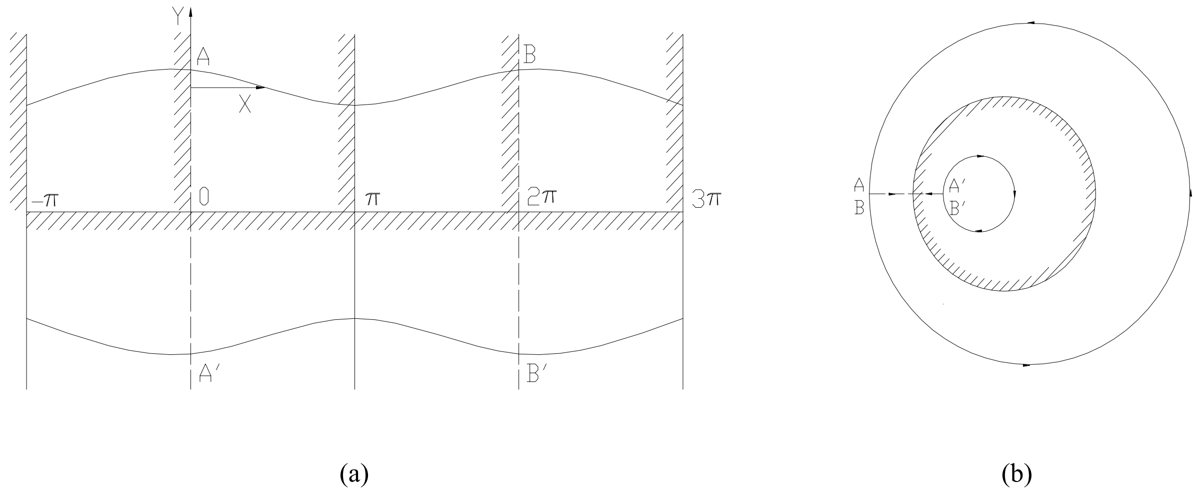

Figure 2. (a) A sketch of the z-plane with a periodic wave and its reflection onto the bed. (b) The corresponding Ω -plane obtained via a conformal mapping.

4. Results

The resulting free surface flow for a box at rest is shown in figure 3. A sinusoidal profile with wave amplitude $A_0 = 0.01$ is used as the initial condition for the fully nonlinear boundary value problem. The non-dimensional time is given by $T(g/L)^{1/2}$. For $T(g/L)^{1/2}=45.0$, a travelling wave can be observed propagating to the right until it encounters the impermeable wall. As time evolves nonlinearity starts to become dominant causing wave breaking at $T(g/L)^{1/2}=54.0$. For smaller wave amplitudes the problem follows linear water wave theory. No wave breaking occurs with a vertical oscillating, steady, free surface flow. In all the computed tests wave energy was conserved.

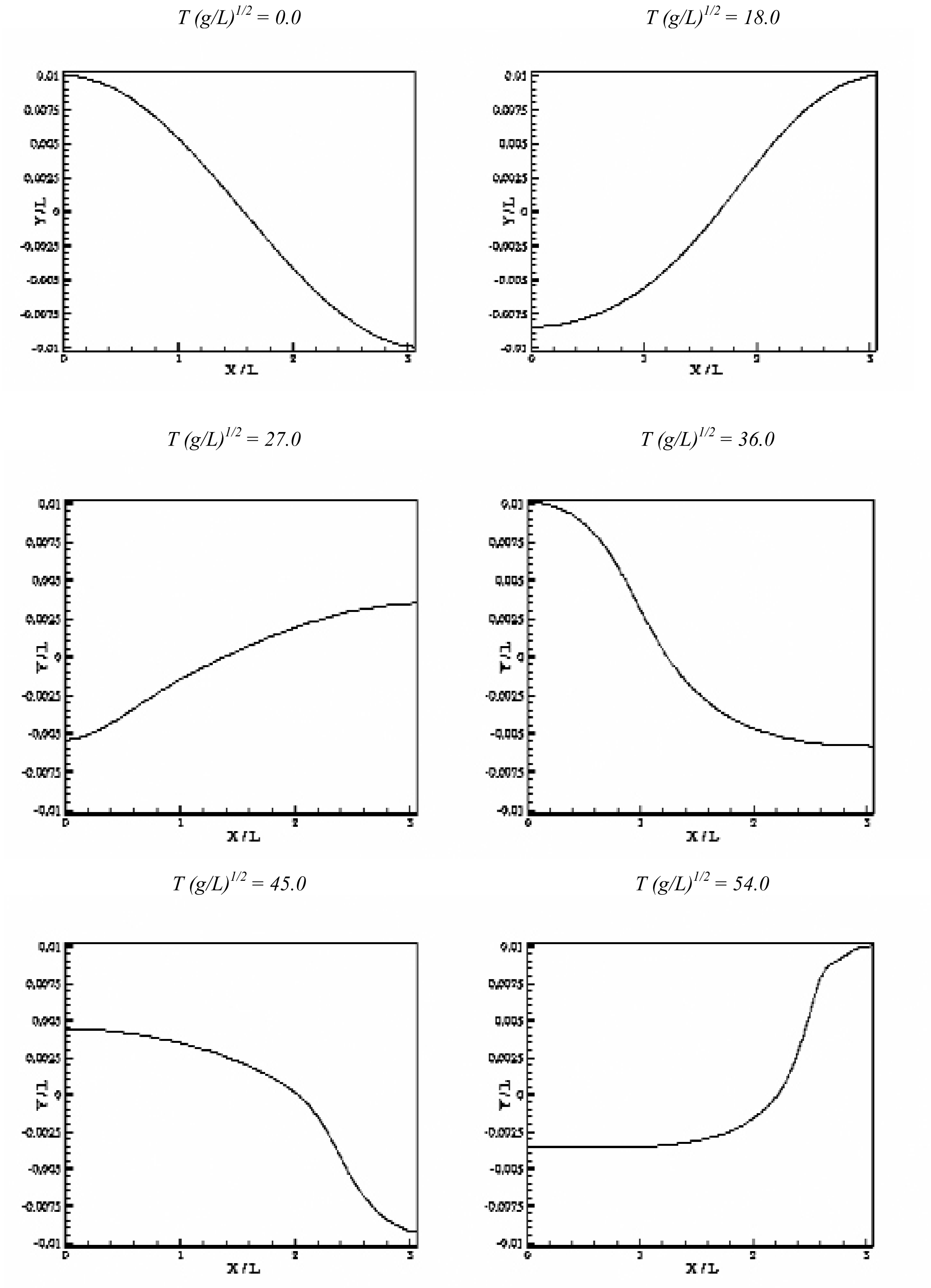

Figure 3. Nonlinear sloshing of gravity waves in a stationary box $(\omega = 0)$. The initial condition corresponds to a sinusoidal wave of amplitude $A_0 = 0.01$. Aspect ratio $h / L = 1$. L is the tank length.

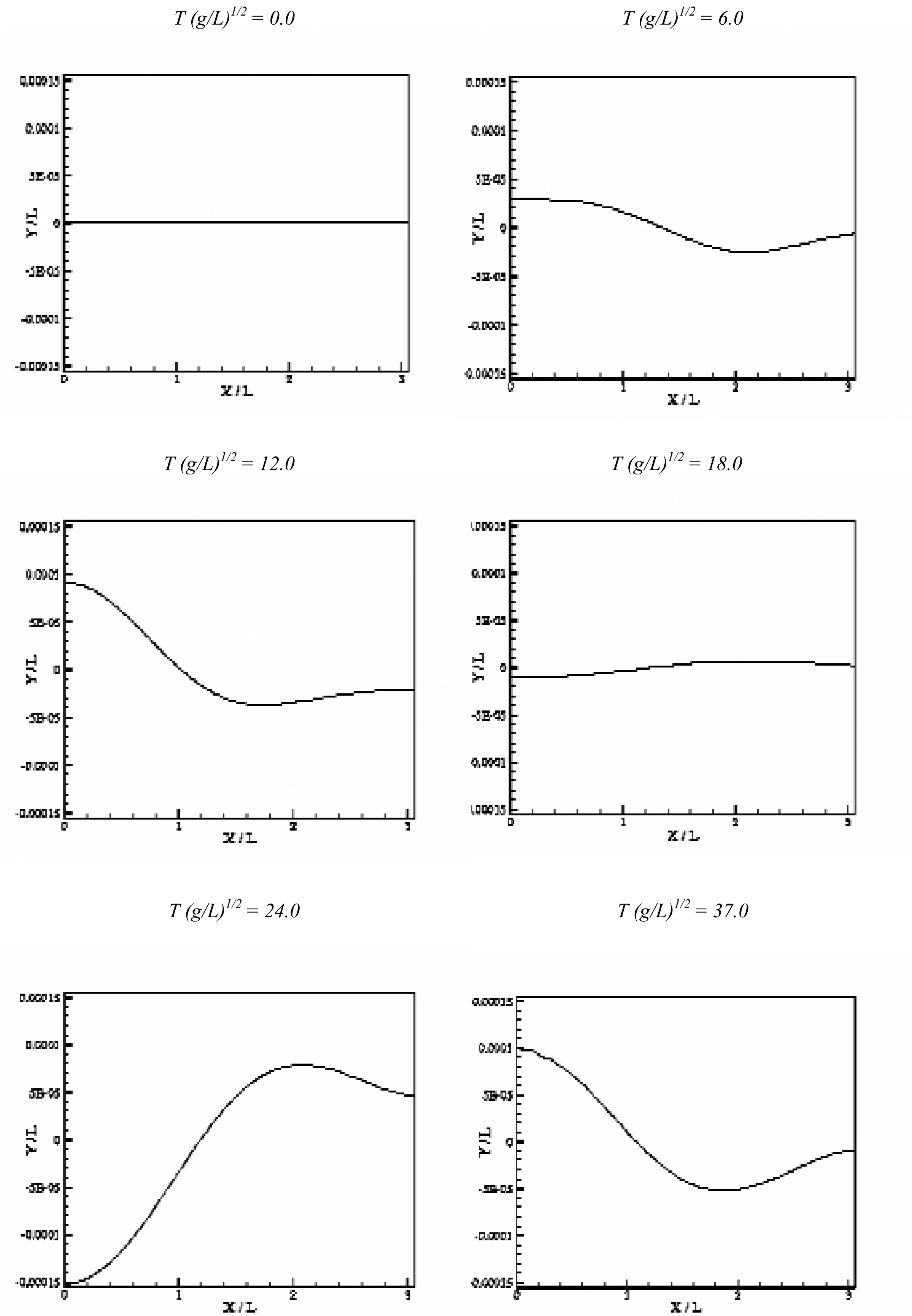

Figure 4. Nonlinear sloshing of gravity waves in a horizontal oscillating box $(\omega = 0.2, \alpha_0 = 0.001)$. Initially the free surface is at rest. Aspect ratio $h/L = 1$. L is the tank length.

Figure 4 shows the fully nonlinear results obtained for a horizontal oscillating box. Initially the free surface is at rest and is disturbed by an imposed harmonic horizontal acceleration of the form,

$$\alpha = \alpha_0 sin(kx + \omega t), \tag{6}$$

where α_0 is the amplitude of the acceleration and k is the wavenumber. For all the computed cases, $k=1$. Figure 4 shows results for $\omega = 0.2$ and $\alpha_0 = 0.001$. A travelling wave can be observed moving to the left when $T(g/L)^{1/2}=24.0$ until wave breaking occurs at $T(g/L)^{1/2}=37.0$.

5. Summary

The efficient and precise algorithm developed by Dold & Peregrine (1986) to compute free surface flows has been successfully extended to simulate fully nonlinear sloshing in a 2D box. Preliminary results show that under certain conditions nonlinearity may become important even for smallish wave amplitudes.

6. Acknowledgement

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7. References

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