THE FREE SURFACE HYDRODYNAMIC IMPACT PROBLEM: A BRIEF REVIEW ON ASYMPTOTIC SOLUTIONS AND EXPERIMENTS WITH A HEMISPHERE

Celso P. Pesce
Leonardo Casetta
Mechanical Engineering Department, Escola Politécnica, University of São Paulo
cppesce@usp.br

Eduardo E. Crivellari
Naval Architecture and Ocean Engineering Department, Escola Politécnica, University of São Paulo

Abstract. *This work addresses the classical problem of the hydrodynamic impacting force acting upon a rigid body, during the water entry phenomenon. A brief review is presented, covering works from Von Karman (1929), Shifman and Spencer (1951), Wagner (1931), Miloh (1981), Moghisi and Squire (1981), Korobkin and Pukhnachov (1988), Cooker and Peregrine (1995), to Wu (1998). Special emphasis is given to asymptotic methods, specially addressing those by Cointe and Armand (1987), Molin et al (1996) and Faltinsen and Zhao (1997). An experimental investigation on an impacting hemisphere has also been carried out and results on measured impacting forces are compared with asymptotic solutions.*

Keywords: Impact, Hydrodynamics, Asymptotic Solutions, Experiments, Hemisphere.

1. Introduction

The water impact and water entry problem of bluff bodies striking a free surface is a classical problem in hydrodynamics. The first analytical studies were performed during the early thirties and were stimulated by an interest in the landing characteristics of seaplanes that were first designed at that time. Hence the classical works of Von Kármán (1929), who approximated the shape of the striking body by a growing flat plate, and that of Wagner (1931), who went a step beyond Kármán's analysis by considering also the water splash, and so including a wetting correction factor. The Second World War also served as an impetus for conducting further research in this field, primarily because of the interest in water entry and water exit of projectiles. The complete treatment of the problem should include the elasticity of the impact body as well as air cushion compressibility effects. Nowadays, the impact problem also belongs to the interest of naval architecture and offshore platform design.

A good and comprehensive review of the hydrodynamic impact problem can be found in Korobkin and Pukhnachov (1988). The simplest problem is the case of the rigid-body impact against a liquid free-surface, in which compressibility effects are not take into account and the flow is considered inviscid and irrotational. The time-scale representing the phenomenon is so small that, during the very starting stage, the free surface can be asymptotically replaced by an equipotential boundary condition, corresponding to the limit of infinity frequency in the sense of the harmonic wave radiation problem. Nevertheless, this approximation is true everywhere except in the near field of the impacting body, where a jet (or spray) is formed.

The correct consideration of the jets is, in essence, the reason for an apparent controversy, noticed by Miloh (1981), but previously touched somehow by Shiffman and Spencer (1951). This apparent controversy points out a significant discrepancy between impacting load calculations when methods based on integration of the pressure field or, alternatively, energy approaches are used instead. Korobkin and Pukhnachov (1988) pointed that half of the kinetic energy is transferred to the jets and half to bulk of the fluid. The same conclusion is drawn by Cooker and Peregrine (1995) through a pressure-impulse theory approach and by Molin, Cointe and Fontaine (1996), based on matched asymptotic expansion solutions obtained by Cointe and Armand (1987) and by Faltinsen and Zhao (1997), for some particular impacting bodies such as cylinders and spheres.

Nonetheless, despite all those previous observations, the apparent discrepancy has been recently claimed non-existent by Wu (1998), on different basis. Although algebraically correct, the analysis conducted by Wu is misleading. It was conducted taking a control volume where the free surface is substituted by an equipotential control surface on which, as an *ad-hoc* boundary condition, the velocity potential time derivative is erroneously assumed null everywhere, even in the neighbourhood of the impacting body. Physically speaking, this assumption can be interpreted as to disregard the jets (or sprays). Wu also presents a consistency analysis through the classical Lagrange Equation approach, as in Lamb, art. 137. But this reasoning is also misleading, once such equation is valid only when the system mass is not explicitly dependent on position (and velocity), hypothesis which would hold only if the jets were not considered apart from the bulk of fluid. Further details about the application of Lagrange equation to mechanical systems with mass explicitly dependent on position can be found in Pesce (2003).

This paper focuses on the discussion of asymptotic solutions, presenting some experimental results. Although not intended to present an extensive review on this very complex problem, the work addresses some problems considered relevant for the present analysis, as the apparent controversy already described. A somewhat broader review is given in Pesce (2000).

2. Some Mathematical Background

As the flow is assumed inviscid and irrotational, a potential scalar function $\phi(x, y, z)$ defines the velocity field. For the sake of simplicity, we consider the fluid at rest before the first contact. Also, the fluid is supposed to be ideal, such that no compressibility effects exist. Laplace equation $\Delta\phi = 0$ is the field equation. The proper boundary conditions, for the surfaces that enclosure the control-volume, are shown in Fig. (1).

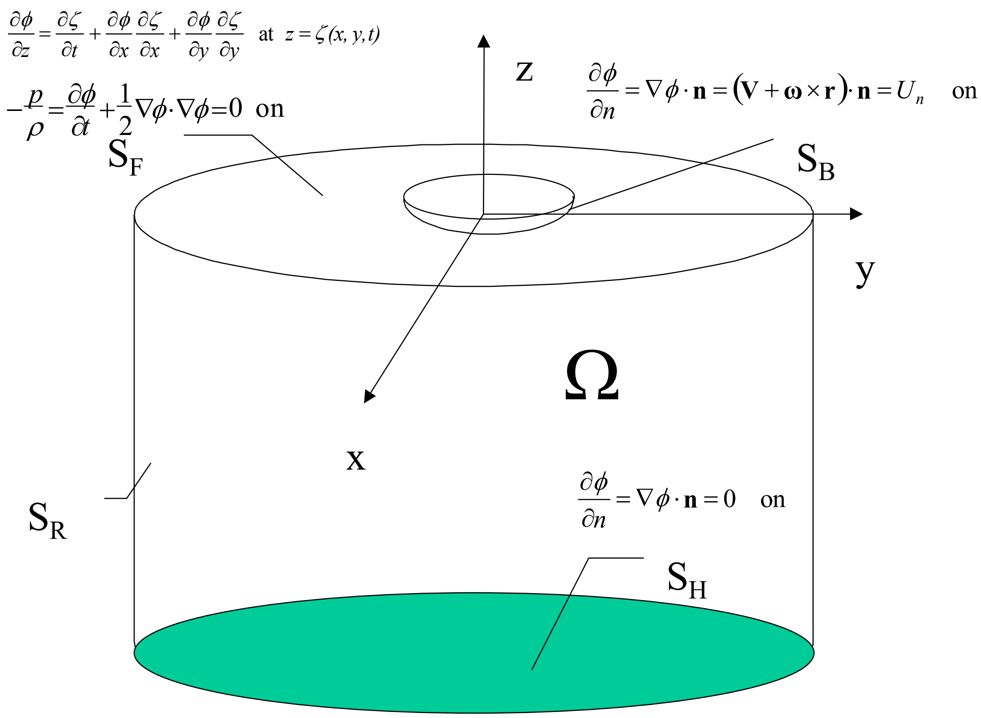

Figure 1 The whole control volume of fluid $\boldsymbol{\Omega}$ in an arbitrary instant of time *t*, and its closure surface $\partial\Omega = S = S_B \cup S_F \cup S_R \cup S_H$.

A usual hypothesis in the theory of hydrodynamic impact is to consider the free surface to be known at $t = 0^+$, such that $\zeta(x, y, t = 0^+) = 0$. This is consistent with conventional mathematical modeling of impact problems in classical mechanics. A Dirichlet type boundary condition is then usually assumed on the free surface $z = \zeta(x, y, t = 0^+) = 0$, namely $\phi(x, y, t = 0^+)\big|_\zeta = 0$, leading to the conclusion that, at $t = 0^+$, kinetic energy transferred to the fluid could be well represented through an integral over the body surface.

This equation satisfies the linearized free surface condition $\phi_{tt} + g\phi_z = 0;\ z = 0$ when the asymptotic limit for the harmonic problem is taken as the frequency tends to infinity. In fact, according to this linearized condition the potential on the free surface behaves like $\phi \approx g\,\phi_z/\omega^2$; $z = 0$ which tends to zero as $\omega \to \infty$, for a finite ϕ_z .

Nevertheless, it should be firstly observed that $\zeta_x\big|_{t=0^+} = \zeta_y\big|_{t=0^+} = 0$ is consistent with the assumption $\zeta(x, y, t = 0^+) = 0$, thus leading to

$$\frac{\partial\zeta}{\partial t} = \frac{\partial\phi}{\partial z} \quad \text{on } z = 0 \text{ at } t = 0^+ \tag{1}$$

on the free surface, *and at least not very close to the intersection between the free surface and the body surface*, where a jet or spray would be formed.

On the other hand, the dynamic free-surface condition is

$$\frac{\partial\phi}{\partial t} = -\frac{1}{2}(\nabla\phi)^2 \quad \text{on } z = \zeta(x, y, t = 0^+) = 0 \tag{2}$$

3. The Pressure Impulse Approach

Cooker and Peregrine (1995) studied the hydrodynamic impact problem through the concept of pressure impulse, where

$$P(\mathbf{x}) = \int_{0^-}^{0^+} p(\mathbf{x},t)\,dt \tag{3}$$

As those authors pointed out, the change in velocity during the impulsive event is supposed to take place over such a short time that the non-linear convective terms in the equation of motion can be neglected compared with the time derivative one, so that

$$\frac{\partial \mathbf{u}}{\partial t} = -\frac{1}{\rho}\nabla p \tag{4}$$

Then, not taking into account the compressibility of the fluid, the equation above yields

$$\Delta P(\mathbf{x}) = 0 \tag{5}$$

and the boundary conditions become

$$\begin{cases} P \equiv 0 \text{ on } S_F^I \\ \dfrac{\partial P}{\partial n} = 0 \text{ on } S_F \\ \dfrac{\partial P}{\partial n} = \rho U_n^- \text{ on } S_F \end{cases} \tag{6}$$

where the superscript I indicates that the corresponding surfaces are seen from the impulsive point of view. With the assumption of impulsive idealization, the rate of change of the kinetic energy can be written as

$$\Delta T^I = \int_{\Omega^I} \frac{1}{2}\rho\left(u^{+2} - u^{-2}\right)dV \tag{7}$$

And, after some algebraic manipulations, using the field equations and the proper boundary conditions,

$$\Delta T^I = -\frac{1}{2}\int_{S_B^I} P U_n^- \,dS \tag{8}$$

This indicates the apparent loss of kinetic energy ever cited. Since the flow is assumed incompressible, inviscid and irrotational, all possible loss of energy is related to its flux through the boundary Ω^I. In the present case, more specifically through the free surface S_F^I , near the impacting body, where the convective terms cannot be neglected. Though mass and momentum fluxes could be negligible through the jets, energy flux would not, representing a considerable part of the energy transferred to the whole fluid.

If a contraposition between momentum and energy approaches is considered, without taking the jets properly into account, different results are also obtained. According to Shiffman and Spencer (1951), the explanation for the difference, encountered when using momentum or energy principles considerations, would be related to a proper added mass definition. This is, actually, equivalent to consider or not the jets as part of the bulk of fluid.

3. Matched Asymptotic Expansion Approach

Faltinsen and Zhao (1997) treated the special case of an axi-symmetric body by using matched asymptotic expansions. Let the generatrix of the body, measured from its vertex, be given by $\eta = \eta(r)$, r measured from the axis of revolution. Let also $r = c(t)$ define the position of the jet root. The jet root surface is given by $S_{RJ}(t) = 2\pi\delta_J(t)c(t)$. In the manner of Faltinsen and Zhao, following Wagner (1931),

$$\eta(r) = \int_0^r \left(w(r;c;W) + W\right)\frac{dt}{dc}\,dc \tag{9}$$

where W is the vertical component of the body velocity, and w is the vertical component of velocity on the control surface $z = 0$, corresponding to a proper outer solution for the potential field. From Faltinsen and Zhao (1997),

$$w(r;c(t);W(t)) = \frac{2W}{\pi}\left[\left(\left(\frac{r}{c(t)}\right)^2 - 1\right) - \sin^{-1}\left(\frac{c(t)}{r}\right)\right] ; r > c \tag{10}$$

is a proper function, valid for $z = 0; r > c$, presenting a vertical dipole-like behavior at infinity, and derived from an outer potential solution. Writing,

$$\mu(c) = W\frac{dt}{dc} \tag{11}$$

Equation (9) is an integral equation,

$$\eta(r) = \int_0^r K(r,c,)\mu(c)dc, \quad \text{with kernel} \quad K(r,c,) = \frac{2}{\pi}\left[\left(\left(\frac{r}{c(t)}\right)^2 - 1\right) - \sin^{-1}\left(\frac{c(t)}{r}\right)\right] + 1 \tag{12}$$

Notice that the jet root velocity is given by

$$V_R(t) = \frac{dc}{dt} = \frac{W(t)}{\mu(c(t))} \tag{13}$$

Once $\mu(c)$ is determined by solving Eq. (12) for a given $\eta(r)$, the jet root velocity is obtained. By matching outer and inner solutions, (equating the inner expansion of the outer solution to the outer expansion of the inner solution), Faltinsen and Zhao, following Armand and Cointe (1987), obtained

$$\delta_J(t) = \frac{W^2}{2\pi}c\left(\frac{dc}{dt}\right)^{-2} = \frac{W^2}{2\pi}c\left(\frac{\mu(c)}{W}\right)^2 = \frac{c(t)}{2\pi}\mu^2(c(t)) = \frac{\mu^2(c)}{2\pi}\int_o^t \frac{W}{\mu}dt \tag{14}$$

and then

$$S_{RJ}(t) = 2\pi\delta_J(t)c(t) = \left(c(t)\mu(c(t))\right)^2 \tag{15}$$

It also follows from the matched asymptotic method that the jet velocity is twice the marching velocity of the jet root on the body surface, i.e.,

$$V_J(t) = 2V_R(t) \tag{16}$$

Consider firstly the normal impact problem, under constant velocity such that $U_n = -Wn_z$. Then from

$$-\frac{1}{2}\rho\int_0^{2\pi} V_j^{\,2}(V_j - V_R)\delta_J r(\theta)\Big|_{\partial C} d\theta \cong -\frac{1}{2}\rho\int_{S_{F0}} (\nabla\phi)^2 \frac{\partial\phi}{\partial n}dS = \frac{1}{2}\int_{S_{B0}} pU_n dS \tag{17}$$

it follows that

$$-\frac{1}{2}\rho V_j^{\,2}(V_j - V_R)S_{RJ} \cong -\frac{1}{2}\rho\int_{S_{F0}} (\nabla\phi)^2 \frac{\partial\phi}{\partial n}dS = -\frac{1}{2}W\int_{S_{B0}} pn_z dS = -\frac{1}{2}WF_z \tag{18}$$

Therefore,

$$\rho 4V_R^{\,3}\left(c(t)\mu(c(t))\right)^2 = WF_z \tag{19}$$

or, using Eq. (13),

$$F_z = 4\rho Wc^2(t)\frac{dc}{dt} = W\frac{d}{dt}\left(\frac{4}{3}\rho c^3\right) \tag{20}$$

or else

$$F_z = 4\rho\frac{W^2}{\mu(c(t))}c^2(t) = 4\rho\frac{W^2}{\mu(c(t))}\left(\int_0^t \frac{W}{\mu(c(t))}dt\right)^2 \tag{21}$$

Equation (20) agrees exactly with Zhao and Faltinsen's (1997) result,

$$F_Z = \rho \frac{2V}{\pi} c \frac{dc}{dt} \int_0^c \frac{2\pi r}{\sqrt{c^2 - r^2}} dr = \rho 4 V c^2 \frac{dc}{dt} \tag{22}$$

derived by pressure integration over the body surface, considering the case of constant velocity. They took the outer solution, $\phi = -(2W/\pi)\sqrt{c^2 - r^2}$; $r < c$, but only retaining in Bernoulli equation the term, $\phi_t|_{W=const} = -\frac{W}{\pi} \frac{2c}{\sqrt{c^2 - r^2}} \frac{dc}{dt}$. In fact, from the pressure integral $F_z = \rho \int_0^c 2\pi r \phi_t dr$, equation (20) is immediately recovered. If the velocity is considered to vary with time, $W = W(t)$, such that

$$\phi_t = -\frac{W}{\pi} \frac{2c}{\sqrt{c^2 - r^2}} \frac{dc}{dt} - \frac{2}{\pi} \sqrt{c^2 - r^2} \frac{dW}{dt} \tag{23}$$

one obtains instead

$$F_z = W \frac{d}{dt}\left(\frac{4}{3}\rho c^3\right) + \frac{4}{3}\rho c^3 \frac{dW}{dt} = \frac{d}{dt}\left(\frac{4}{3}\rho c^3 W\right) \tag{24}$$

Recognizing

$$M_a = \rho \int_0^c 2\pi r \frac{\phi}{W} dr = 4\rho \int_0^c \sqrt{c^2 - r^2}\, r dr = \frac{4}{3}\rho c^3 \tag{25}$$

as the added mass of a disk of radius c at infinite frequency, Eq. (24) can be written

$$F_z(t) = \frac{d}{dt}(M_a W) \quad , \tag{26}$$

claimed by Faltinsen and Zhao (1997) as generally correct. Note that the time derivative of the kinetic energy in the bulk of the fluid is

$$\frac{d}{dt}\left(\frac{1}{2} M_a W^2\right) = \frac{d}{dt}\left(\frac{2}{3}\rho c^3 W^2\right) = \left(2\rho c^2 \frac{dc}{dt} W + \frac{4}{3}\rho c^3 \frac{dW}{dt}\right) W \tag{27}$$

The first term within parenthesis

$$2\rho W^2 c^2(t) \frac{dc}{dt} = \frac{1}{2}\frac{d}{dt}\left(\frac{4}{3}\rho c^3\right) W^2 = \frac{1}{2}\frac{dM_a}{dt} W^2 \tag{28}$$

which corresponds to the time rate of the added mass in the bulk of the fluid, caused by the variation of the wetted surface geometry (the so-called wetted-correction), has exactly the same expression as the flux of kinetic energy through the spray, for a given W, as can be seen from Eq. (17) to Eq. (19). Closing this asymptotic solution, the equation of motion for the body, is then (W has been considered positive downward and the force positive upwards),

$$M \frac{dW}{dt} = -F_z(t) \tag{29}$$

For a sphere Faltinsen and Zhao obtained

$$c(t) = \sqrt{3RWt}\,; \qquad \mu(c(t)) = \frac{2}{3}\frac{c(t)}{R} \tag{29}$$

so that, for the case of constant velocity,

$$F_z = 6\rho W^2 R c(t) = 6\rho W^2 R \sqrt{3RWt} \tag{30}$$

that is exactly the result obtained by Faltinsen and Zhao who, as discussed above, used consistent pressure integration over the body, with the so-called wetted surface correction.

If the velocity is a function of time, $W = W(t)$, the force on the sphere will have an additional term, according to Eq. (24),

$$F_z = 6\rho W^2 R \sqrt{3RWt} + \frac{4}{3}\rho\left(\sqrt{3RWt}\right)^3 \frac{dW}{dt} \tag{31}$$

The equation of motion for the sphere, in the very start of body-surface interaction (such that $W \geq 0$,) reads

$$\begin{cases} \left(M + \frac{4}{3}\rho(3RWt)^{3/2}\right)\frac{dW}{dt} + 6\rho W^2 R\sqrt{3RWt} = 0 \\ W(0) = W_0 \end{cases} \tag{32}$$

One point should be observed here. If the term $-\frac{1}{2}\rho(\nabla\phi)^2$, in Bernoulli equation, is preserved in the inner expansion of the outer solution $\phi \cong -(2W/\pi)\sqrt{(2c)(c-r)}$; $r \to c^-$, gives rise to a logarithmic singularity in the vertical force of the form: $\lim_{r\to c^-} \frac{2}{\pi}\rho W^2 c^2 \log(1-\frac{r}{c})$. Such a singularity does not exist physically. It is a consequence of an infinity velocity derived from the pure outer solution. Remember that the outer solution assumes $\phi = 0$ everywhere on the free surface, what is not valid very close to the body, since this condition does not mimic the zero pressure condition, wherever the convective terms are not small.

4. The Analytic Mechanics Approach

Now, from the work by Pesce (2003), from the point of view of Analytic Mechanics, where the Lagrange equation for systems with explicit dependence of mass with position (and velocity) is properly derived, the force applied on the bulk of fluid is

$$F_Z^{BF} = \frac{d}{dt}\left(\frac{\partial T}{\partial W}\right) - \frac{\partial T}{\partial W} + \frac{1}{2}\frac{dM_a^B}{dz}W^2 + 2\dot{m}v_j \sin\alpha \tag{33}$$

Here, T is the kinetic energy of the bulk of fluid (excluding the jet) and $2\dot{m}v_j \sin\alpha$ corresponds to the reactive force (Metchersky force in Russian scientific literature), being $\dot{m}$ the flux of mass through the jets, v_j the absolute velocity of the fluid particles at the jet root and α the instantaneous angle of the jet with respect to the horizontal. After some algebraic manipulation, it yields

$$F_Z^{BF} = \frac{d}{dt}\left(M_a^B W\right) + 2\dot{m}v_j \sin\alpha \tag{34}$$

This is consistent with Eq. (26), apart the second term, which can be proved to be neglectable. As discussed above, the way to calculate the instantaneous added mass, the flux of mass and the velocity at the jet root can be found in Faltinsen and Zhao (1997), Cointe and Armand (1987). As a matter of fact, the analysis by Molin et al (1996), after asymptotic analysis by Armand and Cointe on the particular and important case of a circular cylinder of radius R, proved that, being $\varepsilon = \sqrt{Wt/R}$ the small parameter, the thickness of the jet root is of order $\varepsilon^3\pi R/4$. And the velocity at the jet root is of order $\varepsilon^{-1}W$. It then follows that mass flux through the jets is of order $\varepsilon^2\pi\rho RLW/2$ and so, $\dot{m}v_j$ is of order $\varepsilon\pi\rho RLW^2/2$, (so is the vertical force applied on the bulk of fluid corresponding to the flux of momentum through the jets), being L the cylinder's length. Contrarily, the energy flux is of order $\pi\rho RLW^3$ and $O\left(\frac{d}{dt}\left(M_a^B W\right)\right) = O\left(\varepsilon^{-2}\pi\rho RLW^2\right)$. So, the impact force on the body, for the case of a cylinder, could be consistently written, according to Pesce (2003) as

$$F_Z = -\frac{d}{dt}\left(M_a^B W\right) \tag{35}$$

On the other hand, if, (see, e.g., Wu (1998)), the third and fourth terms appearing on the right hand side of Eq. 33 were not considered at all, a different assertive would be obtained, according to which $F_Z = -M_a^B\frac{dW}{dt} - \frac{1}{2}W\frac{dM_a^B}{dt}$.

4. Experiments with a Hemisphere

Crivellari (2002) conducted some experiments with a hemisphere impacting on the free surface of water. The hemisphere is made up of acrylic, with a steel plate inside, where a piezoelectric accelerometer is fixed. The mass of the hemisphere may be adjusted by adding steel plates to the original one. The acrylic shell is 6mm thick with radius 175mmm. Standard structural analysis has been conducted to guarantee not only structural integrity but also that the piece is rigid enough to allow deformations to be unimportant. Such analysis considered an initial impacting velocity of 3.0m/s, a maximum value for the mass of 50kg and took the asymptotic model to calculate acceleration and impacting force. A series of structural impact experiments, using an instrumented hammer, has also been conducted, in order to identify modes and natural frequencies of the steel structure supporting the accelerometer, guaranteeing that its response frequency is appropriate.

Figure 2 The acrylic hemisphere.

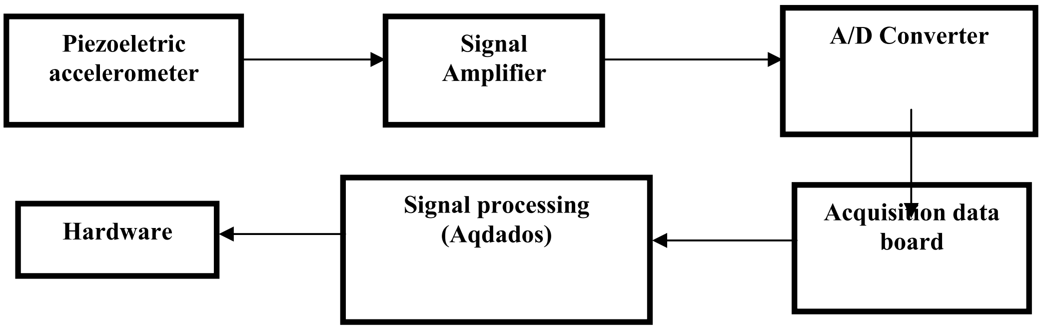

Figure 3 Instrumentation and signal processing.

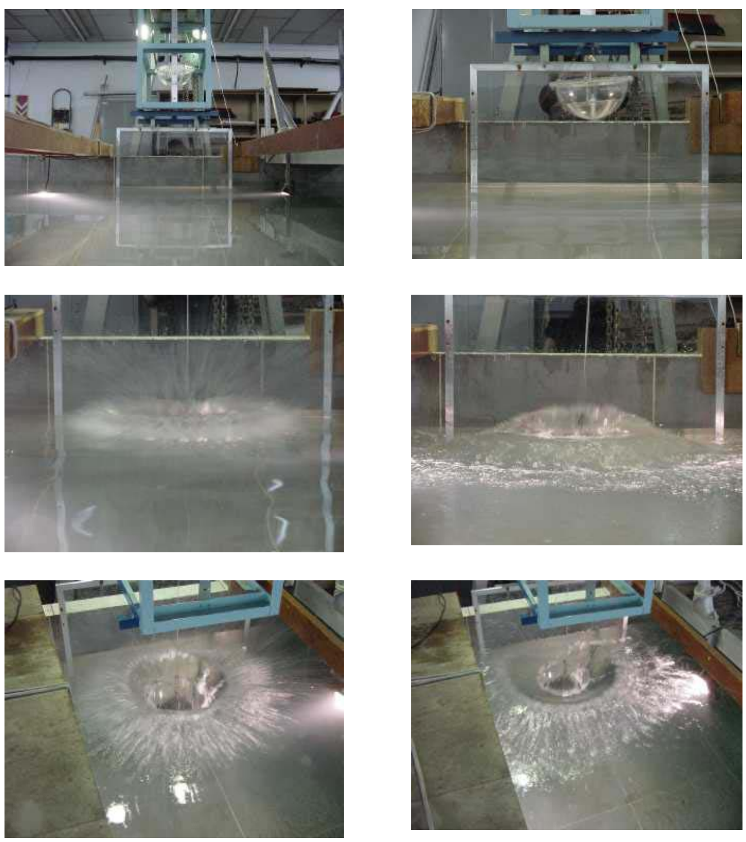

Figure 4 A sequence of an impacting experiment. A hemisphere falls from 1 a height of meter, striking an initially quiescent water free-surface.

The experiments were conducted with 4 different values for the mass: 7.88, 6.81, 5.730 and 4.58 Kg. The hemisphere was left in free-fall, from a height of 1.0 meter, striking an initially quiescent water free surface. Figure 4 shows a sequence of an impacting experiment. Figure 5 presents a typical acceleration record. By analyzing the resulting graphics, it is possible to see that the impact of the hemisphere occurred at $t \approx 3.8s$. However, it can be distinguished two acceleration peaks, Fig. 5. The first one corresponds to the impact of the sphere. But the other represents the impact of the "diametrical web". Only the first peak should be considered, for comparison with analytical results. After this, there is a decaying oscillatory motion, corresponding to the heave movement of the body, once the buoyancy forces start to act together with the gravitational one. Figure 6 shows the comparison between experiments and the analytical model, by integrating numerically Eq. (32). It can be seen that peak determined from the analytical model overestimates, by circa 30%, the experimental measures, Fig. 6. Also, the ramp shown in the analytical estimates are much more abrupt, than those observed through measured acceleration. Reasons for these discrepancies are not well clear and deserve further investigation, even considering the sensor and instrumentation used. Nevertheless, the experiment recovers qualitatively the results by Moghisi and Squire (1981).

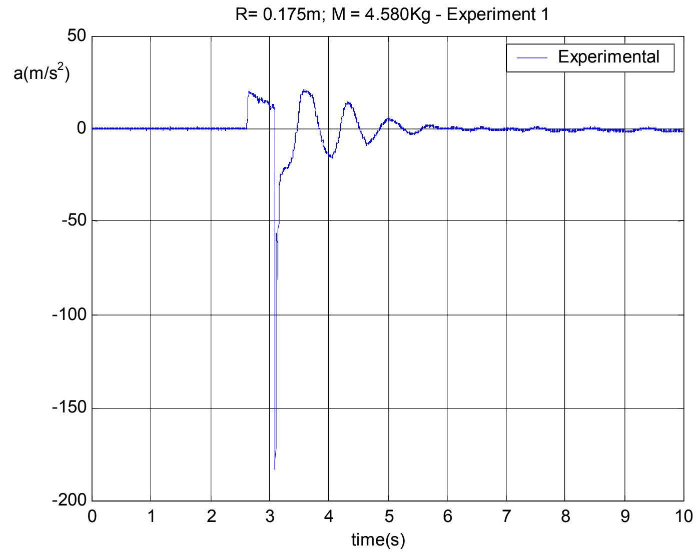

Figure 5 An illustrative acceleration time history for a hemisphere (radius = 0.175m, mass = 4.580Kg).

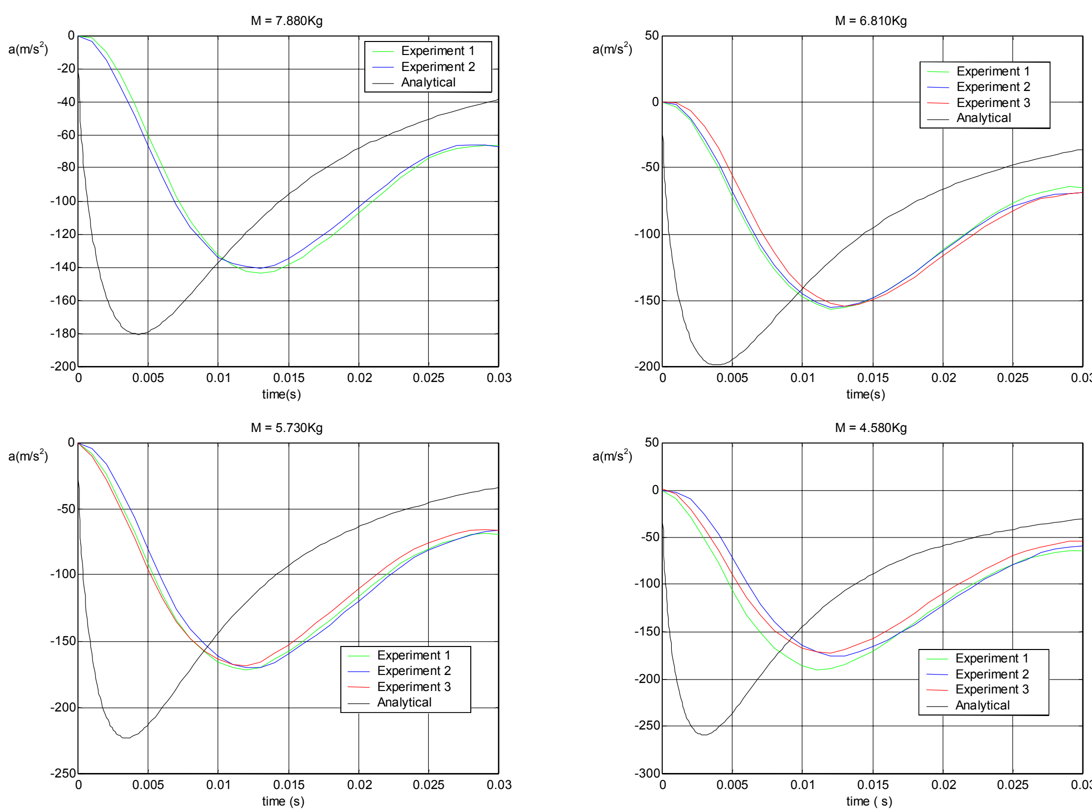

Figure 6 Comparison between experiments and analytical model for a hemisphere (R=0.175m) falling from a height of 1.0 meter, striking the initially quiescent water free surface. Four values of mass are considered.

4. Conclusions

This work addressed the classical hydrodynamic impact problem of a bluff body striking a quiescent free surface, focusing on a brief review of asymptotic methods. The result by Faltinsen and Zhao (1997), concerning a sphere were recovered and expanded to consider the problem of variable velocity. Besides, the analytical formulation for the impact force calculation, shown to be consistent in Pesce (2003), recovers, up to third order in the very small impacting time-scale, the intuitive equation $F_Z = -\frac{d}{dt}\left(M_a^B W\right)$, encountered in many papers, as in Faltinsen and Zhao (1997). This has been discussed, properly, on a physical and mathematical basis and from the point of view of analytic mechanics.

Few experimental results with a hemisphere were presented and compared with the asymptotic solution. Some discrepancies were found. The asymptotic solution overestimates the force peak by circa 30%. Reasons for these discrepancies are not well clear and deserve further investigation, even considering the sensor and instrumentation used. Nevertheless, experimental and analytical results present the same qualitative trend.

3. Acknowledgement

First author acknowledges a research grant, no. 304062/85, from CNPq - Brazilian National Research Council. Second author has been supported by a graduate scholarship from CAPES, Brazil. Third author thanks FAPESP, the S. Paulo Sate Research Foundation, for an undergraduate scholarship, process no. 00/11577-7. First author thanks Professor A. Troesch, University of Michigan, for introducing him to the impact problem, indicating a bench of valuable references and for his time, spent with me in very interesting discussions.

5. References

Cointe, R. And Armand, J.L., 1987, "Hydrodynamic Impact Analysis of a Cylinder". ASME J Offshore Mech Arctic Eng., Vol.109, pp.237-243.

Cooker, M.J. and Peregrine, D.H., 1995, "Pressure-impulse Theory for Liquid Impact problems". J Fluid Mech (1995), Vol.297, pp.193-214.

Faltinsen, O.M. and Zhao, R., 1997, "Water Entry of Ship Sections and Axisymmetric Bodies". Agard Ukraine Inst on Hydromechanics Workshop on High Speed Body Motion in Water, 1997.

Korobkin, A.A. and Pukhnachov, V.V., 1988, "Initial Stage of Water Impact". Ann Review Fluid Mech, Vol.20, pp.159-185.

Lamb, H., 1932, "Hydrodynamics", Dover Publications, N.Y., 6th Ed., 1932, 738 pp.

Miloh, T., 1981, "Wave Slamm on a Sphere penetrating a Free Surface". J Eng Mathematics, Vol.15, pp.221-240.

Moghisi, M. and Squire, P.T., 1981, "An Experimental Investigation of the Initial Force of Impact on a Sphere Striking a Liquid Surface". J Fluid Mech, Vol.108, pp. 133-146.

Molin, B., Cointe, R. and Fontaine, E., 1996, "On Energy Arguments Applied to the Slamming Force". 11th Int Workshop on Water Waves and Floating Bodies, Hamburg.

Pesce, C.P., 2000, "Reviewing Some Fundamental Aspects of The Classical Free Surface Hydrodynamic Impact Problem", Monograph, Escola Politécnica, University of São Paulo, 54 p.

Pesce, C.P., 2003, "The Application of Lagrange Equations to Mechanical Systems with Mass Explicitly Dependent on Position". Journal of Applied Mechanics (to appear).

Shiffman, M. and Spencer, D.C., 1951, "The Force of Impact on a Cone Striking a Water Surface (Vertical Entry)". Communications on Pure and Applied Mathematics, Vol IV, no 4, 1951, pp. 379-418.

Von Kármán, T., 1929, "The Impact of Seaplane Floats During Land". National Advisory Committee for Aeronautics (NACA). Technical Note no. 321.

Wagner, H., 1931, "Landing of Seaplanes". National Advisory Committee for Aeronautics (NACA). Technical Memorandum no. 622.

Wu, G.X., 1998, "Hydrodynamic Force on a Rigid Body During Impact with Liquid". Journal of Fluids and Structures, Vol.12, pp. 549-559.