Mathematical Model to Determine the Hydrodynamic Coefficients on a Semi-submersible Platform

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Abstract. The oil production in deepwater demands a previous knowledge of the floating unit’s behavior when anchored under specific environmental conditions and the programs of numerical simulation are fundamental tools whose working depends upon hydrodynamic coefficients of studied body. As an alternative to calculate these properties of the platforms was proposed the Hooft’s approach (1971). This method consists in subdivide floating unit in simple elements, as cylinders (with circular section or not), plane thin plates or arbitrary elements whose drag and added mass coefficients are known. Based on this model was created a powerful and independent tool to supply information to others programs in order to realize simulations and, because of its facility to use and little details necessary about the final forms of platform, it can even use the Hooft’s model as a first estimative in design stage with an excellent approximation. At last, this work shows that the results obtained from this method to hydrostatic restoring coefficients, added mass coefficients and wave excited forces are very close with respect to those calculated by more sophisticated programs. It is also developed an equationing to estimate the viscous drag coefficients of platforms and the foreseen behavior of floating unity, with or without these data, is compared.

Keywords. Hydrodynamic forces, Added mass, Wave forces, Viscous damping.

1. Introduction

Due to oil exploration in deepwater there are today many numerical simulation programs and everyone need some information about the hydrodynamic properties of platform. To determine this data there are programs that solve the potential problem of flow around the body and calculate the added mass, hydrostatic restoring, potential damping coefficients and the wave exciting force.

However, the licenses of these programs are expensive, their time of making up is long and many details about the floating unit are required. As a solution to these problems was developed a program using the Hooft’s method (1971).

This approach assumes that the platform can be subdivided into simple elements whose added mass and viscous drag are known. This simple elements are usually cylinders, plane plates, spheres and ellipsoids with theirs principal dimensions knew. Also it is assumed that one element does not affect the properties of its neighbors and that the total value of a characteristic of the platform is the total sum of each individual value.

Another advantage of the Hooft’s method is the consideration of viscous damping of floating unit, whose effect is generally in semi-submersible platform much more significant than the potential.

2. Hydrodynamic and hydrostatic forces

The forces exerted over a body immersed in a fluid are present in the classical equation of movement in the six degrees of freedom, where $j=i=x_i$ (see Fig. (1)):

$$\sum_{j=1}^{6} B_{ij} \delta x_j(t) = F_j(t)$$

where:

$$B_{ij} = (\delta_{ij} m_i + a_{ij}) \frac{d^2}{dt^2} + b_{ij} \frac{d}{dt} \frac{d}{dt} + c_{ij}$$

$a_{ij}$: added mass or added moment of inertia in $i$-direction due a motion in $j$-direction

$b_{ij}$: damping in $i$-direction due a motion in $j$-direction
$c_{ij}$: restoring coefficient in $i$-direction due a motion in $j$-direction

$m_i$: mass or moment of inertia of body

$\delta_{ij} = 1$ if $i = j$ or $0$ if $i \neq j$

$s_j$: displacement of unit in $j$-direction

$F_i$: force in $i$-direction

$x_1 = x$: Surge
$x_2 = y$: Sway
$x_3 = z$: Heave
$x_4 = \phi$: Roll
$x_5 = \theta$: Pitch
$x_6 = \psi$: Yaw

Figure 1. Coordinate system and the six directions of movement.

These forces can be divided in two components: the reaction and the wave forces, being the first subdivided in:

- **Hydrostatic forces**: forces in phase with displacement of body (restoring and spring forces).
- **Hydrodynamic forces**: forces in phase with acceleration of body (added mass force) and forces in phase with velocity of body (damping force).

### 2.1. Added mass

The inertial force effects or added mass are defined as the ratio between the force in phase with acceleration and the acceleration ($\ddot{s}$):

$$F_i = F_{ia} \sin(\omega t) + F_{ia}(t)$$

$F_i$: total measured force

$F_{ia}$: force in phase with acceleration

$F_{ia}$: other forces

so

$$a_i = \frac{F_i}{-s_i \omega^2}$$

where the position of platform is $s_i = s_i \sin(\omega t)$ and its acceleration $\ddot{s}_i = -s_i \omega^2 \sin(\omega t)$.

The influence of oscillations was initially neglected and, after new implementations, the system can select the added mass coefficient as function of oscillation frequency.

The geometry of the platform was divided in the following simple elements to perform the calculus of added mass:

#### 2.1.1. Elements of arbitrary form

This elements are usually spheres, final planes of cylinders or other bodies of any form whose added mass in the $x$ ($a_{dx}$), $y$ ($a_{dy}$) and $z$ direction ($a_{dz}$) are known and from that can be determined the rest of them.

It is assumed that their added mass is concentrated in the center of body’s element $(x_{d1}, y_{d1}, z_{d1})$.

Using the process developed by Hoof (1971), the definition of coordinate system of Fig. (1) and considering the center of gravity of unit as $(x_0, y_0, z_0)$, the first line of added mass matrix were calculated:

$$a_{dx} = a_{dys} = a_{dz} = 0$$

$$a_{dy} = a_{dys} = a_{dx} = a_{dz} = 0$$

$$a_{dz} = a_{dys} = 0$$

Other coefficients can be obtained in Hoof (1971).

#### 2.1.2. Circular cylinders

This elements are usually spheres, final planes of cylinders or other bodies of any form whose added mass in the $x$ ($a_{dx}$), $y$ ($a_{dy}$) and $z$ direction ($a_{dz}$) are known and from that can be determined the rest of them.

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$$a_{dz} = a_{dys} = 0$$

Other coefficients can be obtained in Hoof (1971).
To these elements was considered that its added mass per unit length for a flow perpendicular to the longitudinal axis is known and that its cross section is constant over axis. The same development that was presented by Hooft (1971) to an arbitrary flow is used.

To write the equations down it was defined the following variables:

- $C_m$ = added mass coefficient per unit length of the cylinder for a perpendicular arbitrary flow;
- $O_d$ = cross-section area of cylinder;
- $(x_{d1}, y_{d1}, z_{d1})$ and $(x_{d2}, y_{d2}, z_{d2}) = $ coordinates center of the ends of cylinder;
- $(x_{o1}, y_{o1}, z_{o1}) = $ center of gravity of the platform;

- $l_d = \left[ (x_{d2} - x_{d1})^2 + (y_{d2} - y_{d1})^2 + (z_{d2} - z_{d1})^2 \right]^{\frac{1}{2}}$ = length of element;
- $r = \left[ (x_r - x_{d1})^2 + (y_r - y_{d1})^2 + (z_r - z_{d1})^2 \right]^{\frac{1}{2}}$ = distance from end 1 to a transversal section;
- $a_j = \int_0^{r_d} \rho C_{O_d} O_d dr$, $S_d = \int_0^{r_d} r da_d$ and $l_d = \int r^2 da_d$.

$\alpha$, $\beta$ and $\gamma$ are the angles between the element and its projection in the plane YZ, XZ and XY respectively and can be written as:

$$\sin \alpha_d = \frac{x_{d2} - x_{d1}}{l_d}; \quad \sin \beta_d = \frac{y_{d2} - y_{d1}}{l_d}; \quad \sin \gamma_d = \frac{z_{d2} - z_{d1}}{l_d}$$

(9)

The added mass of element $d$ in the $i$-direction due a motion in $j$-direction ($a_{ij}$) can be obtained by:

$$a_{d_{ij}} = a_d \cos^2 \alpha_d$$

$$a_{d_{ij}} = -a_d \sin \beta_d \sin \alpha_d$$

$$a_{d_{ij}} = -S_d \sin \alpha_d \sin \beta_d \sin \gamma_d$$

(10)

Some expressions were omitted here and the complete matrix of coefficients can be found in Hooft (1971), besides there are equations for other elements, as asymmetric or rectangular cylinders, in the same reference. The next three equations were corrected from original paper:

$$a_{d_{ij}} = L_d \sin \alpha_d \sin \beta_d - S_d \left[ \left( x_{d1} - x_0 \right) \sin \beta_d + \left( y_{d1} - y_0 \right) \sin \alpha_d \right] - a_d \left[ \left( z_{d1} - z_0 \right) \left( x_{d1} - x_0 \right) \sin \alpha_d \sin \gamma_d \right] - \left( x_{d1} - x_0 \right) \left( y_{d1} - y_0 \right) \sin^2 \gamma_d \left( z_{d1} - z_0 \right) \sin \alpha_d \sin \beta_d \sin \gamma_d + \left( x_{d1} - x_0 \right) \left( y_{d1} - y_0 \right) \sin \alpha_d \sin \beta_d \sin \gamma_d$$

$$a_{d_{ij}} = -L_d \sin \beta_d \sin \gamma_d - S_d \left[ \left( x_{d1} - x_0 \right) \sin \beta_d + \left( y_{d1} - y_0 \right) \sin \alpha_d \right] - a_d \left[ \left( z_{d1} - z_0 \right) \left( x_{d1} - x_0 \right) \sin \alpha_d \sin \gamma_d \right] - \left( x_{d1} - x_0 \right) \left( y_{d1} - y_0 \right) \sin^2 \alpha_d \left( z_{d1} - z_0 \right) \sin \gamma_d \left( x_{d1} - x_0 \right) \sin \alpha_d \sin \beta_d \sin \gamma_d + \left( x_{d1} - x_0 \right) \left( y_{d1} - y_0 \right) \sin \alpha_d \sin \beta_d \sin \gamma_d$$

$$a_{d_{ij}} = -L_d \sin \gamma_d \sin \alpha_d - S_d \left[ \left( z_{d1} - z_0 \right) \sin \alpha_d + \left( y_{d1} - y_0 \right) \sin \gamma_d \right] - a_d \left[ \left( y_{d1} - y_0 \right) \left( x_{d1} - x_0 \right) \sin \gamma_d \sin \alpha_d \sin \beta_d \sin \gamma_d \right] - \left( z_{d1} - z_0 \right) \left( x_{d1} - x_0 \right) \sin^2 \beta_d \left( y_{d1} - y_0 \right) \sin \gamma_d \left( y_{d1} - y_0 \right) \sin \alpha_d \sin \beta_d \sin \gamma_d + \left( z_{d1} - z_0 \right) \left( y_{d1} - y_0 \right) \sin \alpha_d \sin \beta_d \sin \gamma_d$$

(11)

2.2. Hydrostatic restoring coefficient

The restoring coefficients are defined as ratio between the force in the $i$-direction and the displacement in $j$-direction ($c_{ij} = \frac{F_{ij}}{S_j}$).

In this model, was assumed that the unit is not moored and, therefore, no restoring forces exist in $x$ and $y$-directions or no moment around Z-axis. Considering:

- $(x_{d1}, y_{d1}, z_{d1}) = $ coordinates center of immerged end of element;
- $(x_{o1}, y_{o1}, z_{o1}) = $ center of gravity of platform;
- $(x_{b1}, y_{b1}, z_{b1}) = $ center of buoyancy of platform;
- $l_d = $ length of element (Eq. (6));
- $\gamma = $ angle between the element and its projection on plane XY (Eq. (9));
• \( I_{dx} = \int y^2 dS \) = athwart moment of water plane of cylinder;
• \( I_{dy} = \int x^2 dS \) = longitudinal moment of water plane of cylinder;

Some of the restoring coefficients of the platform are given by Eq. (12), the complete set of equations can be found in Hooft (1971):

\[
c_a = \sum -\frac{\rho g O_c}{\sin \gamma_d} \\
c_{ap} = C_{ap} \sum -\frac{\rho g O_c}{\sin \gamma_d} (y_{d1} - y_0) \\
c_{aq} = C_{aq} \sum \left(\frac{\rho g O_c}{\sin \gamma_d} x_{d1} - x_0 \right) \left(y_{d1} - y_0\right)
\]

(12)

2.3. Damping

The total damping of the unit is composed by a potential parcel and a viscous parcel. The first have its coefficients calculated by WAMIT and, in a semi-submersible platform, its effects are relatively smaller than the viscous, whose development follows down.

\[
F_{dj} = b_j x_j + q_j x_j \left| x_j \right|
\]

(13)

where:
• \( F_{dj} \) = damping force in \( j \)-direction;
• \( b_{ij} \) = potential damping in \( i \)-direction due to a motion in \( j \)-direction;
• \( q_{ij} \) = viscous damping in \( i \)-direction due to a motion in \( j \)-direction;
• \( x_j \) = motion in \( j \)-direction, as defined on Fig. (1).

To this property it is used the same theory of added mass, i.e., the platform was divided in elements of arbitrary form or cylinders with their drag coefficient known.

2.3.1. Elements of arbitrary form

It is assumed that its damping is concentrated in the center of element and that the damping coefficients in \( x \) (\( q_{dx} \)), \( y \) (\( q_{dy} \)) and \( z \)-direction (\( q_{dz} \)) are known. Using the same proceeding of Hooft (1971) and the same definitions of 2.1.1, it is obtained the expressions of first line of entire matrix:

\[
q_{dx} = q_{dy} = 0 \\
q_{dz} = q_{dz} = q_{dz} \left( z_{d1} - z_0 \right) \left| z_{d1} - z_0 \right| \\
q_{dxy} = q_{dxy} = q_{dxy} \left( y_{d1} - y_0 \right) \left| y_{d1} - y_0 \right| \\
q_{dxx} = -q_{dxx} \left( x_{d1} - y_0 \right)
\]

(14)

2.3.2. Cylinders

This element requires previous knowledge of the viscous drag coefficient of a perpendicular flow in three directions of a coordinate system fixed in the longitudinal axis of cylinder by \( x \)-axis, \( y \)-axis horizontal and the other perpendicular to both, respectively \( q_{dx}' \), \( q_{dy}' \), and \( q_{dz}' \). Just some expressions were presented here, for the complete set of equation see Hooft (1971). All elements of type \( q_{dij}' \) are used as damping per unity length.

The drag in the direction of the axis of cylinder is considered null and for circular cylinder the other two are the same value. Using the same definitions of 2.1.2, some equations are developed here and the complete matrix of coefficients can be found in Hooft (1971):

\[
q_{dx} = q_{dx} \left| \cos \alpha_d \right| \\
q_{dy} = -q_{dy} \left| \sin \beta_d \sin \alpha_d \right| \left| \cos \alpha_d \right|
\]

(15)

so \( q_{dij} = I_{ij} \cdot q_{dij}' \), and:

\[
q_{dxx} = q_{dxx} \left( y_{d1} - y_0 \right) - q_{dyy} \left( z_{d1} - z_0 \right) + S_{dxx} \sin \beta_d - S_{dyy} \sin \gamma_d \\
q_{dyy} = q_{dyy} \left( x_{d1} - y_0 \right) - q_{dyy} \left( y_{d1} - y_0 \right) + S_{dyy} \sin \alpha_d - S_{dyy} \sin \beta_d \\
q_{dxx} = q_{dxx} \left( x_{d1} - y_0 \right) - q_{dxx} \left( y_{d1} - y_0 \right) + S_{dxx} \sin \alpha_d - S_{dxx} \sin \beta_d \\
q_{dxx} = q_{dxx} \int\left( y_{d1} - y_0 \right) |y_{d1} - y_0| dr - q_{dxx} \int\left( z_{d1} - z_0 \right) |z_{d1} - z_0| dr
\]

(16)
\[ q_{d,0} = q_{\text{det}} \left( z_0 - z \right) \int_0^\mu \left( z_0 - z \right) dr - q_{\text{det}} \left( x_0 - x \right) \int_0^\mu \left( x_0 - x \right) dr \]

where:

\[ S_{d,j} = \int_0^\mu q_{d,j} \, rdr \]  

(17)

2.4. Wave-excited forces

It was defined a system coordinate \((\xi, \eta, \zeta)\) in which \(\xi\) is coincident with the direction of motion waves (Fig. 2). This new system has its origin coincident with the \((x, y, z)\) system and the \(z\) axis and \(\zeta\) axis have the same direction.

![Figure 2. Definitions of coordinates systems.](image)

A linear potential theory of waves was adopted and the acceleration of water particles can be expressed as:

\[ \ddot{\zeta} = \mu_x \omega^2 \zeta_x \cos(\omega t - \kappa \xi); \quad \ddot{\eta} = -\mu_y \omega^2 \zeta_y \sin(\omega t - \kappa \xi) \]

(18)

where:

- \(\zeta_x\) = wave amplitude;
- \(\kappa = \frac{2\pi}{\lambda}\) = wave number;
- \(\omega\) = wave frequency;
- \(\omega^2 = \frac{g \tanh(\kappa h)}{h}\) = gravity; \(h\) = water depth;
- \(\mu_x = \frac{\sinh[\kappa(h + \zeta)]}{\sinh(\kappa h)}\); \(\mu_3 = \frac{\cosh[\kappa(h + \zeta)]}{\sinh(\kappa h)}\).

The wave exciting force can be divided in two parts, where the first parcel (Froude-krilov force) corresponds to the product of the pressure and the area. To obtain the second parcel (inertia force), the acceleration of the wave particles is computed in \((\xi, \eta, \zeta)\) system, transformed to \((x, y, z)\) and multiplied by the sum of the mass and added mass of the element. To cylinders, it is necessary the integration on the length.

After the compute forces acting on each element of the unit, the total wave exciting force can be obtained by the summation of the contribution of each element.

As used in Hooft (1971) the forces \(X_g, Y_g, Z_g\) and \(K_g, M_g, N_g\) are defined relative to the \((\xi, \eta, \zeta)\) coordinates, while forces and moments without the index \(g\) are related to the \((x, y, z)\) system.

The platform can be divided in elements of which the total mass can be assumed concentrated in one point, elements which can be assumed to be long (cylinders) and planes of which it can be assume that the dimensions are small relative to the wave length.

2.4.1. Small elements

It was used the same procedure presented by Hooft (1971), considering the center of element in \((x_{g1}, y_{g1}, z_{g1})\) and the known added mass of element in the three directions as \(a_{dx1}, a_{dy1}\) and \(a_{dz1}\). The total set of equations can be found in Hooft (1971).

2.4.2. Cylinders
These elements have their diameter considered small relative to the wave length but their own length is not and the forces must be integrated over it.

The force in $\xi$-direction ($X_g$) can be deduced when the components of the acceleration of the water particles along $\xi$- and $\zeta$-direction are known.

$$X_g = X_{\xi g} + X_{\zeta g},$$

(19)

where $X_{\xi g}$ and $X_{\zeta g}$ is obtained multiplying the acceleration by the sum of the mass with added mass in respectively direction.

To obtain the force in $\zeta$-direction ($Y_g$) the same procedure is used and due to the acceleration in $\xi$- and $\zeta$-direction, forces in $\eta$-direction is also generated. In this way, the forces in $x$, $y$ and $z$-directions can be determined by:

$$X = X_g \cos \mu - Y_g \sin \mu; \quad Y = X_g \sin \mu + Y_g \cos \mu; \quad Z = Z_g$$

(20)

Based on this model and adding the corresponding braces, the moments $K$, $M$ and $N$ can be determined. All equations are presented in Hooft (1971).

### 3. The choice of added mass and damping coefficients

The proximity of Hooft’s model to tank’s tests is closely related to the choice of hydrodynamic coefficients for simple forms, as demonstrated by Takagi et al. (1985). Based on this fact, curves of these coefficients as function of oscillatory flow for alternative forms and as function of geometry parameters were researched and implemented.

In development of this work was assumed that the proximity of an element to water surface do not affect the flow around the simple element and that one is not affected by the neighbor element.

For circular cylinders the oscillatory flow was considered as dependent of Kaulegan-Carpenter number ($KC = u_0 T / D$; $T$: oscillation period of cylinder; $D$: diameter of cylinder; $u_0$: amplitude of fluid velocity) and a factor $eta (Re / KC = D^2 / T \nu$; $Re$: Reynolds number, $\nu$: fluid viscosity) representing the relation between flow regime (Re) and oscillation parameters. These curves of added mass and drag coefficients of infinite cylinders were found in books of wave forces theory, as Sarpkaya (1981), while in fluid mechanics books the motions of 2-D bodies are always considered linear and steady flow. For other geometries none information about the dependency of oscillation was found, but, considering that the main effect of waves is concentrated in surface region and that most common columns are circular, no more comprehensive research was performed and a steady flow was assumed. An example of used graphic for $C_D$ (drag coefficient) is showed in Fig. (3).

Another important factor is the influence of geometry proportions in the coefficients. These relationships can be given by diameter-length ratio ($D/L$), smaller length-bigger length sides of cross section ($b/d$ in Fig. (4)) or other ratios that present significant variation of added mass or drag.

For circular cylinders the first ratio is the only important and the correction due this proportion is done multiplying the original drag or added mass coefficient for infinite cylinder by a correction factor which is the coefficient of a finite cylinder divide by the coefficient of an infinite cylinder. The example for $C_D$ coefficient is presented down:

$$y = 52.36x^4 - 62.193x^3 + 25.029x^2 - 4.7735x + 0.9924$$

$$R^2 = 0.9936$$

![Figure 3. Drag coefficient ($C_D$) as function of $KC$ and $\beta$ for infinite circular cylinders, from Sarpkaya and Isaacson (1981), and multiplicative correction factor for drag coefficient of circular cylinder as function of diameter / length ratio ($D/L$). Reference data from Hoerner (1958).](image-url)
For rectangular cross section cylinders the main form coefficients are the ratio between the length of its sides and the ratio curvature of a corner by a prominent dimension. This is done in two stages: first the correction due a different rectangular cross section and second due a curvature of its corners.

For drag coefficient, the original value is corrected due to proportion between its sides as explained above to circular cylinders. This is done multiplying the original coefficient for infinite cylinder with some specific proportions (1:1, 1:2, 2:1), retired from Delany (1953), at a given Reynolds number by a correction factor interpolated by reference data for each case (b/d <1, b/d =1 and b/d >1) and that represents a percentual of original cylinder. This method allows implement only one curve of data and do only corrections in their values.

The added mass is calculated multiplying an imaginary water mass (ρd^2/π; ρ: water density) by a factor interposed from reference and that follows in Fig. (4).

In the second stage, the coefficients are corrected due curvature in the corners of cylinder. This effect is represented by ratio curvature radius of corner / length of side perpendicular to flow (r/d) and the factors that correct the original values are function of this value and Reynolds number.

The cylinders were classified into three categories: b/d <1, b/d =1 and b/d >1, according to adopted by Delany (1953) and implemented in the first correction. For each category the relationship r/d was subdivided in other scale, as presented in Fig. (5).

Based on these classifications several curves of drag correction factors were interposed, one for each b/d and r/d, having as parameter the Reynolds number. It follows down an example of a curve utilized:

\[
y = -9E-31x^5 + 5E-24x^4 - 1E-17x^3 + 1E-11x^2 - 4E-06x + 1,0736
\]

\[
R^2 = 0.9961
\]

For example, rectangular plates were assumed having a \(C_D\) (drag coefficient) independent of Reynolds number in the interval between \(10^5\) and \(10^6\) and given by a function interpolated by data from Hoerner (1958) while its added mass is given by \(C_m^w \rho/4 \pi bd^2\), where \(b\) and \(d\) are defined as for rectangular cross section of cylinders and with b/d < 1, \(\rho\) is the water density and \(C_m^w\) is a coefficient interpolated by experimental data from Sarpakaya (1981).
Circular plates have their $C_D$ for Reynolds number ($Re$) between $10^5$ and $10^7$ equal to 1.17 (Hoerner (1958)) and $C_m$ (added mass value) equal to $\frac{8}{3} \rho a^3$ (Sarpkaya (1981)), independent of $Re$, where $a$ is the radius of plate.

4. Results

Some platforms were modeled and their hydrodynamic properties estimated by Hooft’s method demonstrating good adherence with numerical results generated by more sophisticated programs, as WAMIT.

The example presented in this work is a semi-submersible proposed by ITTC for studies of wave forces. It is composed by eight circular columns, two submersed pontoons and 16 bracings. In the joining of two elements were used circular plates and in the end of pontoons rectangular plates. The pontoons are rectangular and have their corners rounded.

The viscous damping calculated by Hooft’s method and the Wamit’s results are presented in Tab. (1). A linearization process can be done with the quadratic viscous damping showing that it is much more significant in this platform.

### Table 1. Damping coefficients of platform calculated by Hooft and by WAMIT and added mass coefficients wit their percentual difference.

<table>
<thead>
<tr>
<th>Damping</th>
<th>Hooft (viscous)</th>
<th>Wamit (potential)</th>
<th>Added mass</th>
<th>Wamit</th>
<th>Hooft</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b(1,1)$</td>
<td>$177.52$</td>
<td>$0.00039$</td>
<td>$a(1,1)$</td>
<td>$9.866,64$</td>
<td>$9.252,98$</td>
</tr>
<tr>
<td>$b(2,2)$</td>
<td>$1.090,00$</td>
<td>$0.00076$</td>
<td>$a(5,1) = a(1,5)$</td>
<td>$-107.370,00$</td>
<td>$-47.721,50$</td>
</tr>
<tr>
<td>$b(3,3)$</td>
<td>$1.679,00$</td>
<td>$4.708,49$</td>
<td>$a(2,2)$</td>
<td>$26.743,10$</td>
<td>$26.419,03$</td>
</tr>
<tr>
<td>$b(4,4)$</td>
<td>$46.800.000,00$</td>
<td>$5.046.509,84$</td>
<td>$a(4,2) = a(2,4)$</td>
<td>$308.296,00$</td>
<td>$292.718,00$</td>
</tr>
<tr>
<td>$b(5,5)$</td>
<td>$267.000.000,00$</td>
<td>$4.409.034,53$</td>
<td>$a(3,3)$</td>
<td>$43.307,46$</td>
<td>$45.841,00$</td>
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<tr>
<td>$b(6,6)$</td>
<td>$153.000.000,00$</td>
<td>$0.00034$</td>
<td>$a(4,4)$</td>
<td>$41.396.331,25$</td>
<td>$43.742.957,40$</td>
</tr>
<tr>
<td>$b(5,1) = b(1,5)$</td>
<td>$1.011,39$</td>
<td>$-0.00039$</td>
<td>$a(5,5)$</td>
<td>$39.094.993,75$</td>
<td>$46.457.079,75$</td>
</tr>
<tr>
<td>$b(4,2) = b(2,4)$</td>
<td>$14.105,8$</td>
<td>$0.00218$</td>
<td>$a(6,6) = a(4,4)$</td>
<td>$29.237.100,00$</td>
<td>$34.000.973,00$</td>
</tr>
</tbody>
</table>

The added mass coefficients, for infinite period, of ITTC platform calculated by both programs and the difference between the Hooft’s results and the Wamit’s are presented in Tab. (1).

The wave excited forces were calculated for a wave with unitary amplitude and an incident direction with longitudinal equal to 45°. Their amplitudes were compared with the results generated by WAMIT and are showed in Fig. (6).

The restoring coefficients are in conformity with analytical expression and because of this their values are not exposed.

In order to compare the results of numerical simulations, it was used the program NOT (Numerical Offshore Tank) with the same platform described earlier in a ocean with a depth of 594 m. The unity was anchored by eight lines that arrive from the extremes of pontoon, two by two having an angle of 45° between him.

The environmental conditions were that extremes founded in Marlin field, Rio de Janeiro. The current has a helicoidal profile of velocities, i.e., each depth has its own modulus and direction (Tab. (2)). The wind are composed by its modulus and direction (Tab. (2)). All directions are clockwise relative the longitudinal (X-axis).

The waves were represented by Jonswap spectrum, corresponding to their significant height and period between peaks, and their direction (Tab. (2)).

### Table 2. Current, wind and waves conditions.

<table>
<thead>
<tr>
<th>Current</th>
<th>Wind</th>
<th>Waves</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (m)</td>
<td>Velocity (m/s)</td>
<td>Direction (°)</td>
</tr>
<tr>
<td>500</td>
<td>0.54</td>
<td>90</td>
</tr>
<tr>
<td>250</td>
<td>0.64</td>
<td>90</td>
</tr>
<tr>
<td>140</td>
<td>0.74</td>
<td>90</td>
</tr>
<tr>
<td>50</td>
<td>1.1</td>
<td>90</td>
</tr>
<tr>
<td>2</td>
<td>1.6</td>
<td>90</td>
</tr>
</tbody>
</table>
The simulation comprised a period of 2000 seconds and compare the behavior of platform using the hydrodynamic coefficients (added mass, potential damping, hydrostatic restoring and wave exciting force) determined by WAMIT with (damped) and without (undamped) the viscous damping coefficients calculated by Hooft’s method (1971) in the six degrees of freedom.

Figure (7) shows the graphics of results where the position is plotted versus time.

It is observed that the adding of viscous damping have reduced the oscillation peaks in movements of the platform and, in some cases, it have eliminated the oscillation. These results can approximate more the numerical results of tank’s tests and demonstrate that its effects can not be ignored in the dynamic of platforms.

It is also possible to see that the effects of damping are much more significant in direction of movement where the relative velocity is considerable.

5. Conclusion

It may be concluded that, depending of the geometry of the unit, the viscous damping effect can be significant and the estimative of the floating unit behavior can be much imprecise without the consideration of it.

The results obtained to added mass, restoring coefficients and wave exciting force to semi-submersible platform is compared with the obtained with WAMIT program that solve the potential problem. So, considering the computation time to obtain the solution of potential problem, the necessary license to execute WAMIT program and the precision obtained with the process used in this work, it can be more practical the usage of this process and with a low price in precision.
Figure 7. Motions of the platform when the viscous dampings are or not considered.

6. References

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