INVERSE HEAT CONDUCTION TECHNIQUE TO BE APPLIED IN WELDING PROCESS GAS TUNGSTEN ARC WELDING (GTAW)

Cristiene Vasconcelos Gonçalves
School of Mechanical Engineering (FEMEC) –Federal University of Uberlândia (UFU)
cgoncalves@mecanica.ufu.br

Gilmar Guimarães
School of Mechanical Engineering (FEMEC) –Federal University of Uberlândia (UFU)
gguima@mecanica.ufu.br

***Abstract.** Welding is a reliable, cost effective and efficient metal joining Gas Tungsten Arc Welding (GTAW)) process is a widely used welding process in industry. The high temperatures due to the welding process affects the material microstructure and hardness of the regions near the weld, and the residual stresses that will be present in the material after cooling to room temperature. These changes will consequently affect the performance of the welded joint. This work describes the development of an experimental technique based on inverse code to predict the weld pool and the temperature profiles in the workpiece during a welding process. The method uses transient temperature data from the opposite face of the welding bead and a transient thermal model. The model is based on two-dimensional heat conduction transfer with phase change and moving heat source. Influence of the temperature in thermal properties, convection and radiation are also considered. The greatest advantage of the technique is that the liquid-solid interface and the temperature distributions within the solid regions can be obtained from the temperature data at a number of thermocouples located in the solid region, without considering heat transfer and fluid flow in the molten zone. The application of the method is illustrated through numerical experimentation.*

***Keywords**. Inverse problems, welding, simulated annealing, phase change, moving source*

1. Introduction

Gas Tungsten Arc Welding (GTAW) is an arc welding process, in which a voltaic arc is formed between a non-consumable electrode and a metallic workpiece. This process was originally developed for hard-to-well lightweight metals such as aluminum, magnesium and titanium. Many delicate components in aircraft and nuclear reactors are GTAW welded and therefore weld quality is of extreme importance. The joint (weld bead) is achieved trough the fusion of the plate edges and material feeding is optional. For that, a very intense and moving heat source is applied on the workpiece. Part of this heat is used to melt the plate, part is sunk into the workpiece and part is lost to the environment and in the electrode cooling system. The prediction of the way that the heat reaches the plate (intensity, concentration, distribution, temperature gradient) becomes extremely useful for understanding welding phenomena, such as the bead formation (width and penetration depth) or microstructure changes in the heat-affected base metal zone.

Many investigators have studied welding heat flow problems, analytically, numerically, and experimentally. One can find studies about pulsed current GTAW weld pool, (Kim and Na, 1998) and (Tsai and Hou, 1988). The GTAW weld quality is strongly characterized by the weld pool geometry. This is because the weld pool geometry plays an important role in determining the mechanical properties of the weld. Tarng et all (1999), used an optimization algorithm, which the quality of aluminum welds based on the weld pool geometry, was classified and verified by a fuzzy clusters technique.

The majority of the works considers the heat flux input to be known for determining the temperature field. For example, Tsai and Hou (1988) studied the pulsed current GTAW process considering a radial symmetric normal function for the heat transfer rate. Krauss (1986) has formulated the steady state and transient heat transport for a thin-plate GTAW welding using, in the same way, circular or elliptical Gaussian distribution for the welder arc heat flux. Application of this predictions is used on the study of the metallurgical aspects of welds, e.g., grain growth, size, and orientation, and solid-state phase transformations, which in turn leads to the understanding of associated mechanical properties, e.g., residual stress, distortion, ductility, and strength.

In fact, the majority of the assumptions about the distribution are based on the thermal efficiency of the welding process, that is, the net quantity of heat flux input to the workpiece. Usually, the values of this efficiency are either assumed to be known and taken from literature or determined by using calorimetric techniques.

Inverse heat conduction problems (IHCP) have been used recently in the studies of welding processes (Katz and Rubinsky (1984); Hsu et all (1986), Gonçalves et all (2002), Lima e Silva et all (2002) and Al-Khalidy (1997). Katz and Rubinsky (1984) have used a finite element method called border attack with an one-dimensional approach while Hsu et al. (1986), also using finite elements, have considered a quasi stationary two-dimensional model. In that case, the temperature data measured from thermocouple in the solid area are used to calculate the interface solid-liquid position and the temperature field in the solid area of the workpiece. For this, a Newton-Raphson interpolation is used, assuming a stationary welding process. The same stationary welding conditions is also used by Al-Khalidy (1997), but in this work phase change are considered. Another stationary two-dimensional model can be found in the work of Gonçalves et al. (2002). In that case, an analytic model based on Rosenthal (1941) is used for deriving the direct problem equation while the simulated annealing methods is applied for obtaining the heat flux input. It can be observed, however, that in this model the influence of the heat diffusion due to the thickness of the plate is neglected and the heat source moving is considered constant along whole the welding process. Although those considerations are used to reach practical results in GTAW with thin plates, they represent a limitation to the other processes or when welding thicker plates. Lima e Silva et all (2002) have determined a heat flux and temperature field during a GTAW process using a three-dimensional and transient thermal model. For estimating processes is used an inverse technique based on the conjugated gradient optimization method with adjoint equation. The temperature data are measured from the opposite face to welding process. The thermal model considers the heat source moving as boundary condition with time and position dependency. The welded plate is represented by a sample of austenitic steel inoxidable AISI 304 instrumented by ten thermocouples attached at the opposed face of the sample. The use of those temperatures together with the inverse technique allows obtaining the heat flux imposed in the front surface of the plate and consequently of the rate of heat necessary for welding besides the temperature field in the plate. Although the thermal model considers a three-dimensional and transient effect, it does not consider phase change and radiation effects.

This work describes an inverse technique based on simulated annealing method to obtain the temperature field and weld pool shape during a welding process considering a two-dimensional and transient model with phase change. The weld pool shape and location are obtained by using simulated annealing method. The thermal model is based on a solution of a Stefan problem that has been presented by Al-Khalidy (1995). In his work, Al-Khalidy (1995) has obtained the direct solution of a problem that considers phase change, convection and radiation effects in a keyhole plasma arc welding process involving moving heat source.

2. Theoretic fundamentals: direct problem

The thermal problem due to the welding process can be represented by Figure (1). The temperature field can, then, be obtained through the solution of the two-dimensional transient heat diffusion equation, considering as thermal excitation a moving heat source in the direction x with the upper and lower surfaces of plates subject to the thermal convective and/or radiative losses. In this model the 2D system hypotheses is justified by the small plate thickness in comparison with a large width and length of workpiece. It is also assumed that the metal vaporization at the plasma-liquid interface can be neglected and that thermal properties can be considered constant in the liquid region.

.

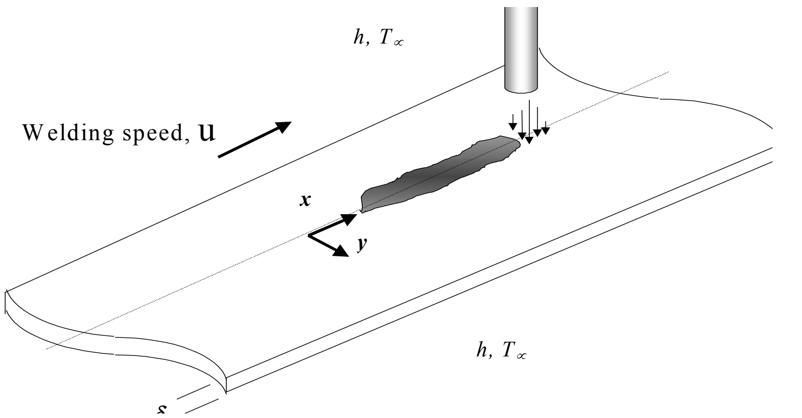


Figure 1 - Welding thermal model

The governing equation of two dimensional melting problem in a region **R** of pure metal can be separated into liquid region ***R$_l$***, and solid region ***R$_s$***. (Al-Khalidy, 1995). The two regions are separated by an interface $\Re$. Al-Khalidy (1995) propose that diffusion equation for the solid region, ***R$_s$***, can be written by

$$\rho_s C_s(T)\frac{\partial T_s(x,y,t)}{\partial t}+\rho_s C_s(T)\,u\,\frac{\partial T_s(x,y,t)}{\partial x}=\nabla k_s \nabla T_s(x,y,t)-\frac{h(T)}{\delta}T_s(x,y,t)\,,\qquad (x,y)\in \boldsymbol{R_s} \tag{1}$$

and for the liquid region, ***R$_l$***,

$$\rho_l C_l(T)\frac{\partial T_l(x,y,t)}{\partial t}+\rho_l C_l(T)\,u\,\frac{\partial T_l(x,y,t)}{\partial x}=\nabla k_l \nabla T_l(x,y,t)-\frac{h(T)}{\delta}T_l(x,y,t)\,,\qquad (x,y)\in \boldsymbol{R_l} \tag{2}$$

where ρ, $C(T)$, T, t, u, $k(T)$, $h(T)$ and δ denote density, specific heat , the difference between the surface and ambient temperature, time, velocity, thermal conductivity, over all heat transfer coefficient and plate thickness. The overall heat transfer coefficient is calculate as follows:

$$h(T) = h_c + h_r \tag{3}$$

where the h_r and h_c are the radiative and convective heat transfer coefficient respectively. The heat balance around the interface (Stefan condition) is as follows

$$k\left(\frac{\partial T}{\partial n}\right)_s - k\left(\frac{\partial T}{\partial n}\right)_l = \rho \lambda u \tag{4}$$

where L represents the total latent heat of fusion, n denotes the normal direction to $\Re$ **,** λ = L during solidification and λ= - L during melting.

Using the enthalpy function (Crank, 1984), Eq.(1) and Eq.(2) can be written together with Eq.(4) as follows

$$\rho\ C\ \frac{\partial T(x,y,t)}{\partial t} + \rho\ C\ U\ \frac{\partial T(x,y,t)}{\partial x} = \nabla k \nabla T(x,y,t) - \frac{h(T)}{\delta} T(x,y,t) + Q \quad (x,y) \in R_s \ and\ R_l \tag{5}$$

where

$$Q = -\rho \frac{\partial \Delta H}{\partial t} \tag{6}$$

U is the welding speed and ΔH is the latent heat component of the enthalpy constrained by the limit (0<ΔH<L) and

$$H(T) = CT \quad when\ T < T_m \ and\ H(T) = CT + \Delta H\ when\ T > T_m \tag{7}$$

The plate is subject to the boundary conditions

$$\frac{\partial T_l}{\partial y} = 0;\ \frac{\partial T_s}{\partial y} = 0 \qquad \text{at} \qquad y = 0 \tag{8}$$

$$T_s(x,y,t) = 0 \qquad \text{at} \qquad x \to \infty\ and\ \ y \to \infty \tag{9}$$

and initial condition

$$T(x,y,0) = 0 \qquad \text{at} \qquad (x,y) \in R \tag{10}$$

The interface must satisfies the isothermal condition

$$T(z,y,t) = T_m - T_\infty \qquad \text{at} \qquad (x,y) \in \Re \tag{11}$$

where T_m is the melting temperature and T_∞ is the ambient temperature.

The Eq.(5) to Eq.(11) represents the direct solution of the problem when Q is known at any time and location. The direct problem is, then, solved by using numerical algorithms based on the finite volumes. To obtain the numerical solution of the melting problem the domain has been discretized using a grid system of 60 x 20 nodes covering the entire workpiece. A step time of 0.01s is used.

3. Theoretic fundamentals: Inverse Problem

The inverse phase-change problem is stated as an optimization problem. The shape of phase change boundary as well as the temperature field within the solid region is identified without solving heat transfer and fluid flow equation in the liquid region.

An arbitrary location of phase-change boundary is assumed in order to correct this location afterwards using the correction-prediction method (Al-Khalidy, 1997). The location of the interface is assumed arbitrary to satisfy the interfacial condition, Eq. (11). An appropriate number of control nodes should be assumed to form the interface shape.

The radial distance, r, for each control node from the weld center is calculated after assuming the angular direction γ, Fig. 2. The coordinates of these nodes with respect to weld center coordinate are given by

$$x_{ri} = r_i \cos(\gamma_i) \tag{12}$$

$$y_i = r_i \, sin(\gamma_i) \tag{13}$$

where x_r is the x coordinate of the node on a guessed interface position and i = *1,2,...,n*. The coordinate with respect to the global coordinate can be found:

$$x_i = x_m + x_r \tag{14}$$

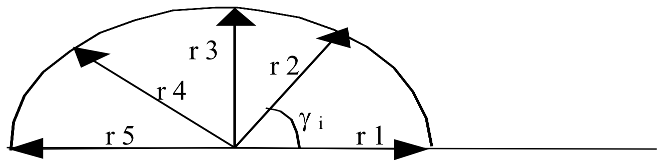


Figure 2. Coordinate system for the liquid-solid interface

where x_m is the location of the heat source.

The results are optimized with respect to the least-square errors between measured and estimated values of temperature at the number of thermocouples located in the solid region. The objective function, *F*, can be defined as the least-square errors *F* between the computed temperature T_{ji} and the measured temperature Y_{ij},

$$F(x) = \sum_{j=1}^{J} \sum_{i=1}^{M} \left(Y_{ji} - T_{ji}\right)^2 \tag{15}$$

where i represents the index for the thermocouple position and *j* the index for measurement time.

3.1 Simulated Annealing Algorithm

There are several inverse techniques that can solve this welding thermal problem. It can be cited, for example, the conjugate gradient with adjoint equation method (Ozisik and Orlande, 2000), parameter estimation approaches (Beck e Arnold, 1977), sequential time domain (Beck et all, 1985), genetic algorithm (Gonçalves, 2000), or Powell algorithm, (Al-Khalidy, 1997). The choice of Simulated Annealing is due the fact that no function to be smoothed (differentiable) are required in contrast of the gradient methods. Hence, it lends itself to ill-posed or ill-conditioned problems how can be see in Gonçalves et. all (2002).

Simulated Annealing's root is in thermodynamics, where the thermal energy of a material is studied. Annealing is a thermal process for obtaining low energy states in a material. First, the temperature is increased to a value at which a solid melts, afterwards the temperature is slowly decreased until all particles have arranged themselves in the ground state of the solid. The particles arrange themselves randomly, if cooling of the material is sufficiently slow. These random fluctuations will allow the material to escape from local energy minimum and to achieve a global energy minimum. If the cooling process is too fast, *i.e.*, the material is quenched, the particles will not escape the local energy minimum and are frozen in a configuration with some residual energy that characterize the thermal stresses in the material, (Jensen, 2002)

The Simulated Annealing approach is an analogue to the annealing process. It can be performed in optimization by randomly perturbing the decision variable and keeping track of the best objective function value for each randomized set of variables. After many trials, the set that produced the best objective function value, Eq. (15), is designed to be the center, over which perturbation will take place for the next "temperature". The "temperature", that in this technique is the standard deviation of the random number generator $\in[0,1]$, is then reduced and news trials performed.

Let each configuration be defined by the set of atom positions where E represents the energy of the configuration and T is the "temperature". In each step of the configuration, an atom is given a small random displacement and the resulting change, ΔE, in the energy of the system is computed. When the processes generating new states, this states is either accepted or rejected, according to the metropolis criterion (Metropolis et all, 1956): if $\Delta E \leq 0$, the displacement is accepted and this configuration is used as the starting point of the next step. If $\Delta E \geq 0$, the probability that the configuration is accepted is given by the following equation:

$$P(\Delta E) = e^{(-E/K_b T)} \tag{16}$$

where K_b is the Boltzmann´s constant. The choice of the probability function given by Eq. (16) has the consequence that the system evolves, according to a Boltzmann distribution. A random number P'∈ [0,1] is generated. If P' < P, then the configuration is accepted and the function moves uphill. The decision variables are not accepted as a new optimum value, they are merely used as the points from where the next random trial points are generated.

4. Simulated experiment

Experimental results are supposed to be simulated by using numerical data. The procedure used here simulates these measured temperatures introducing random errors to the exact temperatures as

$$Y_{ij} = T_{exa} \cdot (1 + \tau w) \tag{17}$$

where the exact temperature T_{exat} is determined from the solution of the direct problem, Eq. (5) to Eq.(11), by using the thermal properties and geometrical dimensions presented in Tab. (1). The value of τw is the error term, w is the percent disturbance into the data and τ is a random number.

As mentioned before, in order to solve the Eq. (5) to Eq. (11) the knowledge of the weld pool shape is necessary. A weld pool configuration, characterized by the parameter r and γ must be, then, defined once the experimental data are being simulated. This configuration is shown in Tab.(2) and must be recovered by the inverse problem. The shape adopted considers the radio of the weld pool, r_i constant and equal 0.005 m.

Table 1. Simulated experiment configuration

Material	AISI 304 stainless steel
Thickness, δ, [m]	0.00635
Length, L, [m]	0.05
Width, W, [m]	0.025
Welding speed [m/s]	0.0083
Grid system (nodes)	60x20
Specific heat - liquid [J/(kgK)]	812.0
Specific heat - solid [J/(kgK)]	$484.6 + 0.159T + 18.07e^{-6}T^2$
Melting temperature, T_m, [°C]	1427
Surface emissivity	0.95
Latent heat of fusion [J/kg]	265,200
Viscosity [kg/(ms)]	0.00642
Density [kg/m³]	7,200
Stefan-Boltzmann constant	5.669E-8
Thermal conductivity - liquid [W/(mK)]	31.5
Thermal conductivity - solid [W/(mK)]	$14.42 + 0.0169T - 2.44\,e^{-6}T^2$

Table 2. Polar coordinates used to identify the phase-change boundary

i	γ_i[rd]	proposed value r_i [m]	initial guess r_i [m]	search range
1	0.0	0.005	0,0001	W x L
2	π/4	0.005	0,0001	W x L
3	π/2	0.005	0,0001	W x L
4	3π/2	0.005	0,0001	W x L
5	π	0.005	0,0001	W x L

The temperature Y_{ij} are supposed to simulate temperature measured located in the opposite face of solid region. 10 thermocouple are simulated.

4. Inverse solution procedure

The straightforward solution is not possible for this problem because of the unknown location of the solid-liquid interface. The solution is, then, obtained through of an iterative process. To start the iterations an initial estimation is made from the r_i configuration and $\Delta H = \Delta H_0$ which may be chosen as zero. The direct problem is solved and $T(x, y, t)$ is computed as follows:

1. Assume an arbitrary shape and location of the phase-change boundary, it means, initial guess to the radius $\boldsymbol{r_i}$ as shown in Tab. (2) for a constant time step. The location of the interface is assumed arbitrary to satisfy the interface condition given by Eq. (11)
2. Adopt $\Delta H = \Delta H_0 = 0$
3. Solve Eq. (5) to Eq. (11).
4. Optimize the results with respect to the least-square errors given by Eq. (15) using simulated annealing method.
5. Identify the shape of the interface through the new estimated values of r_i for the current step time. The control volume in which phase change occurs is then identified.
6. The latent heat source is recalculated and a new value of C and k is found.
7. The steps 3 to 6 are repeated until convergence is reached.
8. The obtained values of the temperatures and ΔH at any time are assumed to be the previous value in the next time step.
9. The calculation for new time step is repeated.

5. Results and discussion

Figure (3) presents a comparison between the experimental simulated data obtained from the direct solution with noise addition, Y_{ij}, and the temperature estimated by inverse code, T_{ij}. At distances faraway from moving heat source, an observer does not feel any significant temperature rise in his surroundings. The temperature reaches its peak at the point where the heat source passes by the observer. As the heat moves away from the observer, a sharp decay in temperatures is observed for times before 4s. It is interesting to note that after this time graduate, decay appears. The heat diffusion and phase change effects are responsible for this thermal behavior. It can be observed that the agreement between simulated data and model-predicted results degrades for times after 4s. The lower sensitivity in cooling process can justify this behavior. However, one of the main objectives in welding process is prediction of maximum temperatures so that material failure can be avoided. In this case the estimated process can be considered with success.

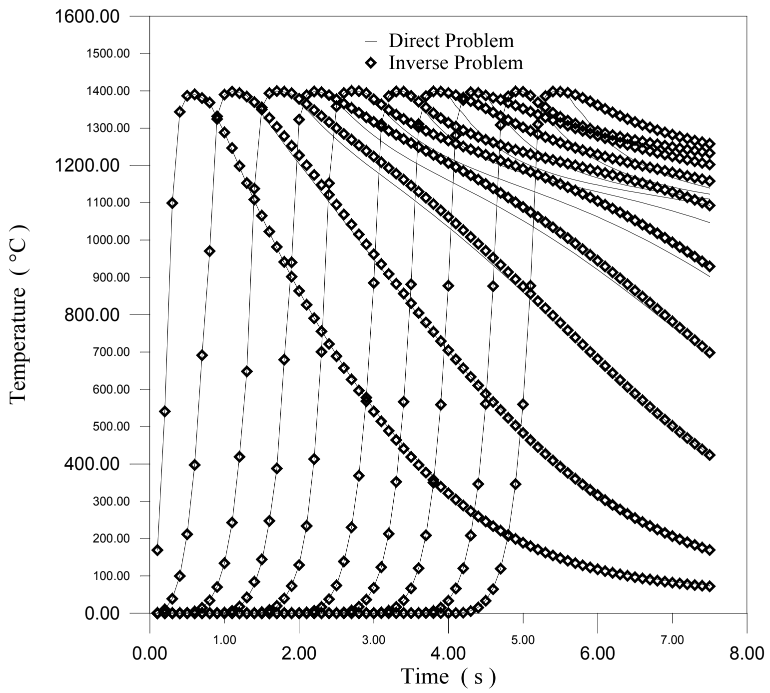


Figure 3. Comparison between estimated and experimental temperatures at the opposite face of the welding bead

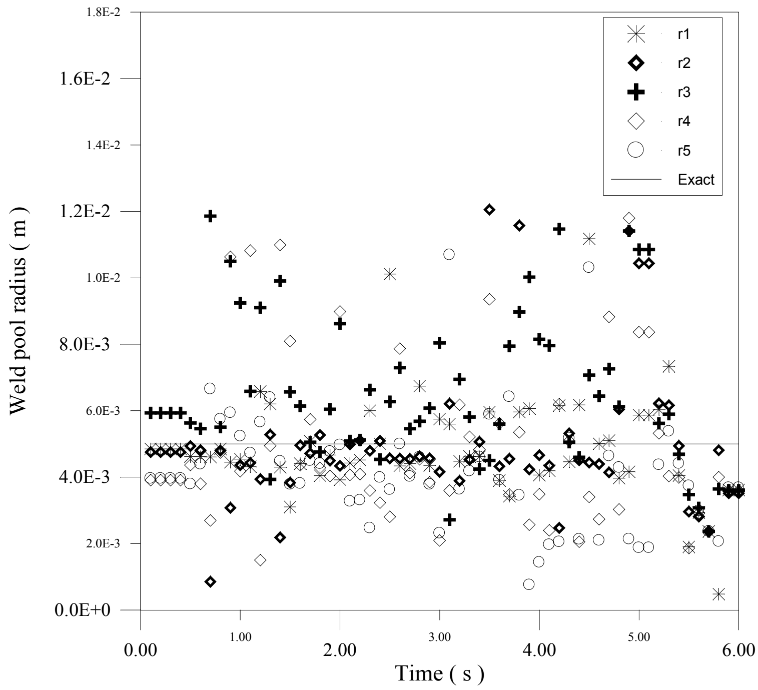


Figure 4 – Weld pool radius obtained by SA

Figure (4) presents all estimated radius for each step time calculus. Once the exact value for the radius is 0.005m, a better comparison of the results can be obtained trough an average radio for these times. Figure (5) presents this comparison. It can be observed that the estimated values oscillate about the expected value 0.005m over the total time.

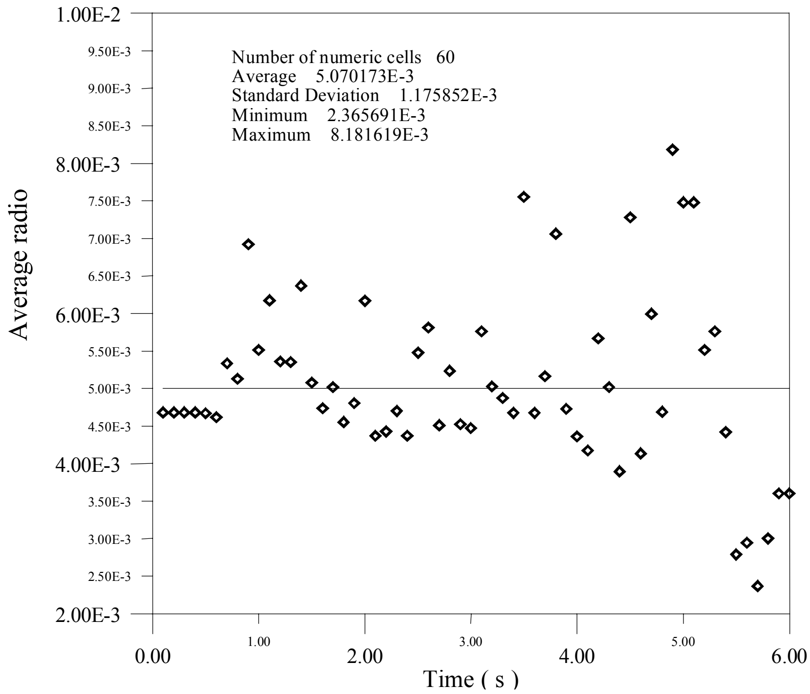
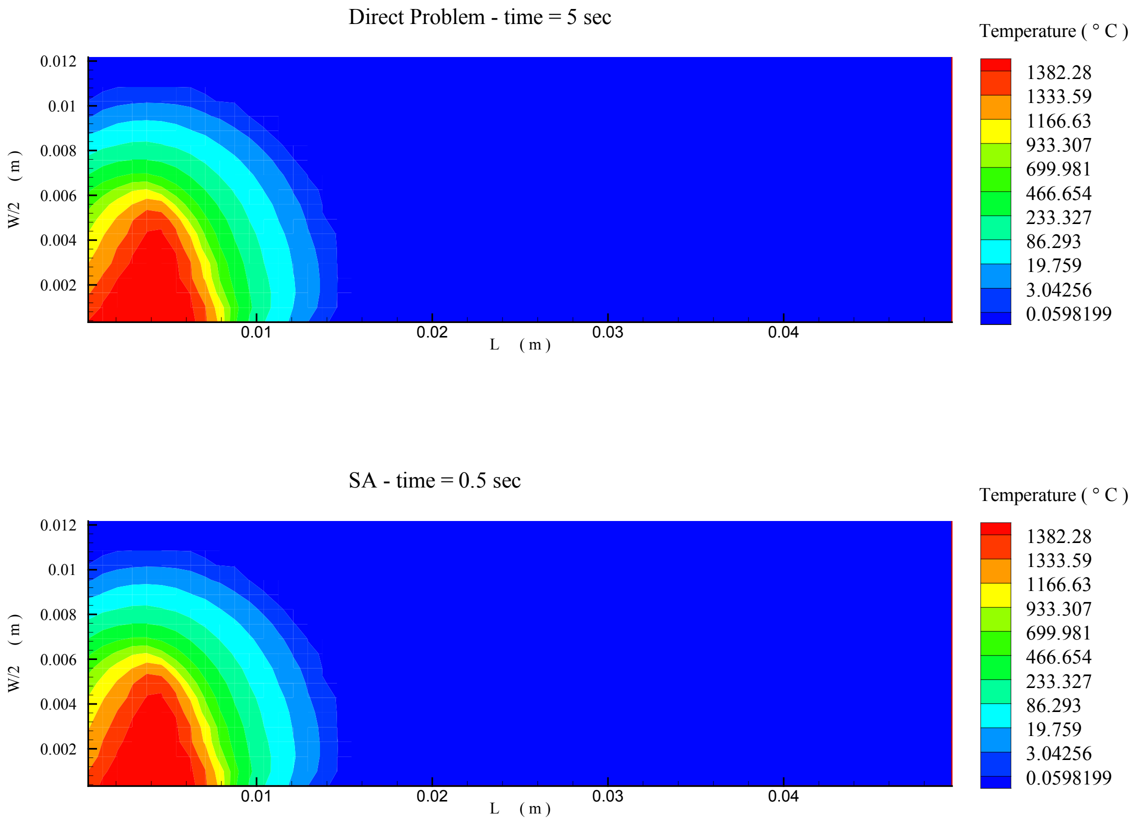
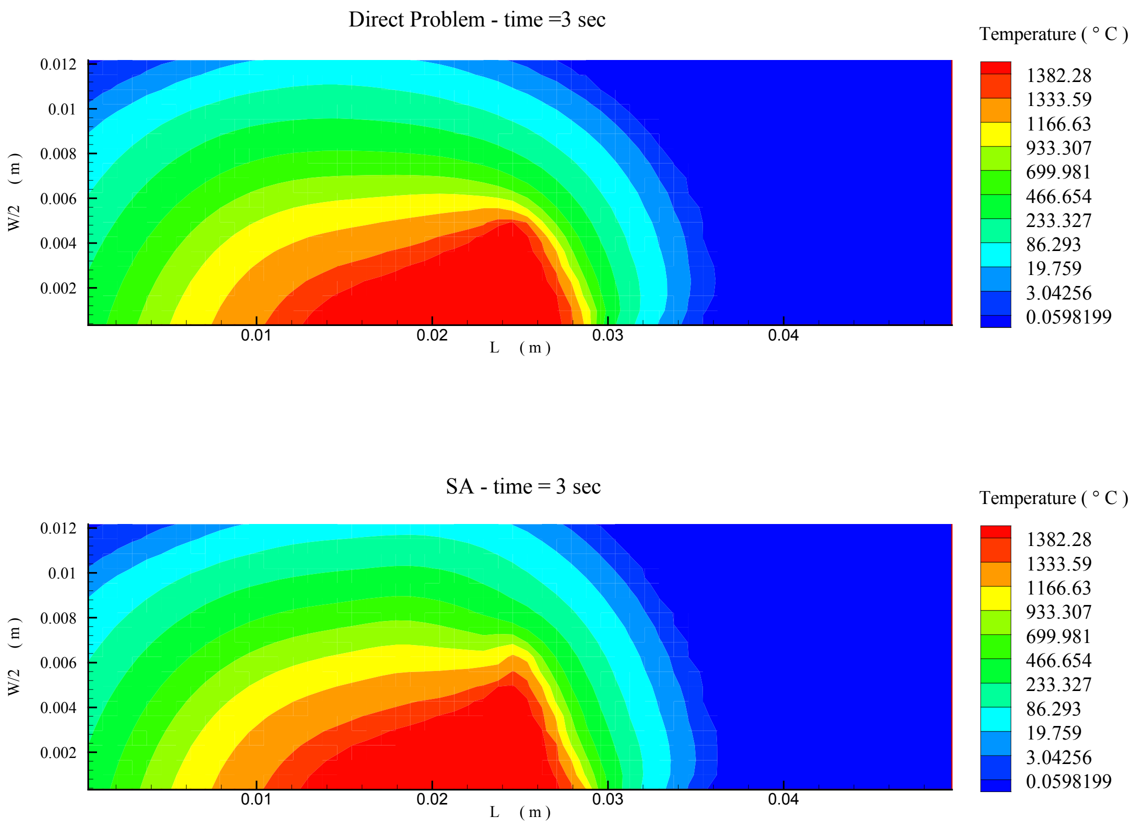


Figure 5. Average weld pool radio for each step time

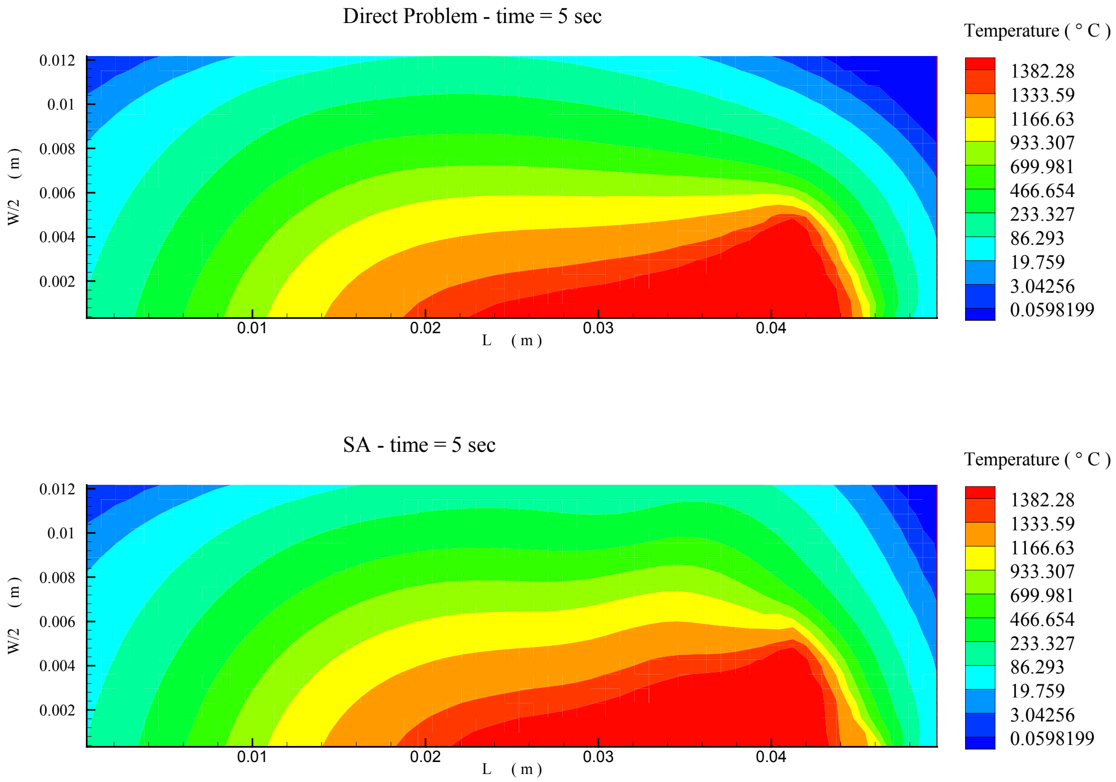
Once the radius have been obtained by SA alghoritm, the temperature field over all plate can be, now, obtained. The temperature fields for 3 different times are compared with respective exact solution considering the configuration given by Tab. (1) and Tab.(2). These results are shown in Fig. (6). It can be observed the behavior between the exact and estimated temperature field is very similar.


Direct Problem - time = 5 sec
Temperature (° C)
1382.28
1333.59
1166.63
933.307
699.981
466.654
233.327
86.293
19.759
3.04256
0.0598199
W/2 (m)
0.012
0.01
0.008
0.006
0.004
0.002
L (m)
0.01
0.02
0.03
0.04
SA - time = 0.5 sec
Temperature (° C)
1382.28
1333.59
1166.63
933.307
699.981
466.654
233.327
86.293
19.759
3.04256
0.0598199
W/2 (m)
0.012
0.01
0.008
0.006
0.004
0.002
L (m)
0.01
0.02
0.03
0.04

a)


Direct Problem - time =3 sec
Temperature (° C)
1382.28
1333.59
1166.63
933.307
699.981
466.654
233.327
86.293
19.759
3.04256
0.0598199
W/2 (m)
0.012
0.01
0.008
0.006
0.004
0.002
L (m)
0.01
0.02
0.03
0.04
SA - time = 3 sec
Temperature (° C)
1382.28
1333.59
1166.63
933.307
699.981
466.654
233.327
86.293
19.759
3.04256
0.0598199
W/2 (m)
0.012
0.01
0.008
0.006
0.004
0.002
L (m)
0.01
0.02
0.03
0.04

b)



c)

Figure 5. Comparisons between exact and estimated temperature field for different times a) 0.05s b) 3s c) 5s

The great problem with the use of random techniques as simulated annealing technique is the computational elevated time. In this work, an adaptation process was realized in order to reduce the interval of search in each interaction. The procedure was the use of adaptive bands, where the upper and lower limits were reevaluated when the objective function goes to a lower value.

6. Conclusion

In the present work a computational inverse code was carried out to predict the development of the weld pool and the temperature histories in the workpiece during the welding process. The model has considered a two-dimensional heat conduction transfer with phase change and moving source. The convection and radiation effects have also been considered. The optimization technique, which is the *simulated annealing method*, has shown very efficient in weld pool estimation and consequently in temperature field estimation. What a great advantage of the procedure proposed here is that the liquid-solid interface and the temperature field can be obtained just using temperature data from the solid region, without considering heat transfer and fluid flow in a molten zone. The results can be considered satisfactory.

7. Acknowledgement

The authors thank to the fomentation organs Fapemig and CNPq.

8. References

Al-Khalidy, Nehad, 1995, “Enthalpy Technique for Solution of Stefan Problems: Application of the Keyhole Plasma Arc Welding Process involving moving heat source, Int. Comm. Heat Mass Transfer, Vol. 22, No. 6, pp. 779-790.

Al-Khalidy, Nehad, 1997, “Application of Optimization Methods for Solving Inverse Phase-Change Problems, Numerical Heat, part B, pp. 477-497.

Beck, J. V. e Arnold, K. J., 1977, “Parameter estimation in engineering and science”*, John Wiley and Sons Inc*., New York, 501p.

Beck, J. V., B. Blackwell and C. R. St. Clair, 1985, “Inverse Heat Conduction, Ill-posed Problems”, Wiley Interscience Publication, New York (1985)

Gonçalves, C. V., Scotti, A. and Guimarães, G., 2002, “Simulated Annealing Inverse Technique applied in Welding: A Theoretical and Experimetal Approach”, proceedings of 4th International Conference on Inverse Problems in Engineering, Rio de Janeiro, Brazil.

Gonçalves, C. V., Silva, L. A., Silveira Neto, A., Rade, D. A. and Guimarães, G., 2000, "An inverse Technique Applied to Natural Convection Over a Heated Vertical Plate", proceedings of ASME National Heat Transfer Conference, Pittsburg, USA

Jensen, E. A., 2002, "Determination of discrete relaxation spectra using Simulated Annealing", J. Non-Newtonian Fluid Mech., Vol. 107, pp.1-11.

Hsu, Y. , Rubinsky, B., and Mahin, K., 1986, "An Inverse Finite Element Method for the Analysis of Stationary Arc Welding Process", ASME J. Heat Transfer, Vol. 108, pp. 734-741

Katz, M. and Rubinsky, B., 1984, "An Inverse Finite Technique to Determine the Change of Interface Location in One-Dimensional Melting Problem", Numerical Heat Transfer, Vol. 7, pp. 269-283.

Kim, W.-H., Na, S.-J., 1998, "Heat and Fluid Flow in Pulsed Current GTA Weld Pool", Int. J. Heat and Mass Transfer, Vol. 41, pp. 3213-3227

Kraus, H. G., 1986, "Thermal Finite Element Formulation and Solution Versus Experimental Results for Thin-Plate GTA Welding" , Journal of Heat Transfer, Vol. 108, pp. 591-596.

Lima e Silva, S. M.M., Vilarinho*, L. O,* Scotti, A. and Guimarães, G., 2002, Heat flux determining in GTA welding process by using a three dimensional modelling in inverse heat conduction problems, ECTP - European Conference on Thermophysical Properties 2002 (England)

Metropolis, N., Rosenbluth, A. W., Rosenbluth, M. N. and Teller, A. H., 1953, "Equations of State Calculations by

Ozisik, M. N. and Orlande, H. R. B, 2000., "*Inverse Heat Transfer"*, Taylor & Francis, NY, USA

Rosenthal, D., 1941, "Mathematical theory of heat distribution during welding and cuting.", Welding Journal 20(5), pp. 220s –234s.

Tsai, C. L. and Hou, C. A., 1988, "Theoretical Analysis of Weld Pool Behavior in the Pulsed Current GTAW Process", Journal of Heat Transfer, Vol. 110, pp. 160-165

Tarng, Y. S., Tsai, H. L. and Yeh, S. S., 1999, "Modelling, optimization and classification of weld quality in tungsten inert gas welding", International Journal of Machine Tools & Manufacture, Vol. 39, pp. 1427-1438.