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PRATICAL NEW TECHNIQUES FOR SOLVING NONLINEAR SYSTEM PROBLEMS

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Summary

This lecture reviews the basic of multiple-input/single-output (MI/SO) techniques that provide an accurate practical method to analyze and identify the dynamic properties of nonlinear physical systems. The Direct MI/SO Technique is applicable to nonlinear systems with specified parallel linear and nonlinear transformations. The Reverse MI/SO Technique is applicable to nonlinear systems with feedback. Excitation and response properties of measured random data can have arbitrary probability and spectral features. Each of the identified nonlinear components can be evaluated at any desired frequency with separate coherence functions. Thus these techniques represent a significant advance in using real measured data to help improve the design and understanding of nonlinear physical systems. Material for this lecture is taken from the latest book by J. S. Bendat, Nonlinear System Techniques and Applications, Wiley-Interscience, New York, 1998.

1- Features of Direct and Reverse MI/SO Techniques

This presentation states some of the main features of the Direct and Reverse MI/SO Techniques that are not available in other nonlinear methods such as least-squares time-domain procedures or in higher-order Volterra series. In particular, a key result developed in this work is how to convert single-input / single-output (SI/SO) nonlinear models with feedback into equivalent multiple-input/single-output (MI/SO) linear models without feedback. Each of the other features are also important such as requiring only functions of one frequency and removing restrictions regarding the probability or spectral natures of the excitation and response data. These practical techniques apply to Gaussian or to non-Gaussian data with arbitrary spectral density functions. Also, confidence limits on the linear and nonlinear terms can be obtained easily at each frequency using coherence functions.

The features of Direct and reverse MI/SO techniques are:

1. Single-input/single-output (SI/SO) nonlinear models without feedback can be converted into equivalent direct dynamic multiple/input/single-output (MI/SO) linear models without feedback.
2. Single-input/single-output (SI/SO) nonlinear models with feedback can be converted into equivalent reverse dynamic multiple/input/single-output (MI/SO) linear models without feedback.
3. An exact nonlinear representation is obtained using a linear frequency response function of one frequency for each nonlinear component.
4. The nonlinear system amplitude properties can be identified as well as the physical parameters with frequency-dependent coefficients.
5. There are no restrictions on the probability or spectral natures of the excitation or response data.
6. All results are easy to compute, simple to interpret, and can be evaluated at every frequency for both linear and nonlinear terms using appropriate coherence function.

2- Zero-Memory Nonlinear System and Finite-Memory Nonlinear System

A zero-memory nonlinear system is a system that acts on an input x(t) in a nonlinear fashion g{x(t)} so as to produce an output y(t) at the same instant of time. There is no weighting of past inputs to give the present output y(t). A simple example is a square-law device where $y(t) = x^2(t)$ as shown in figure 1. A finite-memory nonlinear system is a zero-memory nonlinear system that is followed or preceded by a linear system. In figure 2 the zero-memory nonlinear system is followed by a linear system defined by a frequency response function A(f) using A for “after”. One could also have a zero-memory nonlinear linear system preceded by a linear system defined by a B(f) using B for “before”. It turns out that general results for many nonlinear physical problems can be obtained only for cases where the linear systems follow the zero-memory nonlinear operations, so these cases are discussed here. Many engineering applications of these matters are covered fully in Reference {1}.

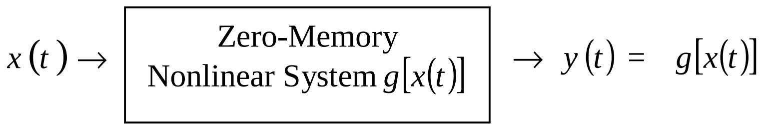


Figure 1. Zero-Memory Nonlinear System

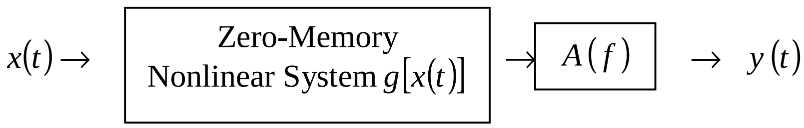


Figure 2. Finite-Memory Nonlinear System

3- Volterra Series of Linear, Bilinear and Trilinear Systems

A Volterra series, also called a “power series with memory” is a functional extension of linear (first-order) systems to bilinear (second-order), trilinear (third-order) and to higher-order systems as shown in figure 3. These extensions require knowledge of multi-dimensional frequency response functions H(f,g) to describe the bilinear system and H(f,g,h) to describe the trilinear system in place of the simple of the simple one-dimensional frequency response H(f) for the linear system. Given the input excitation data and the output response data, general input/output formulas to identify the linear, bilinear and trilinear frequency response functions can be solved only for Gaussian input data, see [1]. These higher-order functions are difficult to obtain in practice because they require large amounts of data are complicated to interpret.

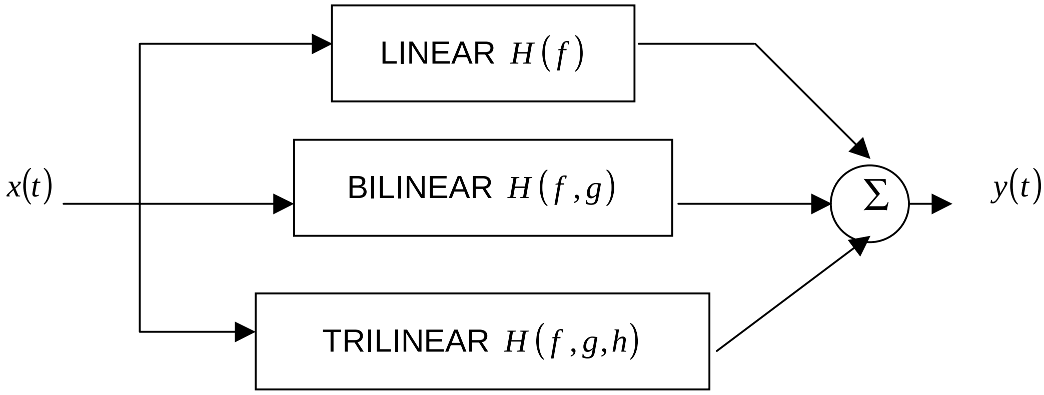


Figure 3. Volterra Series of Linear, Bilinear and Trilinear System

4- Nonlinear System of Linear, Squarer and Cuber Systems

A special Volterra series is shown on this slide consisting of a linear system $A_1(f)$ in parallel with two nonlinear systems: a squarer followed by a different linear system $A_2(f)$, and a cuber followed by another linear system $A_3(f)$ as shown in figure 3. The squarer path represents a special bilinear system and the cuber path represents a special trilinear system. The output $x_2(t)$ from the squarer is $x^2(t)$ and the output $x_3(t)$ from the cuber $x^3(t)$. This model can be used as a third-order approximation for various types of nonlinear transformations such as $x(t)|x(t)|$, see [1].

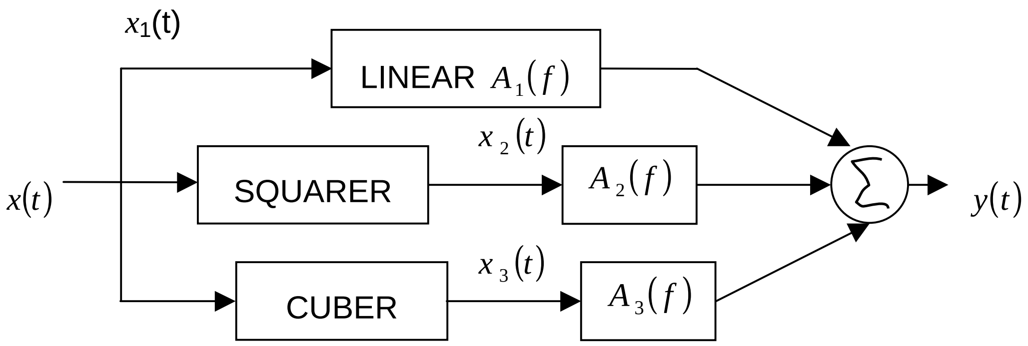


Figure 4. Nonlinear System of Linear, Squarer and Cuber System

5- General single-input/single-output nonlinear system

A very general single-input / single-output nonlinear system is shown on figure 5, where in place of the square, there is an arbitrary known or assumed zero-memory nonlinear system $g_2[x(f)]$ with output $x_2(t)$, and in place of the cuber, there is an arbitrary known or assumed zero-memory nonlinear system $g_3[x(f)]$ with output $x_3(t)$. The nonlinear model in Slide 5 will fit many more physical nonlinear problems than the nonlinear model in slide 4 and is the one recommended to use in practice. Given the input excitation data $x(t) = x_1(t)$ and the output response data $y(t)$, the system identification problem here is to identify the three linear frequency response functions $A_1(f)$, $A_2(f)$ and $A_3(f)$. This is easy to solve by well-established techniques in [2,3] if one recognizes that this nonlinear model can be replaced by an equivalent three-input / single-output linear model where the three inputs are the known $x_1(t)$, $x_2(t)$ and $x_3(t)$ and the output is the known $y(t)$.

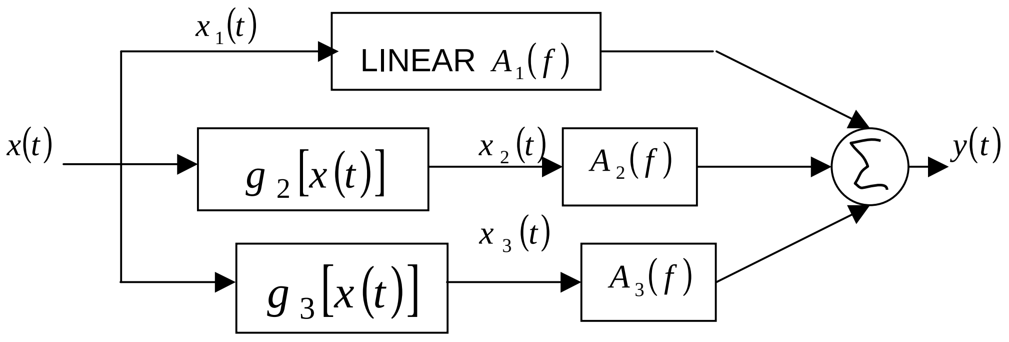


Figure 5. General Single-Input/Single-Output Nonlinear System

6- Multiple-input/single-output linear system with correlated input records and Multiple-input/single-output linear system with uncorrelated input records

Figure 6 shows the three-input/single-output linear system with possible correlated input records that is 100% equivalent to the general single-input / single-out nonlinear system in figure 5. The identification of the three linear frequency response functions $\{A_I(f)\}$ requires one to replace the three correlated input records $\{x_I(t)\}$ in Slide 6 with three uncorrelated input records $\{u_I(t)\}$ as shown in figure 7, where the notation "u" is used here to indicate "uncorrelated". Also, the three original linear frequency response functions in Slide 6 have to be replaced by new linear frequency response functions $\{L_i(f)\}$ as shown in Slide 7 so as to preserve the same output noise data $n(t)$ in both figures 6 and 7.

It is straightforward to compute de uncorrelated input records from the original input records. The first uncorrelated input record $u_1(t)$ is the same as $x_1(t)$. The second uncorrelated input record $u_2(t)$ is the result obtained by removing the linear effects of $x_1(t)$ from $x_2(t)$. The third uncorrelated input record $u_3(t)$ is the result obtained by removing the linear effects of both $x_1(t)$ and $x_2(t)$ from $x_3(t)$. The $\{L_i(f)\}$ system functions can be computed by three separate simple input/output spectral calculations, and the $\{A_I(f)\}$ functions by algebraic equations. This is the direct MI/SO Technique for Nonlinear System Identification!

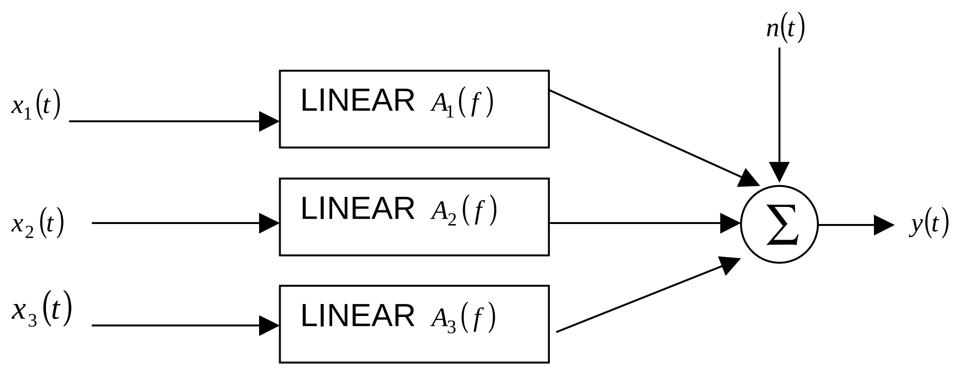


Figure 6: Multiple-input/single-output linear system with correlated input records

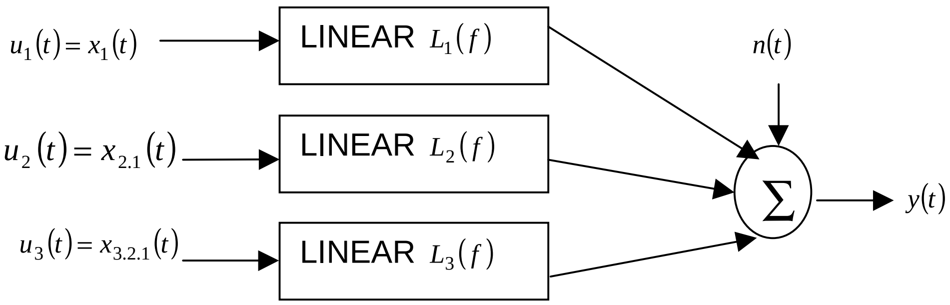
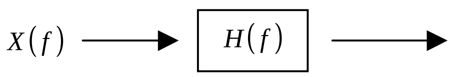


Figure 7: Multiple-input/single-output linear system with uncorrelated input records

7- Equivalent Forward and Reverse Linear Systems

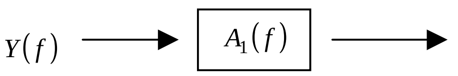
Figure 8 shows equivalent "forward" and "reverse" linear systems that can represent single-input/single-output problems. In the reverse linear system, the mathematical input is the real measured physical response record, and the mathematical output is the real measured physical excitation record. The linear frequency response function in the reverse system is the reciprocal of the linear frequency response function in the forward system.

The "forward" single-input/single-output linear system is



$Y(f)=H(f)X(f)$

The "reverse" single-input/single-output linear system is



$X(f)=A_1(f)Y(f)$ where $A_1(f)=[H(f)]^{-1}$

$X(f)$= physical excitations = mathematical output in reverse system
$Y(f)$= physical response = mathematical input in reverse system

Figure 8. Equivalent Forward and Reverse Linear System

8- SDOF Nonlinear Systems and Equations

This figure 9 shows a typical mechanical picture of a SDOF nonlinear system with a nonlinear restoring force $p(u;\dot{u},t)$ that can be a function of $u,\dot{u}$ and t. Here, $F(t)$ is a force input and $u(t)$ is a displacement output. The total restoring force is defined by $F_R(u,\dot{u},t)$. Two physical examples of nonlinear systems are given by;

$p(u;\dot{u},t)=u^3(t)$, a nonlinear cubic stiffness Duffing force

$p(u;\dot{u},t)=\dot{u}(t)|u(t)|$, a nonlinear viscous damping force

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) + p(u,\dot{u},t) = F(t)$$

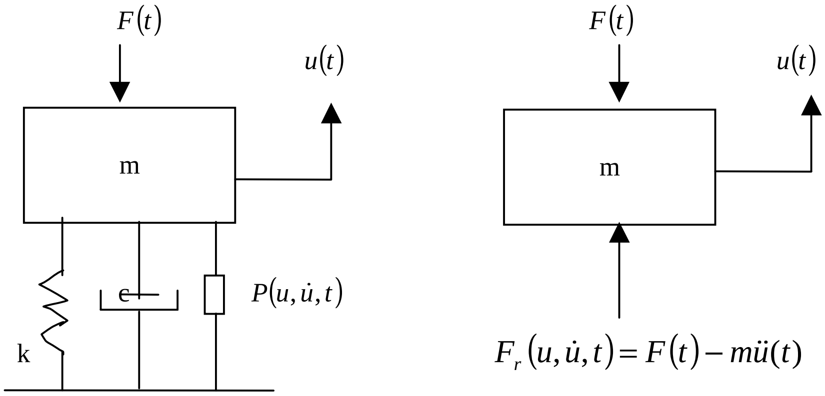


Figure 9: SDOF nonlinear systems and Equations

9- Single-input/single-output nonlinear model with feedback

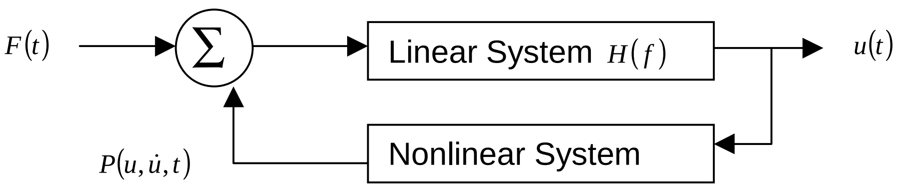
The single-input/single-output nonlinear model with feedback in this figure10 is the same as the mechanical system shown in figure 9. This nonlinear model with feedback is quite difficult to analyze for:

(1) prediction of the system response from knowledge of the force input plus the linear and nonlinear system properties, or

(2) system identification of the linear and nonlinear system properties from knowledge of the input and output records.

Complicated time domain iterative procedures are usually employed with restricted applications due to assumptions that are made on either data or the systems.

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) + p(u,\dot{u},t) = F(t)$$



$F(t)$= force input

$P(u,\dot{u},t)$= nonlinear term

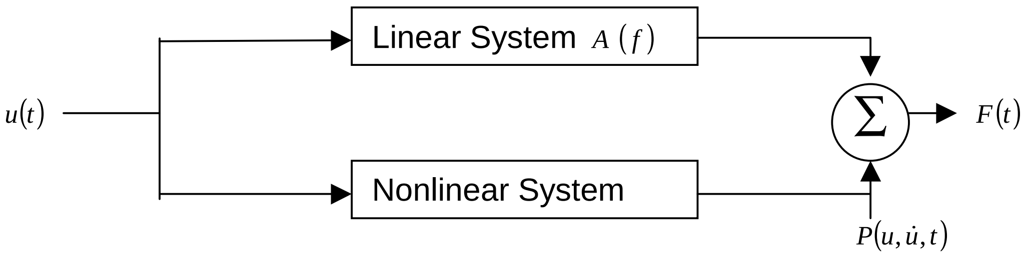
$u(t)$= displacement output

Figure 10. Single-Input/Single Output Nonlinear Model with Feedback

10- Reverse single-input/single-output nonlinear model without feedback

The reverse single-input/single-output nonlinear model without feedback is illustrated on this figure 11 where the previous physical force input $F(t)$ is now considered to be the mathematical output, and the previous physical displacement output $u(t)$ is now considered to be the mathematical input. The same nonlinear differential equation of motion applies to this model as before. Now, however, it is simple to solve this model because the nonlinear system can often be replaced by the general nonlinear system model in figure 5 with suitably defined nonlinear terms, and then solved by the standard MI/SO procedures outlined in figures 6 and 7. This is the basis of the Reverse MI/SO Technique for Nonlinear System Identification !.

$$m\ddot{u}(t) + c\dot{u}(t) + ku(t) + p(u, \dot{u}, t) = F(t)$$



$u(t)$ = displacement input $P(u, \dot{u}, t)$ = nonlinear term

$F(t)$ = force output

Figure 11 Reverse Single-Input/Single-Output Nonlinear Model without Feedback

11- Reverse MI/SO Technique for nonlinear system identification

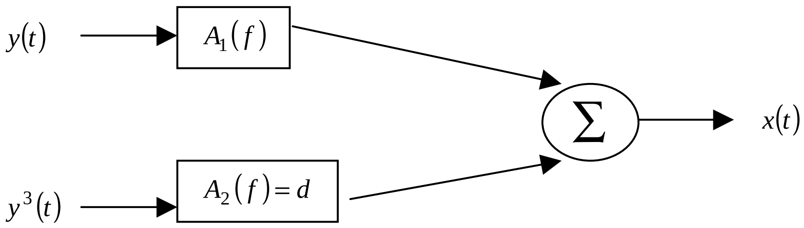
The Reverse MI/SO Technique for Nonlinear System Identification is illustrated in figure 12 for the Duffing single-input/single-output nonlinear system where the nonlinear restoring force is given by $p(u; \dot{u}, t) = du^3(t)$ when d is a constant coefficient. This involves the reversal of the physical excitation force input $x(t) = F(t)$ and the physical output $y(t) = u(t)$ to create the two-input/single-output linear mathematical model shown, where $x(t)$ is now the mathematical force output with $y(t)$ as the first mathematical input and $y^3(t)$ as the second mathematical input. This two-input/single-output linear model to identify the frequency response functions $A_1(t)$ and $A_2(t)$ can be solved easily by the standard procedures in [1,2].

The Duffing single-input/single-output nonlinear system is

$$m\ddot{y}(t)+c\dot{y}(t)+ky(t)+dy^{3}(t)=x(t)$$

where
$x(t)$ = physical excitation = mathematical output
$y(t)$ = physical response = first mathematical input
$y^{3}(t)$=nonlinear term = second mathematical input

The equivalent two-input/single-output linear system is here



This MI/SO linear system can be solved easily by kown procedures
Figure 12. Reverse MI/SO Technique for Nonlinear System Identification

12- Conclusions from computer studies and from laboratory test programs

Some main conclusions from computer studies and from laboratory test programs are stated on this figure 12. These results prove conclusively that these new practical Direct and Reverse MI/SO Techniques for Nonlinear System Analysis and Identification are significant developments to help solve many nonlinear system engineering and scientific problems.

1. Direct and Reverse MI/SO techniques can be implemented using established procedures and computer programs by changing SI/SO nonlinear models into equivalent MI/SO.
2. These MI/SO techniques can be used with simulated or measured data to identify each linear and nonlinear term in proposed nonlinear integro-differential equations of motion.
3. Nonlinear system amplitude properties can be determined as well as the frequency properties of coefficients for the linear and nonlinear system physical parameters. The spectral contributions from each linear and nonlinear term can be evaluated using appropriate coherence functions.
4. Computed cumulative coherence functions with simulated or measured data give improved results over wider frequency ranges when nonlinear terms are included, and show the particular frequencies where the nonlinear terms are important. The computed coefficients with measured data are usually frequency-dependent.
5. These features make the Direct and Reverse MI/SO Techniques the most accurate practical methods available today to analyze and identify the dynamic properties of nonlinear physical systems based upon simulated or measured data.

References:

1. Bendat, J. S., Nonlinear System Techniques and Applications, Wiley-Interscience, New York, 1998.
2. Bendat, J. S. , and Piersol, A. G., Engineering Applications of Correlation and Spectral Analysis, 2nd Ed., Wiley-Interscience, New York, 1993.
3. Bendat, J. S. and Piersol, A. G., Random Data: Analysis and Measurement Procedures, 2nd Ed., Wiley-Interscience, New York, 1986.