Analysis of asymmetric radial deformation in pipe with local wall thinning under internal pressure using strain energy method

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Abstract

Structural integrity of pipe has been an interesting area for many researchers in last years. To avoid the problems and the routine procedure of inspection and maintenance several works have been proposed. Metals like Carbon steel are used extensively in pipeline industries due to simplicity and economy. However, flaws like as cracks, pitting, local wall thinning can be generated by corrosion, erosion, and environmental exposure to various substances. It is very important to evaluate the strength of flaw to maintain the integrity of the pipeline systems. Many woks on local wall thinning have been developed focusing the mechanical behavior of pipe under combined loading. In this paper, an analysis of asymmetric radial deformation in pipe with local wall thinning under internal pressure is presented. The asymmetric radial displacements are estimated using strain energy method based on the Castigliano's theorem. Finally, the results are compared and valided using a commercially available finite element code.

Keywords: structural integrity, energy method, local wall thinning.

1 Introduction

Recently, there has been growing interest in structural integrity of pipe. A large number of the pipes are made of metals, like carbon steel, and they are used extensively in the petrochemical, refinery, and pipeline industries [1]. The reason for this is simply economic. It is widely available, inexpensive, and maintainable. However, this type of material is susceptible to flaws like as cracks, pitting, local wall thinning, which can be generate by corrosion, erosion, and environmental exposure to various substances [2–4]. Therefore, it is very important to evaluate the strength of pipe with local wall thinning to maintain the integrity of the pipeline systems. Many woks [5–7] on local wall thinning have been

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developed focusing the mechanical behavior of pipe under combined loading. Some literatures [2, 4, 8] using finite element method to investigate the mechanical behavior of pipe with defect. Nevertheless, in many cases the analyses expend a lot of computing time. The purpose of this paper is to analyze the asymmetric radial deformations in pipe with local wall thinning under internal pressure. The analytical radial displacements are estimated using the classical theorem of Castigliano [9–11], and the results are compared and valided with finite element (FE) analyses.

2 Circular pipe with local wall thinning subject to internal pressure

Considerer a pipe like a thin-walled cylinder of mean radius r and thickness t, as shown in Fig. 1(a). The pipe with internal defect (local wall thinning) is submitted to the action of uniformly distributed internal pressure of intensity p. The defect is represented by an angle of 2θ and a depth of d. In this work, for simplicity, the defect is assumed to be symmetric along the circumferential axis and it is extend throughout the entire length of the cylinder. The problem can geometrically be interpreted as shown in Fig. 1 (b). In order to solve this statically indeterminate the problem the Castigliano's theorem is used.

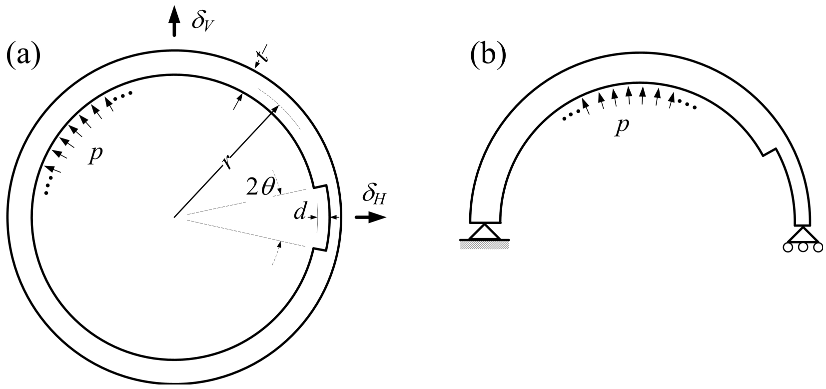

Figure 1: Geometry of pipe with local wall thinning.

In the present analysis, it will be considered that the radial displacements, δ_H and δ_V, which are associated the strain energy, are only due to bending moment and longitudinal force acting on the cross section. This hypothesis is reasonable because the strain energy due to shearing force is smaller when compared with others, thus it can be neglected.

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2.1 Bending moments and reaction forces

In the first, the magnitude of the bending moments, M_2 and M_1, and reaction forces, V_1 and V_2, will be determinate by means of Castigliano theorem. Let us consider the bending moment of curved bar at any cross section, as illustrated in Fig 2., it can be given by

$$M(\xi,\phi) \equiv \begin{cases} M_\rho = M(\rho,\phi), & 0 \leqslant \phi < \theta \\ M_r = M(r,\phi), & \theta \leqslant \phi \leqslant \pi \end{cases} \tag{1}$$

where the mean radiuses in non-defect and defect region are $\rho = r_i + (d+t)/2$, $r = r_i + t/2$, respectively. The bending moments in two regions can be written as

$$M_\rho = M_2 - \left(V_2\rho - p\rho^2\right)(1-\cos\phi) \tag{2}$$

$$M_r = M_2 - V_2(\rho - r\cos\phi) - pr\rho\cos\phi + \frac{p}{2}\left(r^2+\rho^2\right) \tag{3}$$

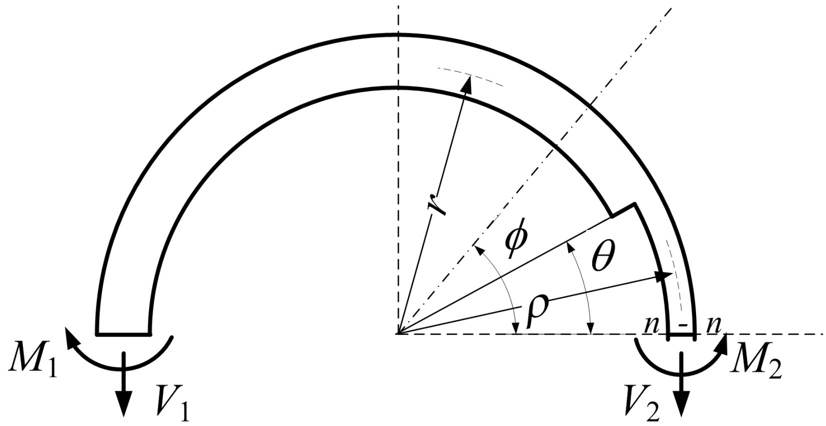

Figure 2: Scheme representative of half pipe.

In order to determinate V_2 and M_2, it can be assumed two hypotheses: (i) due the symmetry, the cross section $n-n$ does not rotate during the bending of the pipe and (ii) the displacement in the direction of V_2 at cross section $n-n$ is zero. Then, from the first hypothesis we have

$$\frac{dU}{dM_2} = 0 \tag{4}$$

in which U is the strain energy of half pipe which we are considering. Substituting the eqs. (2) and (3) into (4), we find the following expression

$$\frac{\rho}{I_\rho}\int_0^\theta \frac{\partial M_\rho}{\partial M_2} M_\rho d\phi + \frac{r}{I_r}\int_\theta^\pi \frac{\partial M_r}{\partial M_2} M_r d\phi = 0 \tag{5}$$

where, $\frac{\partial M_\rho}{\partial M_2} = \frac{\partial M_r}{\partial M_2} = 1$, and the moments of inertia are $I_\rho = \frac{(t-d)^3}{12}$ and $I_r = \frac{t^3}{12}$ then,

$$\frac{\rho}{I_\rho}\int_0^\theta M_\rho d\phi + \frac{r}{I_r}\int_\theta^\pi M_r d\phi = 0 \tag{6}$$

by the second hypothesis, following the same idea

$$\frac{dU}{dV_2} = 0 \tag{7}$$

where, $\frac{\partial M_\rho}{\partial V_2} = -\rho\left(1-\cos\phi\right)$, $\frac{\partial M_r}{\partial V_2} = -\left(\rho - r\cos\phi\right)$ then,

$$\frac{\rho}{I_\rho}\int_0^\theta M_\rho \rho\left(1-\cos\phi\right)d\phi + \frac{r}{I_r}\int_\theta^\pi M_r\left(\rho - r\cos\phi\right)d\phi = 0 \tag{8}$$

From (5) and (8), it is possible to obtain the reaction V_2 and M_2. In addition, the bending moment M_1 here are determined substituting $\phi = \pi$ into eq. (3), then

$$M_1 = M_r^{\phi=\pi} = M_2 - V_2\left(\rho + r\right) + \frac{p}{2}\left(\rho + r\right)^2 \tag{9}$$

and using the equilibrium equation, is ease to find the reaction force V_1 given by

$$V_1 = p\left(\rho + r\right) - V_2 \tag{10}$$

2.2 Stress distribution analysis

In this work, it is assumed that the thickness of the wall is small in comparison with the radii, i.e., a case of thin-walled tube. Thus, the stress state can be expressed in circumferential and longitudinal components. The circumferential stress distribution can be divided in two parts, one associate with bending moment and other with normal tension,

$$\sigma_\varphi\left(\xi,\varphi\right) = \begin{cases} \sigma_\varphi^\rho\left(\rho,\varphi\right) = -\frac{M_\rho}{I_\rho}\left(\rho' - \rho\right) + \frac{p\rho_i}{t-d}, & \rho_i < \rho' < \rho_o \;\; and \;\; 0 < \varphi < \theta \\ \sigma_\varphi^r\left(r,\varphi\right) = -\frac{M_r}{I_r}\left(r' - r\right) + \frac{pr_i}{t}, & r_i < r' < r_o \;\; and \;\; \theta < \varphi < \pi \end{cases} \tag{11}$$

In order to determine the longitudinal component let us assume that the cylinder is subject to a plane strain distribution. Hence, using generalized Hooke's law, we have

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$$\varepsilon_r = \frac{\sigma_\varphi}{E}\left[-\nu\left(1+\nu\right)\right]$$
$$\varepsilon_\varphi = \frac{\sigma_\varphi}{E}\left[1-\nu^2\right] \tag{12}$$
$$\sigma_z = \nu\sigma_\varphi$$

Figure 3 shows the circumferential and longitudinal components of stress distribution as a function of the angle φ. In this analysis were considered the following parameters: Young's modulus, E, equal to 200 GPa; Poisson's coefficient, ν, equal to 0.3; inner pressure, p, equal to 1 Mpa; inner radius, r_i, equal to 50 mm; wall thickness, t, equal to 2 mm; half angle of total defect, θ, equal to 60 degree; depth of local wall thinning, d, equal to 1 mm.

As illustrated in Fig. 3, there is a significant discontinuity in stress distribution due to defect. The maximum external stress occur when the angle φ is equal to zero, on the other hand, the maximum internal stress is observed in the discontinuity, when the angle of defect is assumed at limit value, i.e., θ equal to 60 degree. It can also be observed the good agreement between analytical results and numerical using finite element method

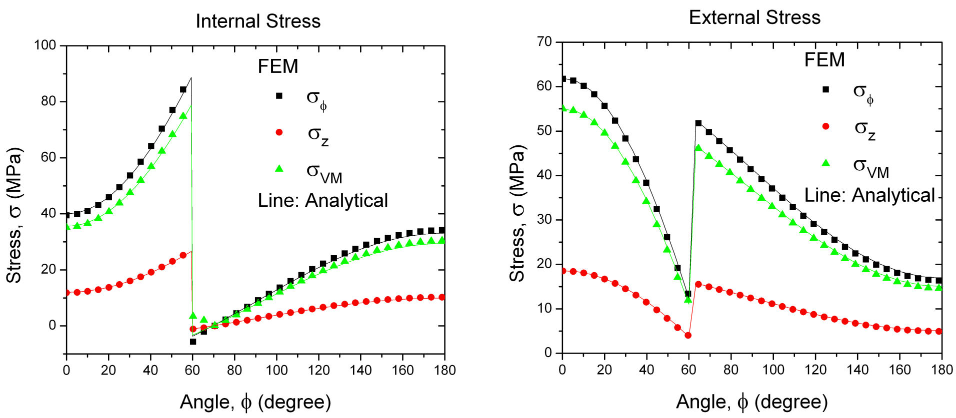

Figure 3: Internal and external stress distribution in pipe with local wall thinning: defect angle of 60 degree and depth of 1 mm.

In the present analyses, it is taken into account the linearly elastic behavior of materials, thus it is necessary to impose a criteria for initial yielding in pipe with local wall thinning to validate the results. The equivalent stress criterion based on Von Mises is considered and the equivalent stress can be defined as

$$\sigma_{VM} = \sqrt{\frac{1}{2}\left[\left(\sigma_\phi - \sigma_z\right)^2 + {\sigma_\phi}^2 + {\sigma_z}^2\right]} = \sigma_\phi\sqrt{\left(1+\nu^2-\nu\right)} \leqslant SMYS \tag{13}$$

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As can be seen in Fig. 3, the equivalent stress is also plotted and it is observed the good agreement between the results. In order to guarantees the best performance of the analytical results, the stress value is taken less than the specified minimum yield strength of the material (*SMYS*).

A carbon steel pipe with the specified minimum yield strength of the material (*SMYS*) equal to 250 MPa is assumed in the present analysis. This type of pipe is commonly used in the piping system. Fig. 4 illustrate the maximum value of equivalent stress as a function of the depth of defect for different angle of defect. These results were obtained considering the same parameters introduced before.

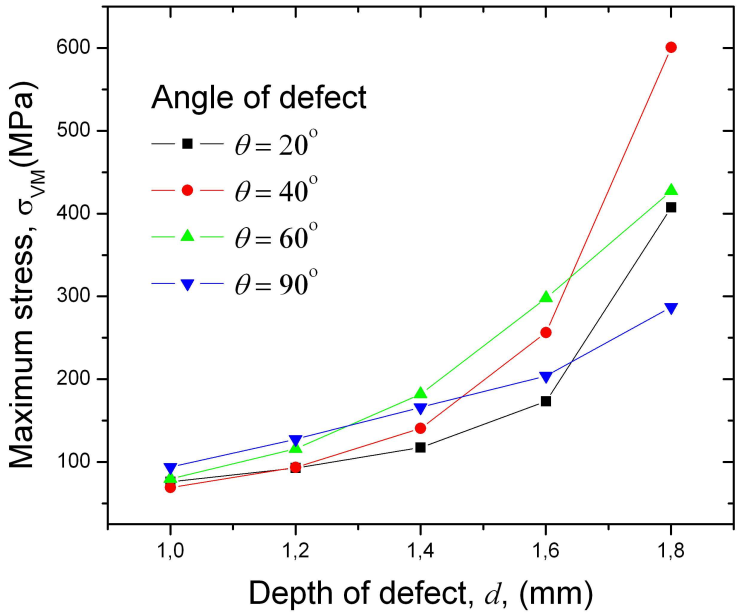

Figure 4: Maximum stress associated with Mises yield criterion.

The results presented in Fig. 4, show that for a depth of defect smaller than 1.5 mm, for all angle of defect, the maximum value of equivalent stress is linear elastic behavior of material.

2.3 Asymmetric displacement analyses

In the following discussion it is assumed that the pipe is a curved bar as shown in Figs. 1 and 2. In order to calculate the deflections of the curved bar, will be used strain energy method, in particular the Castigliano theorem. Hence, the total strain energy stored in the an elastic solid occupying a region Ω is then given by the integral over the domain

$$U = \int_{\Omega} U_0 d\Omega \tag{14}$$

where, considering the linearly elastic behavior of isotropic materials, the strain energy density is defined by

$$U_0 = \frac{1+\nu}{E}\left(\sigma_\phi^2 + \sigma_z^2\right) - \frac{\nu}{2E}(\sigma_\phi + \sigma_z)^2 \tag{15}$$

To determine the horizontal and vertical displacements, the half pipe, represented by the curved bar, can be divided in two quadrants to simplify the problem. The solution of this problem is developed in the following item.

2.3.1 Horizontal displacements analyses of the pipe with local wall thinning

The total horizontal displacement is composed by the two components associated to each quadrant. To determine them, each horizontal displacement is calculated separately, and to make this, it is necessary introduction imaginary forces, H_1 and H_2, as illustrated in Fig. 5 and 6. The total displacement can be given by

$$\delta_H = \delta_{H1} + \delta_{H2} \tag{16}$$

using the eqs. (14) and (15), we have

$$\delta_{H_1} = \left.\frac{\partial U}{\partial H_1}\right|_{H_1=0} = \frac{1-\nu^2}{E}\left[\int_{r_i}^{r_o}\int_{\pi/2}^{\pi}\left[\sigma_\varphi^r \frac{\partial \sigma_\varphi^r}{\partial H_1}\right]_{H_1=0} r'\, d\varphi\, dr'\right] \tag{17}$$

where the circumferential stress, presented in eq. (11), can be rewritten adding the term of imaginary force H_1, that it is ease to take from Fig. 5. Then,

$$\sigma_\varphi^r = \sigma_\varphi^r - \frac{(r'-r)}{2I_r} H_1 r \sin\varphi|_{H_1=0} \tag{18}$$

following the same idea, considering the quadrant with local wall thinning,

$$\delta_{H_2} = \left.\frac{\partial U}{\partial H_2}\right|_{H_2=0} = \frac{1-\nu^2}{E}\left[\int_{\rho_i}^{\rho_o}\int_{0}^{\theta}\left[\sigma_\varphi^\rho \frac{\partial \sigma_\varphi^\rho}{\partial H_2}\right]_{H_2=0} \rho'\, d\varphi\, d\rho' + \int_{r_i}^{r_o}\int_{\theta}^{\pi/2}\left[\sigma_\varphi^r \frac{\partial \sigma_\varphi^r}{\partial H_2}\right]_{H_2=0} r'\, d\varphi\, dr'\right] \tag{19}$$

$$\sigma_\varphi^r = \sigma_\varphi^r - \frac{(r'-r)}{2I_r} H_2 r \sin\varphi|_{H_2=0} \tag{20}$$

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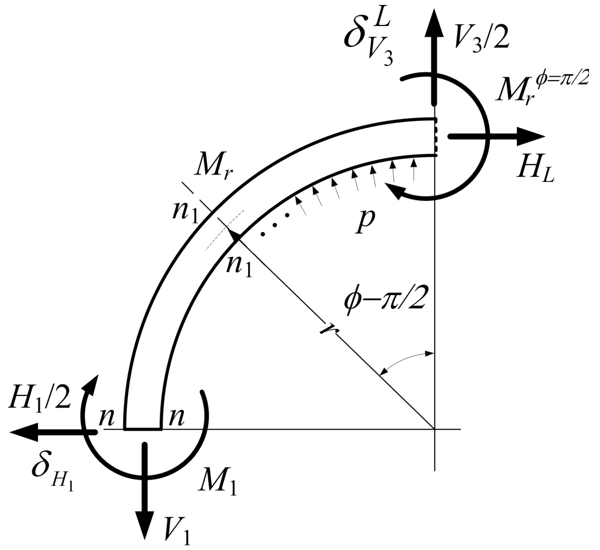

Figure 5: Geometry considering one quadrant of the pipe.

2.3.2 Vertical displacements analyses of the pipe with local wall thinning

For the total vertical displacement, defined by

$$\delta_V = \delta^L_{V_3} + \delta^R_{V_3} \tag{21}$$

we have, following the same idea,

$$\delta^L_{V_3} = \left.\frac{\partial U}{\partial V_3}\right|_{V_3=0} = \frac{1-\nu^2}{E}\left[\int_{r_i}^{r_o}\int_{\pi/2}^{\pi}\left[\sigma^r_\varphi \frac{\partial \sigma^r_\varphi}{\partial V_3}\right]_{V_3=0} r'\, d\varphi\, dr'\right] \tag{22}$$

$$\sigma^r_\varphi = \sigma^r_\varphi + \frac{(r'-r)}{2I_r} V_3 r \cos\varphi|_{V_3=0} \tag{23}$$

$$\delta^R_{V_3} = \left.\frac{\partial U}{\partial V_3}\right|_{V_3=0} = \frac{1-\nu^2}{E}\left[\int_{\rho_i}^{\rho_o}\int_{0}^{\theta}\left[\sigma^\rho_\varphi \frac{\partial \sigma^\rho_\varphi}{\partial V_3}\right]_{V_3=0} \rho'\, d\varphi\, d\rho' + \int_{r_i}^{r_o}\int_{\theta}^{\pi/2}\left[\sigma^r_\varphi \frac{\partial \sigma^r_\varphi}{\partial V_3}\right]_{V_3=0} r'\, d\varphi\, dr'\right] \tag{24}$$

$$\sigma^r_\varphi = \sigma^r_\varphi - \frac{(r'-r)}{2I_r} V_3 r \cos\varphi|_{V_3=0} \tag{25}$$

In order to evaluate the asymmetric radial displacements in pipe with local wall thinning, the analyses are initially conducted by assuming an inner pressure p equal to 1MPa, diameter-thickness

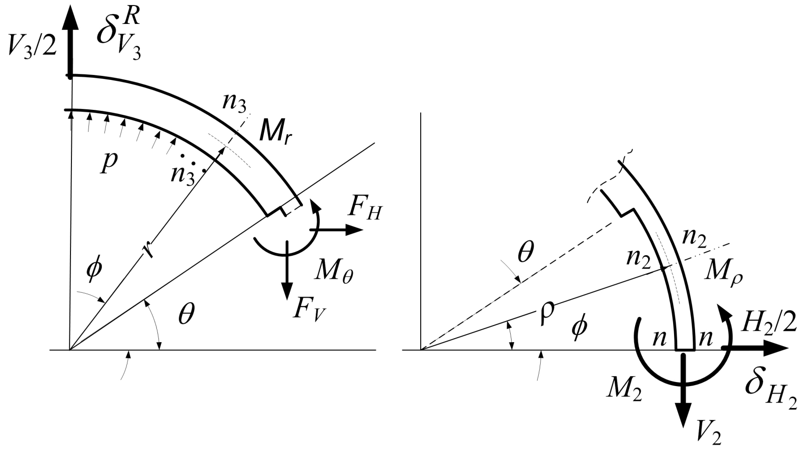

Figure 6: Geometry considering the local wall thinning.

radio D/t > 20 and different defect forms. In Figure 7, the horizontal and vertical displacements are obtained taken into account a pipe with inner radius equal to 50 mm, a pipe wall thickness equal to 2 mm, and for each half angle of total defect, 20, 40, 60 and 90 degree, the depth of local wall thinning is assumed of 0 to 1.8 mm.

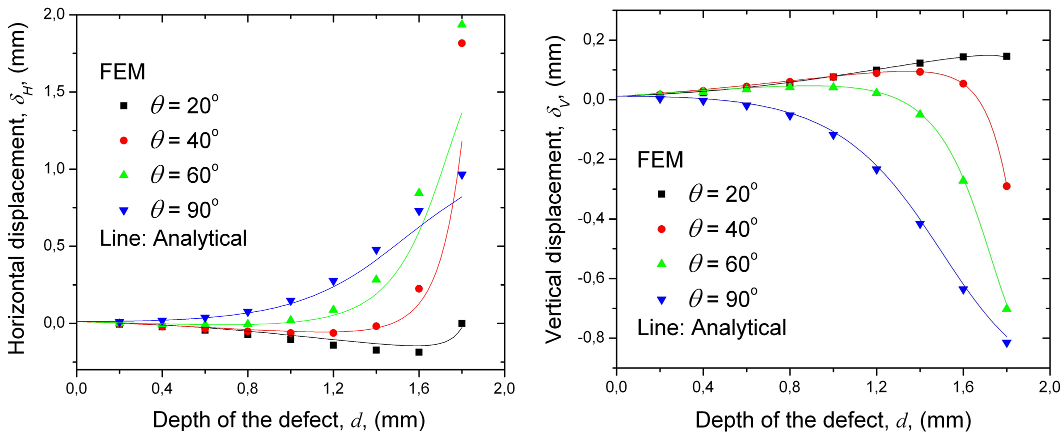

Figure 7: Horizontal and vertical displacement in pipe with internal defect subject a internal pressure.

It can be observed from Fig. 7, that the analytical results obtained using the analyses developed here is in agreement with simulated results using finite elements method. It is important to observe that, these results are valid for depth of the defect value smaller than 1.5 mm due to elastic limit.

3 Conclutions and comments

This study was designed to analyze the asymmetric radial deformation generated by local wall thinning in pipe under internal pressure. The idea of these preliminary results is to provide an alternative means of estimating the internal defect in pipe by means measurement of diameter variations. To solve this, a simple and inexpensive method to obtain asymmetric radial deformation was suggested based on classical Castigliano theorem. Taking into account the approximations, it can be considered a very good agreement between the analytical results and numerical results using finite element method. If the radial displacements are well known it is possible to estimate the defect dimension and consequently to predict failure in pipe. The authors expected that by mean of radial measurements, associate with present analysis, it will be possible to replace or add inspection like pig scan. It is important to emphasize that pig scan is only used in pipeline, does not being used in tub system. For future work the authors aim to approach the problem using experimental results.

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