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A MULTI-OBJECTIVE OPTIMIZATION DESIGN FOR PARALLEL STRUCTURES

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Abstract. *In this paper, a multi-objective optimization process is proposed in order to enhance the design of parallel structures. Manipulators with parallel architecture have inherent advantages in some applications with respect to serial manipulators, like high stiffness, accurate positioning and high movement velocities. Therefore, they address great interest in some industrial applications and medical fields. Main characteristics of parallel structures like workspace, singularities and compliant displacements are considered in order to propose design criteria obtaing a computationally efficient objective functions. The proposed procedure has been applied to a 5R Symmetric Parallel Manipulator.*

Keywords: *Parallel Manipulators, Singularities, Compliant displacements, Multi-objective optimization.*

1. INTRODUCTION

A parallel manipulator typically consists of a moving platform that is connected to a fixed base by several serial chains, called limbs. Features of such system can present better stiffness and payload capacity with respect to the serial architectures, and high velocity and acceleration during the operation. Furthermore, errors in the joints are not cumulative, which contributes for its overall accuracy. Due to their characteristics they have been studied extensively both from theoretical and practical viewpoints. Prototypes have been conceived and built together with the development of theoretical investigations on kinematics and dynamics. The attention are focused to a number of possible industrial applications such as manipulation (Gonçalves and Carvalho, 2008a; Macho, et al., 2008), packing and assembly/disassembly machines (Figielski et al., 2007), motion simulation (Stewart, 1965), milling machines (Hess-Coelho et al., 2001), toys and sensors. However, they have some disadvantages such as small and complex workspace with internal singularities and the complexity of their forward kinematics (Gosselin and Angeles, 1990; Macho et al., 2008; Gonçalves and Carvalho, 2008b).

Optimization methodologies have long been applied to mechanism synthesis in order to obtain high performances and suitable mechanism dimensions. Several performance criteria could be taken into account for design purposes, as for example workspace, singularities and stiffness.

The workspace of parallel kinematic mechanisms has in general a complex volume shape. Discretization algorithms are usually used to determine workspace of manipulators. They consist in discretizing the 3-dimensional space, solving the Inverse Kinematics for each point, and verifying the constraints that limit the workspace (Ceccarelli et al, 2005). Such discretization algorithms are used by most of researchers: they are general and can be applied to any type of architecture.

One of the important limitations of parallel mechanisms is that they may lead to singular configurations in which the stiffness of the mechanism is compromised. The physical meaning of a singularity in kinematics refers to those configurations in which the number of degree of freedom (dof) of the mechanism changes instaneously. The concept of singularity has been extensively studied and several classification methods have been defined. Gosselin and Angeles (1990) suggested a classification of singularities for parallel manipulators into three main groups. The first type of singularity occurs when the manipulator reaches internal or external boundaries of its workspace and the output links loses one or more dof. Second type of singularity is related to those configurations in which the output link is locally movable even if all the actuated joints are locked. Third type is related to linkage parameters and occurs when both first and second type of singularities is involved. Tsai (1990) classify the tree type of singularity by: inverse singularity; direct singularity and combined singularity respectively. Their method is based in find the roots of the determinant of the manipulator's Jacobian matrices. However, obtain analytical expressions for the singularity loci for mechanisms with more than three dof are more difficult because the complexity of the determinant. Another alternative approach to obtain singular configurations for parallel architectures is based in the analysis of stiffness matrix (Gonçalves and Carvalho, 2009).

Stiffness can be defined as the capacity of a mechanical system to sustain loads without excessive changes of its geometry (Rivin, 1999). These produced changes on geometry, due to the applied forces, are known as deformations or compliant displacements.

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Compliant displacements in a parallel robotic system produces negative effects on static and fatigue strength, wear resistance, efficiency (friction losses), accuracy, and dynamic stability (vibration). The growing importance of high accuracy and dynamic performance for parallel robotic systems has increased the use of high strength materials and lightweight designs improving significant reduction of cross-sections and weight. Nevertheless, these solutions also increase structural deformations and may result in intense resonance and self-excited vibrations at high speed (Rivin, 1999). Therefore, the study of the stiffness becomes of primary importance to design multibody robotic systems in order to properly choose materials, component geometry, shape and size, and interaction of each component with others. Some examples of design procedures based on stiffness analysis can be found in (Yoon et al., 2004; Deblaise et al., 2006)

The overall stiffness of a manipulator depends on several factors including the size and material used for links, the mechanical transmission mechanisms, actuators and the controller (Tsai, 1999). In general, to realize a high stiffness mechanism, many parts should be large and heavy. However, to achieve high speed motion, these should be small and light. Moreover, one should point out that the stiffness is greatly affected by both the position and the values of the mechanical parameters of the structure parts (Yoon et al., 2004).

There are three main methods have been used to derive the stiffness model of parallel manipulators (Deblaise et al., 2006). These methods are based on the calculation of the *Jacobian matrix* (Company et al., 2005); the *Finite Element Analysis* (FEA) (Bouzgarrou et al., 2004) and the *Matrix Structural Analysis* (MSA) (Deblaise et al., 2006; Przemieniecki, 1985; Dong et al., 2005; Gonçalves and Carvalho, 2008a).

The methods based on calculation of the Jacobian matrix are simple and they supply one initial estimation of the stiffness matrix. The uses of Finite Element Analysis models are reliable, but these models have to be remeshed over again, involving very tedious and time-consuming routines. However these models are well adapted to validate analytical models, or some experimental results. Methods based on matrix structural analysis are simple and easy for computational implementation.

Using the matrix structural analysis the stiffness matrix of each beam element and joint is obtained. Then, the elemental matrices are grouped in order to obtain the structure stiffness matrix. In singular position the stiffness is compromised, and the inverse stiffness matrix of the whole structure in this configuration is badly scaled, identified using a condition number. A large condition number indicate a nearly or singular position.

Obtaining high performances requires the choice of suitable mechanism dimensions especially as there is much larger variation in the performances of parallel architectures according to the dimensions than for classical serial ones.

Indeed, with the development of manipulators for performing a wide range of tasks, the introduction of performance indices or criteria, which are used to characterize the manipulator, has, became very important. A number of different optimization criteria for manipulators may be appropriate depending on the resources and general nature of tasks to be performed. Consequently, one of the problems facing the designer is how to choose performance criteria and justify the optimality of different designs.

Only recently, it has been possible to consider simultaneously several design aspects in design procedures for manipulators. Modern design procedures make use more and more of the formulation of optimization problems that can be solved by using well-established mathematical techniques in commercial software packages (Ceccarelli et al., 2005).

In this paper is presented a formulation for optimum design of parallel structures that considers the workspace, singularities and stiffness. This methodology is applied on 5R symmetric parallel manipulator, in order to obtain design parameters of the structure. As the workspace of 5R symmetric parallel manipulator has a complex format, this workspace has been represented through an equivalent area. The analysis of stiffness and singularity consider the methodology proposed by Gonçalves and Carvalho (2008a, 2009) to formulate objective functions.

2. THE 5R SYMMETRIC PARALLEL MANIPULATOR

A five-bar manipulator is a typical parallel manipulator with the minimal degrees of freedom, which can be used for positioning a point on a region of a plane. A 5R parallel manipulator consists of five bars that are connected end to end by five revolute joints, two of which are connected to the base and actuated, as shown in Fig. 1. Such a manipulator with a symmetric structure has attracted many researchers, who have investigated its position analysis (Liu et al, 2006; Alici and Shirinzadeh, 2004), workspace (Macho, et al., 2008), assembly modes (Cervantes-Sánchez et al., 2001, singularity (Macho et al., 2008; Mbarek et al., 2007; Figielski et al, 2007; Gonçalves and Carvalho, 2009a), performance atlases (Liu et al, 2006) and kinematic design (Cervantes-Sánchez et al., 2001; Alici and Shirinzadeh, 2004).

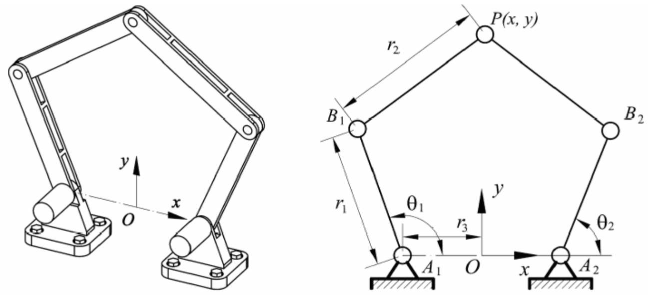

Figure 1. The 5R Parallel Manipulator (Liu et al., 2006).

A kinematics model of the manipulator is developed as shown in Fig. 1. Each actuated joint is denoted as A_i (i = 1, 2), the other end of each actuated link is denoted as B_i and the common joint of the two legs is denoted as P, which is also the output point. A fixed global reference system O_{xy} is located at the center of A_1A_2 with the *y* axis normal to A_1A_2 and the *x* axis directed along A_1A_2. For the structure symmetry, one have $OA_1 = OA_2 = r_3$, $A_1B_1 = A_2B_2 = r_1$ and $B_1P = B_2P = r_2$.

2.1. Backward Kinematics

The input angles θ_1 and θ_2 can be obtained, by using the backward kinematics, when the position of point *P* is known by considering the reference frame O*xy* and the following restriction:

$$\left|pb_i\right| = r_2 \,,\quad i = 1,2 \tag{1}$$

or in scalar form

$$(x - r_1\cos\theta_1 + r_3)^2 + (y - r_1\sin\theta_1)^2 = r_2^2 \tag{2}$$
$$(x - r_1\cos\theta_2 - r_3)^2 + (y - r_1\sin\theta_2)^2 = r_2^2 \tag{3}$$

If the output point *P* is known, the inputs angles for reaching this position can be obtained as

$$\theta_i = 2\tan^{-1}(z_i)\,,\quad i = 1,2 \tag{4}$$

where

$$z_i = \frac{-b_i \pm \sqrt{b_i^2 - 4a_ic_i}}{2a_i}\,,\ i = 1,2 \tag{5}$$

and

$$\begin{aligned}
a_1 &= r_1^2 + y^2 + (x + r_3)^2 - r_2^2 + 2(x + r_3)r_1 \\
b_1 &= -4yr_1 \\
c_1 &= r_1^2 + y^2 + (x + r_3)^2 - r_2^2 - 2(x + r_3)r_1 \\
a_2 &= r_1^2 + y^2 + (x - r_3)^2 - r_2^2 + 2(x - r_3)r_1 \\
b_2 &= b_1 = -4yr_1 \\
c_2 &= r_1^2 + y^2 + (x - r_3)^2 - r_2^2 - 2(x - r_3)r_1
\end{aligned} \tag{6}$$

There are four solutions for backward kinematic problem of 5R manipulator from Eq. (4).

2.2. Forward Kinematic

The forward kinematic problem consists in obtaining the coordinates of point *P* with respect to a set of given inputs angles θ_1 and θ_2. From Eqs. (2) and (3) one can write

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$$x^2 + y^2 - 2(r_1 \cos\theta_1 - r_3)x - 2r_1 \sin\theta_1 y - 2r_1 r_3 \cos\theta_1 + r_3^2 + r_1^2 - r_2^2 = 0 \quad (7)$$

$$x^2 + y^2 - 2(r_1 \cos\theta_2 + r_3)x - 2r_1 \sin\theta_2 y + 2r_1 r_3 \cos\theta_2 + r_3^2 + r_1^2 - r_2^2 = 0 \quad (8)$$

The Equations (7) and (8) yield to

$$x = ey + f \quad (9)$$

where

$$e = \frac{r_1(\sin\theta_1 - \sin\theta_2)}{2r_3 + r_1 \cos\theta_2 - r_1 \cos\theta_1} \quad (10)$$

$$f = \frac{r_1 r_3(\cos\theta_2 + \cos\theta_1)}{2r_3 + r_1 \cos\theta_2 - r_1 \cos\theta_1} \quad (11)$$

The y coordinate can be obtained by substituting Eq. (9) into Eq. (8) as

$$dy^2 + gy + h = 0 \quad (12)$$

so

$$y = \frac{-g \pm \sqrt{g^2 - 4dh}}{2d} \quad (13)$$

where

$$\begin{aligned} d &= 1 + e^2 \\ g &= 2(ef - er_1 \cos\theta_1 + er_3 - r_1 \sin\theta_1) \\ h &= f^2 - 2f(r_1 \cos\theta_1 - r_3) - 2r_1 r_3 \cos\theta_1 + r_3^2 + r_1^2 - r_2^2 \end{aligned} \quad (14)$$

Equations (9) and (13) provide two solutions for the forward kinematic problem of the 5R manipulator.

3. A MULTI-OBJECTIVE OPTIMIZATION DESIGN FOR PARALLEL STRUCTURES

Once the numerical technique is chosen or is advised for solving the proposed multi-objective optimization problem, the main efforts can be addressed to the formulation of common algorithms for numerical evaluation of optimality criteria and design procedure constraints. In the following, main aspects are overviewed by emphasizing the common numerical evaluations for parallel manipulators in terms of workspace, singularity, and stiffness.

3.1 General concepts

A multi-objective optimization problem can be written in the form

$$\min\left[f_1(x), f_2(x), \ldots, f_k(x)\right]$$

for k objective functions $f_i : \Re^n \to \Re$ subject to equality and inequality constraints. For the vector of decision variables, $x = \left[x_1, x_2, \ldots, x_n\right]^T$, the task is to determine the set F of all vectors which satisfy the constraints and the particular set of optimal values $x^* = \left[x_1^*, x_2^*, \ldots, x_n^*\right]^T$.

As soon as there are several objectives to be optimized simultaneously, usually there is no longer a single optimal solution but rather a whole set of solutions. When several objectives are optimized at the same time the search space becomes partially ordered. To obtain the optimal solution there will be a set of optimal trade-offs between the conflicting objectives.

In this context, best solution means a solution not worst in any of the objectives and at least better in one objective than the other. An optimal solution is the solution that is not dominated by any other solution in the search space. Such

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an optimal solution is called a Pareto-optimal and the entire set of such optimal trade-offs solutions is called a Pareto- optimal set.

The publication of Kuhn–Tucker (1951) is one of the first rigorous mathematical treatments about Pareto Optimality. The work of Koopmans (1951) initiated the use of Pareto optimality in operations research. Today a number of papers and books on the subject are found.

Even though there are several ways to approach a multi-objective optimization problem, most work is concentrated on the approximation of the Pareto set.

Given a set of alternatives, the problem of choosing the best alternative depends on the way the data is classified. One of the most popular evaluation methods is to associate to each alternative a real value, and the best alternative is chosen as the one with the largest or the smallest value.

In a higher dimension the notion of the smallest and the largest values is not available. In this case, the concept of partial order in a multidimensional space can be applied.

The Pareto cone: Let $\Re_+^n$ be the positive octant of the n-dimensional Euclidean space. Then, for two vectors $x=(x_1, \ldots, x_2)$, $y=(y_1, \ldots, y_n)$ in $\Re^n$, one has $x \le y$ if and only if $x_i \le y_i$, $i=1,\ldots,n$. The cone $\Re_+^n$ is called the Pareto cone because the original Pareto optimality is defined by the order generated by this cone. When $n=1$, the usual order of real numbers is exactly this order. The order is total in the sense that any two numbers x and y are comparable: either $x \ge y$ or $y \ge x$. On the other hand, when $n > 1$ this order is not total.

By using the concept of partial order it is possible to define the concept of optimal solution. However, in a real world situation, a decision making (trade-off) process is also useful to evaluate optimal solutions. Therefore, in this paper a procedure to determine the Pareto frontier is presented. An up-to-date discussion about this subject is presented by Pardalos and Du (2008).

3.2. Optimum workspace for planar parallel manipulators

It is possible to compute the workspace of a parallel manipulator through a geometrical approach. The corresponding procedure is described by Gonçalves et al. (2007). This formulation uses the volume of solids of parallelepiped, cylindrical or spherical geometry. In such case a fixed volume is the goal to be achieved. As this work is carried out to study the planar mechanism this methodology is modified to maximize the area of the workspace.

It is possible to demonstrate the relationship between such approach and the maximization of the perimeter of a planar figure, Fig. 2. In the present paper the objective is to maximize the workspace, A_p, and not only achieve a given area, A. Therefore, if x and y are the sides of a rectangle, the objective of maximum area can be achieved through the expression

$$f_1 = x.y \tag{15}$$

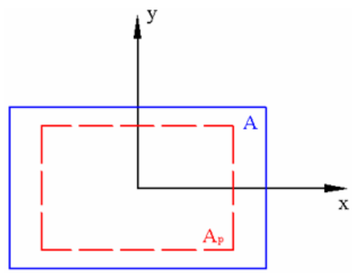

Figure 2. The rectangle geometry.

3.3. Optimum stiffness and singularity for planar parallel manipulators

In this paper, the stiffness matrix is obtained from the method *Matrix Structural Analysis* (*MSA*), also known as the *displacement method* or *direct stiffness method* (*DSM*). The methods of structural analysis is based on the idea of breaking up a complicated system into component parts, discrete structural elements, with simple elastic and dynamic properties that can be readily expressed in a matrix form. The discrete structure is composed by elements which are joined by connecting nodes. When the structure is loaded each node suffers translations and/or rotations, which depend on the configuration of the structure and the boundary conditions. For example, in a fixed linkage no displacement occurs. The nodal displacement can be found from a complete analysis of the structure. The matrices representing the beam and the joint are considered as building blocks which, when fitted together in accordance with a set of rules derived from the theory of elasticity, provide the static and dynamic properties of the whole structure (Przemieniecki, 1985).

The stiffness matrix of the three-dimensional straight bar with uniform cross-sectional area is

$$k_j = \begin{bmatrix} k_{bj} & -k_{bj} \\ -k_{bj} & k_{bj} \end{bmatrix} \quad (16)$$

where k_{bj} is given by:

$$k_{bj} = \begin{bmatrix} \frac{A_j E_j}{L_j} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12 E_j I_{zj}}{L_j^3} & 0 & 0 & 0 & \frac{6 E_j I_{zj}}{L_j^2} \\ 0 & 0 & \frac{12 E_j I_{yj}}{L_j^3} & 0 & -\frac{6 E_j I_{yj}}{L_j^2} & 0 \\ 0 & 0 & 0 & \frac{G_j J_j}{L_j} & 0 & 0 \\ 0 & 0 & -\frac{6 E_j I_{yj}}{L_j^2} & 0 & \frac{4 E_j I_{yj}}{L_j} & 0 \\ 0 & \frac{6 E_j I_{zj}}{L_j^2} & 0 & 0 & 0 & \frac{4 E_j I_{zj}}{L_j} \end{bmatrix} \quad (17)$$

On Equation (17) E_j and G_j are, respectively, the modulus of elasticity and the shear modulus of element *j; I_{yj}, I_{yz}* are the moment of areas about the *Y* and *Z* axes, respectively. J is the Saint-Venant torsion constant and A_j is the cross-sectional area.

The stiffness of a joint is given by (Gonçalves and Carvalho, 2008a):

$$k_{joint} = \begin{bmatrix} k_c & -k_c \\ -k_c & k_c \end{bmatrix} \quad (18)$$

Where $k_c = diag(k_{tx}, k_{ty}, k_{tz}, k_{rx}, k_{ry}, k_{rz})$; k_{tx}, k_{ty}, k_{tz} are the translation stiffness and k_{rx}, k_{ry}, k_{rz} the rotational stiffness along the axes.

For application of *MSA* is necessary to write the stiffness matrices of all elements in the same reference frame. This transformation, element by element, must be held before the assembly of the stiffness matrix of the structure. This transformation matrix, T_j, can be obtained from algebra linear.

Thus, the stiffness matrix of the elements in a common reference frame (elementary stiffness matrix), for segments, k_j^e , and for joints, k_{joint}^e , are:

$$[k_j^e] = [T_j][k_j][T_j]^T \quad (19)$$

$$[k_{joint}^e] = [T_j][k_{joint}][T_j]^T \quad (20)$$

After obtaining the stiffness matrix of each beam and joint in a common reference frame, the stiffness matrix of whole structure can be obtained using the *MSA*. Based on how the structure elements are connected, from their nodes, it is possible to define a connectivity matrix. As each segment and joint stiffness are known, the global stiffness matrix is obtained by a superposition procedure. This global stiffness matrix is singular because the system is free. After application of the boundary conditions, for example, where the displacements are known, the new matrix is invertible and the compliant displacements can be done by:

$$\{U\} = K^{-1} \{W\} \quad (21)$$

Where *U* are the compliance displacements and *W* are the external wrenches applied. This procedure is described in detail in (Gonçalves, 2009).

In a singular position the stiffness is compromised, and the inverse stiffness matrix of the whole structure, Eq. (21), in this configuration is badly scaled, identified by using a condition number. A large condition number indicate a nearly or singular position.

The condition number, *cond*, of a square matrix is the product of the norm of the matrix and the norm of its inverse (Meyer, 2000), Eq. (22).

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$$cond(K) = \|K\| * \|K^{-1}\| \tag{22}$$

There are different ways to evaluate the matrix norm (|| . ||). In the present text the norm is calculated by:

$$\|K\|_1 = \underset{j}{Max} \sum_{i=i}^{n} \|k_{ij}\| \quad \text{(Absolute maximum sum of columns of K)} \tag{23}$$

As a result, a general computational routine for the mapping of the workspace of the parallel robotic structure is given, since the stiffness matrix is dependent of the configuration of the structure. Simultaneously with the mapping of workspace, the method *MSA* is applied to obtain the stiffness matrix of structure and the computation of the condition number.

The Figure 3 shows the model *MSA*. The links are given by nodes: 1-2; 3-4; 5-6 and 7-8. The revolute joints are given by nodes: 2-3; 4-5 and 6-7. The model has 8 nodes.

The segments are built with steel ($E = 2 \times 10^{11}$ N/m^2 and $G = 0.8 \times 10^{11}$ N/m^2); the cross-sectional area is circular with $0.005m$ diameter and $r_1 = 0.1m$; $r_2 = 0.1m$ and $r_3 = 0.1m$. The boundary conditions are given by actuators considered as blocked in nodes 1 and 8. The external force and torque are applied on node 5, which is the center of the end-effector. The others joints are passive and modeled with $k_{tx} = k_{ty} = k_{tz} = 2 \times 10^{11}$ N/m; $k_{rx} = k_{ry} = 2 \times 10^{11}$ N/rad and $k_{rz} = 0$ N/rad like proposed by Gonçalves (2009).

Applying the methodology *MSA* for the 5R manipulator is possible to map the stiffness, Eq. (21) simultaneously with calculation of the singularities positions, Eq. (22).

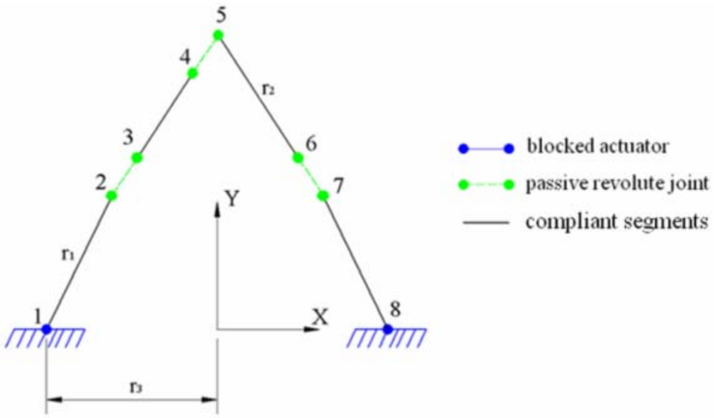

Figure 3. Model *MSA* of 5R mechanism.

The objective function for the analysis of the singularity is

$$f_2 = \frac{cond(K)}{[cond(K)]_0} \tag{24}$$

where $[cond(K)]_0$ is the initial value.

The objective function that evaluates the stiffness, obtained by Eq. (21), considers the compliance displacements of point *P*, node 5, corresponding the linear compliant displacement *x* and *y*, and the rotational compliance displacement about axis *z*. The completed procedure for obtained the compliant displacements is describe in Gonçalves (2009) and Gonçalves and Carvalho (2008a). The corresponding objective functions are:

$$f_{3\_1} = \frac{(f_{3\_x})_d}{(f_{3\_x})_g}\ ;\ f_{3\_2} = \frac{(f_{3\_y})_d}{(f_{3\_y})_g}\ ;\ f_{3\_\phi} = \frac{(f_{3\_\phi z})_d}{(f_{3\_\phi z})_g} \tag{25}$$

where $(f_{3\_x})_d$, $(f_{3\_y})_d$ and $(f_{3\_\phi z})_d$ are the compliant displacements obtained by Eq. (21) and the $(f_{3\_x})_g$, $(f_{3\_y})_g$ and $(f_{3\_\phi z})_g$ are the initial values desired. The initial values of $(f_{3\_x})_g$, $(f_{3\_y})_g$ and $(f_{3\_\phi z})_g$ may be different for each problem. The criterion adopted is explained on the next section.

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3.4 Weighting Objective Formulation

To formulate the performance criterion that takes into account all the objective functions in such a way that an overall multi-criterion objective function can be written, the Weighting Objective Method is used. The minimization process leads to a Pareto optimal solution or, alternatively, to a set of optimal solutions. The scalar objective function that represents the performance criteria altogether is written as:

$$f(x) = \sum_{i=1}^{k} \alpha_i f_i(x) \tag{26}$$

where $\alpha_i \geq 0$ are weighting coefficients that represent the relative importance of each separate criterion. From the numerical point of view the minimization process depends also on the numerical values that express the objective functions. Due to scaling problems, the numerical values that express the objective functions should be adjusted. Otherwise, α_i will not represent the relative importance of the objective functions (Deb, 2001). Consequently, Eq. (26) should be rewritten as follows:

$$f(x) = \sum_{i=1}^{k} c_i f_i(x) \tag{27}$$

where c_i are scaling factors. Usually, satisfactory results are obtained if $c_i = \frac{\alpha_i}{f_i^0}$, where f_i^0 represents the minimum of the objective function f_i calculated separately (Eschenauer *et al*, 1990). Eq. (27) was used in the optimization processes shown in this paper.

4. NUMERICAL RESULTS

Numerical computations were performed to evaluate the proposed objectives in a unified approach. The maximum area f_1, the singularity avoidance f_2 and the maximization of the stiffness f_3 are considered in a multi-objective formulation.

Without loss of generality, a minimization objective is given by

$$F = -\left(\frac{f_1}{f_1^0}\right)^2 + \left(\frac{f_2}{f_2^0}\right)^2 - \left(\sum_{i=1}^{3} \frac{f_{3,i}}{f_{3,i}^0}\right)^2 \tag{28}$$

The proposed formulation can be solved as a maximization problem by multiplying this objective function by (-1).

The design variables are the radius R of the links, r_1 is the length of the first link, r_2 is the length of linkage bar, and r_3 is the length of the basis, Fig. 1. The feasible value of each design variables is bounded by $0.001 < R < 0.05$ m, $0.01 < r_1 < 1$ m, $0.01 < r_2 < 1$ m and $0.01 < r_3 < 1$ m, respectively.

Since an initial design (R_0, $r_{1,0}$, $r_{2,0}$, $r_{3,0}$) is given, weighting factors need to be determined. To obtain an appropriate value, the objective function is evaluated without such constants, that is, $(f_i^0 = 1)$. The values of the objective functions when using such design are set as weighting factors f_i^0.

Deterministic and heuristic optimization methods were used to find the optimal solution of the problem.

For a deterministic evaluation, a Sequential Quadratic Programming (SQP) was adopted, since it belongs to the state of the art in nonlinear programming methods (Powell, 1978). At the major iterations, a positive definite quasi-Newton approximation of the Hessian of the Lagrangian function is calculated using the BFGS method.

Results obtained by this methodology are presented in Table 1.

Table 1. Optimal design provided by a deterministic procedure.

Initial design	f_1^0	f_2^0	$f_{3,1}^0$	$f_{3,2}^0$	$f_{3,3}^0$	Optimal design
(0.005, 0.1, 0.1, 0.1)	2.7e-12	3.3e+18	9.4e+3	0.3	2.0e+5	(0.05, 0.01, 0.01, 1.00)
(0.01, 0.3, 0.3, 0.3)	2.4e-11	8.0e+17	4.0e+4	2.4	1.4e+5	(0.037, 0.010, 0.031, 0.010)
(0.03, 0.01, 0.01, 0.01)	2.7e-14	5.4e+20	7.9343	3.3e-004	4.1e+5	(0.05, 0.01, 0.13, 1.00)

The problem under consideration is highly nonlinear. It follows that small deviation in the design values may lead to big deviations in the objectives. In this context the deterministic optimization is well suited to perform a fine tuning of the design parameters aiming to improve the overall performance of the system. The interpretation of this behavior is: if the small decrease of any performance will lead to a bigger increase of other performance, this new configuration will be preferred.

The second analysis was carried out by means of a heuristic optimization methodology, the so called Differential Evolution Methodology. This strategy is based on genetic algorithm and has been proven to be suitable to deal with a number of problems.

A feature of such methodology is that initial design is not required. Furthermore, local minima are not a problem, since the search direction does not requires information about the gradient of the objective function.

Without loss of generality, weighting parameters were chosen the same than those used in the first experiment. It should be pointed out that different values may influence the optimal design. This behavior can be used to provide a higher priority to some objective against others.

The optimal results are presented in the Table 2 below. Different evolution criteria can be set in the procedure. The current implementation provides nine strategies of evolution (lines 1 to 9, respectively). Each strategy was evaluated three times (columns 1st , 2nd and 3rd respectively).

Table 2. Optimal design provided by a heuristic procedure.

	Optimal design											
Strategy	R			r1			r2			r3		
	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd	1st	2nd	3rd
1	0.0410	0.0305	0.0453	0.8700	0.7141	0.6138	0.0936	0.2295	0.6215	0.4058	0.1262	0.8608
2	0.0018	0.0044	0.0345	0.9842	0.3264	0.5511	0.1755	0.5356	0.4315	0.1152	0.6579	0.6480
3	0.0318	0.0345	0.0010	0.7743	0.7070	0.8668	0.9335	0.4479	0.6164	0.9730	0.0294	0.9901
4	0.0071	0.0454	0.0392	0.4955	0.5384	0.3442	0.8545	0.1181	0.6118	0.8752	0.8276	0.7438
5	0.0063	0.0217	0.0476	0.6354	0.3660	0.4495	0.1352	0.5627	0.0694	0.1430	0.7451	0.8681
6	0.0120	0.0105	0.0148	0.3798	0.8969	0.7337	0.0966	0.1081	0.1464	0.6437	0.0537	0.8384
7	0.0159	0.0193	0.0206	0.0557	0.5973	0.8336	0.2035	0.8738	0.1430	0.7230	0.9342	0.0699
8	0.0311	0.0080	0.0011	0.8205	0.5638	0.1973	0.8874	0.0145	0.1511	0.9318	0.7690	0.2754
9	0.0015	0.0205	0.0455	0.5370	0.3810	0.5567	0.2866	0.1398	0.0426	0.9468	0.4407	0.0633

In this case the method is not attracted by a local minimum. Differences on the optimal design are justified by a highly nonlinear nature of the problem and a high sensitivity of the objectives regarding small changes on the design variables.

5. CONCLUSION

In this paper a methodology to obtain design parameter of a parallel robotic structure was presented.

First, backward and forward kinematic equations of a 5R symmetric parallel manipulator were presented. It was followed by general concepts of multi-objective optimization concepts, optimum workspace formulation, stiffness and singularity analysis.

A key point to evaluate multi-objective problems is the computation of weighting factors to correctly express objective priorities.

The current study considered objectives with the same priority. It was achieved by using results of an initial design as weighting factors.

Two strategies were used for the optimization process. The first strategy was a Quadratic Sequential Programming. The method was able to improve the initial design by means of a local optimization. This method is recommended when a fine tuning of design variables is required. The improvement of the overall performance index is sometimes achieved by means of the penalization of individual objectives. The second strategy consists in a Differential Evolution Methodology. This heuristic method is able to search for a global optimum. Different evolution parameters were considered in multiple runs.

The result shows that there is no unique solution for this problem. It follows that, multiple designs lead to similar objective values.

From the results is possible to conclude that: the proposed formulation is able to deal with the complexity of the parameters evaluated; this problem is highly nonlinear and coupled; there are optimization methods suitable for fine tuning of the parameters (deterministic approach) and evaluation of a wide design scenario (heuristic approach).

Future research includes the analysis of the Pareto frontier when a qualitative analysis is considered and the use of stochastic optimization methods to consider uncertainties in the parameters.

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