# MODELING AN OPTICAL DIGITIZER ROBOTIC FOR READING THREEDIMENSIONAL OBJECTS 

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Abstract. In several technological and artistic areas it is necessary to obtain the dimensions of three-dimensional objects for its graphical representation and/or reproduction. A general and unique method does not exist to do this task due to the object form, its physical composition and his dimensions that make difficult the reading process. To take the object dimensions there are a variety of system, manual or robotics, known as scanners or digitizers, which each of them has its own methods to read dimensions and process then. In this paper will be presented the development of a three-dimensional digitizer using a robotic structure of five degrees of freedom. In order to reduce the amount of data, diminish the error sources on the measurements and to reduce the computational costs it was used a methodology, based on the triangulation method, that pursue the normal vector of the three-dimensional surface.

Keywords: Digitizers, Optical digitizer, Reverse Engineering, Robotics, Surface digitalization.

## 1. INTRODUCTION

It is clear the increasing interest for using computational tools and CAD/CAM softwares to design and produce three dimensional objects. In some cases the physical object exists and must be reproduced. Then its dimensions must be known.

The first step of reconstructing a model is to acquire data of the real object. This can be done by using manual and/or automatic equipment named scanner or digitizer which the output is a collection of points describing the original surface. In the acquisition data process, thousands or millions of data must be acquired due to the resolution and correctness of the final model is highly dependent on the density of distribution of the points. In general the data set is a non-organized cloud of points, with non-uniform sampling density, with gaps and noise. In this way two kinds of equipments have been developed acquire the surface measures: those whose use measurement techniques with contact and those without contact. Measurements techniques without contact use several physical methods such as optical, magnetic, and others. The techniques with contact in general use a contact sensor to take the surface coordinate when the sensor touches it. The process for reconstruction a real object is known as reverse engineering (Bajaj et al., 1995), (Larsson and Kjellander, 2006), (Moccozet et al., 2004), (Müller et al., 2000), (Remondino, 2003) and (Várady et al., 1996).

Then, a big challenge for a surface reconstruction is to acquire on "ideal" set of points that represents more faith fully to the real object. Thus, the digitizer plays an important role in the reconstruction process.

In this paper it has been described a robotic system to use as digitizer. The system works as a left and right hand where the left hand take the object and the right one manipulates an optical sensor to "read" the surface coordinates. After the system description, its kinematic model and workspace are analyzed, the error analyses of the structure is presented with numerical simulations based on standard components used to construct it.

## 2. THE DIGITIZER DESCRIPTION

The digitizer is composed by a mechanical structure and a reading system. The reading system is made up by a laser beam source and a CCD (Charge-Coupled Device) as receiver, embedded in a compact unit. The reader system is based on the triangulation active spatial method to take the surface measures (Everett, 1995).

The mechanical structure of the digitizer has 5 dof and is composed of two kinematic chains, where one is used to move the laser sensor and the other one is used to take up the object. The structure which moves the laser sensor is composed of two revolute joints and one prismatic joint (a RRP structure), and the structure which takes up the object to pose it during the reading process is composed of a prismatic joint and a revolute ones where a table is assembled to support the object (a PR structure).

Using a RRP + PR structure can be justified because it respects the restrictions imposed by the triangulation method, it can be constructed in a robust way, presents good repeatability and the kinematical model can be obtained in a closed form which enable takes up the surface coordinates in function of structure point coordinates. In the fig. 1 an illustration
of the digitizer is presented.


Figure 1. Mechanical structure of digitizer

## 3. KINEMATIC MODEL

In order to obtain the kinematic model must be considered that for any point $Q$ of the surface object its normal vector $\boldsymbol{N}$ must be oriented to the laser beam to enable the reading process. Then, the $Q$ point and its normal $\boldsymbol{N}$ must be put in the lecture plane defined by the laser beam. This re-positioning process is done by the PR structure.

In figure 2 is represented the used inertial reference frame ( $O_{o} X Y Z$ ) a generic $Q$ point with its normal which its orientation is defined by angles $\eta$ and $\gamma$.

To pose the $Q$ point and its normal in the $X Y$-plane firstly the prismatic joint moves by $q_{5}$ and then the revolute joint turns by $q_{4}$. After the both motion the point $Q$ is in the $X Y$-plane with the denomination $Q_{1}$, fig. 2 and 3 .

Using the classical homogeneous transformation matrix and equipollent reference frames, the homogeneous matrix between references frames $\left(O_{o} X Y Z\right)$ and $\left(O_{5} x_{5} y_{5} z_{5}\right)$ is given by

$$
T_{05}=\left[\begin{array}{ccc:c}
c\left(q_{4}+\gamma\right) \cdot c \eta & -c\left(q_{4}+\gamma\right) \cdot s \eta & s\left(q_{4}+\gamma\right) & x_{Q} \cdot c q_{4}-z_{Q} \cdot s q_{4}  \tag{1}\\
s \eta & c \eta & 0 & y_{Q} \\
-s\left(q_{4}+\gamma\right) \cdot c \eta & s\left(q_{4}+\gamma\right) \cdot s \eta & c\left(q_{4}+\gamma\right) & -x_{Q} \cdot s q_{4}-z_{Q} \cdot c q_{4}+q_{5} \\
\hdashline 0 & 0 & 0 & 1
\end{array}\right]
$$

Where $s q$ represents sinq and $c q$ is cos $q$.
By considering the new situation of the $Q$ point in $X Y$-plane, defined by point $Q_{1}$, and which normal is defined by $\bar{N}$, de homogeneous matrix between reference frames ( $O_{o} X Y Z$ ) and ( $O_{5^{*}} x_{5^{*}} y_{5^{*}} z_{5^{*}}$ ), fig.3, is given as

$$
T_{05^{*}}=\left[\begin{array}{ccc:c}
c \eta & -s \eta & 0 & x_{Q_{1}}  \tag{2}\\
s \eta & c \eta & 0 & y_{Q_{1}} \\
0 & 0 & 1 & 0 \\
\hdashline 0 & 0 & 0 & 1
\end{array}\right]
$$



Figure 2. References frames for PR structure


Figure 3. Reference frames for the new position of point $Q$ in $X Y$-plane, defined by $Q_{l}$ point.
From equations (1) and (2) the coordinates of the new situation of the point $Q$ in the $X Y$-plane, defined by the point $Q_{l}$ can be given by

$$
\begin{align*}
& x_{Q}=x_{Q_{1}} \cdot c q_{4}+q_{5} \cdot s q_{4}  \tag{3}\\
& y_{Q}=y_{Q_{1}}  \tag{4}\\
& z_{Q}=x_{Q_{1}} \cdot s q_{4}-q_{5} \cdot c q_{4} \tag{5}
\end{align*}
$$

And the motions of the PR structure defined by the $q_{4}$ and $q_{5}$ joint coordinates can be written as

$$
\begin{align*}
& q_{4}=\left\{\begin{array}{l}
-\gamma\left(-\frac{\pi}{2}<\gamma \leq \frac{\pi}{2}\right) \\
\pi-\gamma\left(\frac{\pi}{2}<\gamma \leq 3 \cdot \frac{\pi}{2}\right)
\end{array}\right.  \tag{6}\\
& q_{5}=x_{Q} \cdot s q_{4}-z_{Q} \cdot c q_{4} \tag{7}
\end{align*}
$$

The kinematic analysis of the RRP structure can be done by considering it in an initial configuration as sketched in fig. 4, where the $Q_{l}$ point and its normal $\bar{N}$ are represented too


Figure 4. Reference frames of the RRP kinematic chain on its initial configuration.
In order to point out the laser beam the normal direction the RRP configuration must be changed as sketched in fig. 5, where the following structural parameters are used: $d$ is the radius of the ring; $a$ is distance between the centers of the second revolute joint and the prismatic one at, the initial configuration, and $b$ corresponds to the length of the laser beam (given by the sensor), figs 4 and 5.

From figures 4 and 5 using the homogeneous matrices we can write

$$
\begin{align*}
& T_{09}=\left[\begin{array}{ccc:c}
c\left(q_{1}+q_{2}\right) & -s\left(q_{1}+q_{2}\right) & 0 & d \cdot c q_{1}+\left(a+b+q_{3}\right) \cdot c\left(q_{1}+q_{2}\right) \\
s\left(q_{1}+q_{2}\right) & c\left(q_{1}+q_{2}\right) & 0 & d \cdot s q_{1}+\left(a+b+q_{3}\right) \cdot s\left(q_{1}+q_{2}\right) \\
0 & 0 & 1 & 0 \\
\hdashline 0 & 0 & 0 & 1
\end{array}\right]  \tag{8}\\
& \hdashline T_{09^{*}}=\left[\begin{array}{ccc:c}
-c \eta & -s \eta & 0 & x_{Q_{1}} \\
s \eta & -c \eta & 0 & y_{Q_{1}} \\
0 & 0 & 1 & z_{Q_{1}} \\
\hdashline 0 & 0 & 0 & 1
\end{array}\right] \tag{9}
\end{align*}
$$



Figure 5. The RRP structure pointing up the laser beam to the normal $\bar{N}$ and the used reference frames.
From equations (8) and (9) can be obtained the following relations

$$
\begin{align*}
& x_{Q_{1}}=d \cdot c q_{1}+\left(a+b+q_{3}\right) \cdot c\left(q_{1}+q_{2}\right)  \tag{10}\\
& y_{Q_{1}}=d \cdot s q_{1}+\left(a+b+q_{3}\right) \cdot s\left(q_{1}+q_{2}\right)  \tag{11}\\
& z_{Q_{1}}=0 \tag{12}
\end{align*}
$$

and then

$$
\begin{align*}
& q_{1}=\operatorname{arcsen}\left[\frac{1}{d}\left(-x_{Q_{1}} \cdot s \eta+y_{Q_{1}} \cdot c \eta\right)\right]+\eta  \tag{13}\\
& q_{2}=\pi+\eta-q_{1} ;\left(-\frac{\pi}{2}<\eta \leq 3 \cdot \frac{\pi}{2}\right)  \tag{14}\\
& q_{3}=-x_{Q_{1}} \cdot c \eta-y_{Q_{1}} \cdot s \eta-d \cdot c q_{2}-a-b \tag{15}
\end{align*}
$$

Then, if the joint coordinates $q_{1}, q_{2}, q_{3}, q_{4}$ and $q_{5}$ are known, given by the digitizer system, the position of a point $Q$ of the 3D surface and its normal direction, defined by angles $\eta$ and $\gamma$ can be obtained from eqs. (3) to (15).

Considering a ring which radius is $175,5 \mathrm{~mm}$, the workspace of the RRP structure as a function of the normal orientation $\eta$ is represented in fig. 6 , where $0 \leq \eta \leq \pi$. The workspace is the region between the upper surface, defined for a minimum value of $q_{3}\left(q_{3}=0 ; a+b=57,5 \mathrm{~mm}\right)$ and the lower surface, defined for a maximum value of $q_{3}\left(q_{3}=120 \mathrm{~mm}\right)$.

## 4. ERROR ANALYSES OF THE STRUCTURE

The characteristic parameters of the structure are submitted to errors affecting the final reading of the position and orientation of the vector $\boldsymbol{N}$. The errors are due to the reading system and the mechanical system. The errors from the reading system can be adjusted by a calibration that is made by programming. The errors due to the mechanical system are caused by the adjustments of the parts (geometric errors) and certains not geometric phenomena as: hysteresis, the joint friction and temperature; affecting with this the repetibilidade of the reached positions (Moring and Pack, 1986).


Figure 6. Workspace of the RRP structure in function of $\eta$.
The accuracy of a structure is the total deviation of a pre-established situation, considered as the correct situation, and the reached situation (Dominique, 2006).

The accuracy can be obtained by the Jacobian of the structure by the expression

$$
\delta_{x}=J \cdot \delta q
$$

Where $\delta_{x}$ represents the deviation of the situation of the end-effector and $\delta_{q}$ the errors due to the join coordinates of the structure.

In this paper the errors of RRP structure had been analyzed by using the equations obtained from the kinematic model. In this case the position of a point $Q_{1}$ are given by eqs. (10) and (11) and the normal direction is given by $\eta=\pi-q_{1}-q_{2}$, fig. 5 and eqs. (8) and (9).

Then, the deviation can be obtained by considering errors $\delta q_{1}, \delta q_{2}$ and $\delta q_{3}$, in joint coordinates $q_{1}, q_{2}$ and $q_{3}$, respectively. The theoretical situation of the point $Q_{1}$ and the normal are given by

$$
\begin{align*}
& x_{Q_{1}}=d \cdot c q_{1}+\left(a+b+q_{3}\right) \cdot c\left(q_{1}+q_{2}\right)  \tag{16}\\
& y_{Q_{1}}=d \cdot s q_{1}+\left(a+b+q_{3}\right) \cdot s\left(q_{1}+q_{2}\right)  \tag{17}\\
& \eta=\pi-\left(q_{1}+q_{2}\right) \tag{18}
\end{align*}
$$

Considering the errors $\delta q_{1}, \delta q_{2}$ and $\delta q_{3}$, eqs (16), (17) and (18) becomes:

$$
\begin{equation*}
x_{Q_{1}^{\prime}}=d \cdot c\left(q_{1}+\delta q_{1}\right)+\left(a+b+q_{3}+\delta q_{3}\right) \cdot c\left(q_{1}+\delta q_{1}+q_{2}+\delta q_{2}\right) \tag{19}
\end{equation*}
$$

$y_{Q_{1}^{\prime}}=d \cdot s\left(q_{1}+\delta q_{1}\right)+\left(a+b+q_{3}+\delta q_{3}\right) \cdot s\left(q_{1}+\delta q_{1}+q_{2}+\delta q_{2}\right)$

$$
\begin{equation*}
\eta^{\prime}=\pi-\left(q_{1}+\delta q_{1}+q_{2}+\delta q_{2}\right) \tag{21}
\end{equation*}
$$

Then, the deviations in position and the normal orientation can be written as

$$
\begin{align*}
& \delta x=x_{Q_{1}^{\prime}}-x_{Q_{1}}  \tag{22}\\
& \delta y=y_{Q_{1}^{\prime}}-y_{Q_{1}}  \tag{23}\\
& \delta \eta=\eta^{\prime}-\eta \tag{24}
\end{align*}
$$

And the total error for position is

$$
\begin{equation*}
e=\sqrt{\delta x^{2}+\delta y^{2}} \tag{25}
\end{equation*}
$$

A sensibility index $\mu$ represents the influence of a joint error and $\delta_{q}$ in the end-effector situation. Then, the following relation can be written

$$
\mu_{i}=\frac{e_{i}}{\delta q_{i}}
$$

Where $i$ represents the joint coordinate. In this paper $i=1,2,3$.
Numerical simulations have been made by considering commercial components will be used to construct a prototype. In figure 7 is represented what happens when errors $\delta q_{1}, \delta q_{2}$ and $\delta q_{3}$ are considered in joint coordinates $q_{1}$, $q_{2}$ and $q_{3}$, respectively


Figure 7. Sketch of the theoretical position of a point $Q_{1}$ and the real one $Q{ }^{\prime}{ }_{1}$ by considering errors $\delta q_{1}, \delta q_{2}$ and $\delta q_{3}$ in joint coordinates $q_{1}, q_{2}$ and $q_{3}$, respectively.

As example, in table 1 are presented results of simulation for a known position of point $Q_{l}$, given by its $x_{Q I}$ and $y_{Q I}$ coordinates and the normal orientation $\eta$. In order to analyze the deviations it has been used for actuators of the revolute joints, $q_{1}$ and $q_{2}$, a resolution of 0,35 arc-seconds and repeatability of 5 arc-seconds. The actuator of the linear joint defined by $q_{3}$, a resolution of $0,00635 \mathrm{~mm}$ and for the laser sensor a resolution of $0,1 \mathrm{~mm}$. These actuator characteristics enable to obtain the errors $e_{1}, e_{2}$ and $e_{3}$ in position of point $Q_{1}$ due to the errors in the $q_{1}, q_{2}$ and $q_{3}$ joint coordinates, respectively, and its correspondent sensibility index.

We can note that: the joint coordinate $q_{3}$ does not affect the normal direction, because the $\eta$ angle does not depend on it, as shown in eq.(18); from simulations it has been shown that the joint coordinate $q_{2}$ strongly affects the normal orientation, and both $q_{2}$ and $q_{3}$ joint coordinates must have excellent resolution in order to obtain all points of a 3D surface. If this is true, the obtained cloud of points is uniform and without gaps well representing the 3D-surface of the object.

The high values of $\mu_{2}$, in table 1 show the strong sensibility when errors exist in the $q_{2}$ joint coordinate.

Table 1. Results of error analysis

| $\eta\left[{ }^{\circ}\right]$ | $x_{O_{I}}[\mathrm{~mm}]$ | $y_{O_{I}}[\mathrm{~mm}]$ | $x_{O^{\prime}}[\mathrm{mm}]$ | $y_{O^{\prime}}[\mathrm{mm}]$ | $e_{I}[\mathrm{~mm}]$ | $\mu_{I}$ | $e_{2}[\mathrm{~mm}]$ | $\mu_{2}$ | $e_{3}[\mathrm{~mm}]$ | $\mu_{3}$ |
| :---: | ---: | ---: | ---: | ---: | ---: | :---: | :---: | :---: | :---: | :---: |
| 163,1380 | $-17,8486$ | $-2,1431$ | $-17,8474$ | $-2,1399$ | 0,0004 | 17,9768 | 0,0039 | 158,8915 | 0,1064 | 1,0000 |
| 188,2021 | $-21,9436$ | $-19,1299$ | $-21,9437$ | $-19,1268$ | 0,0007 | 29,1115 | 0,0036 | 150,3386 | 0,1064 | 1,0000 |
| 76,5300 | $-7,6595$ | $-29,2281$ | $-7,6623$ | $-29,2291$ | 0,0007 | 30,2150 | 0,0036 | 148,2800 | 0,1064 | 1,0000 |
| 38,2582 | 6,2223 | $-17,5231$ | 6,2203 | $-17,5260$ | 0,0005 | 18,5951 | 0,0039 | 159,4838 | 0,1064 | 1,0000 |
| $-79,5429$ | 18,2350 | $-13,3539$ | 18,2018 | $-13,3542$ | 0,0005 | 22,6018 | 0,0038 | 158,3717 | 0,1064 | 1,0000 |
| $-6,0891$ | 32,4402 | $-1,1226$ | 32,4399 | $-1,1253$ | 0,0008 | 32,4597 | 0,0035 | 143,1083 | 0,1064 | 1,0000 |
| 75,0658 | 20,3964 | 12,8703 | 20,3998 | 12,8698 | 0,0006 | 24,1176 | 0,0038 | 157,0410 | 0,1064 | 1,0000 |
| 43,7973 | 7,1528 | 16,4398 | 7,1551 | 16,4371 | 0,0004 | 17,9284 | 0,0038 | 158,8227 | 0,1064 | 1,0000 |
| 67,4168 | $-5,3932$ | 28,4863 | $-5,3905$ | 28,4848 | 0,0007 | 28,9924 | 0,0036 | 150,5456 | 0,1064 | 1,0000 |
| 162,5900 | $-21,4196$ | 21,4534 | $-21,4191$ | 21,4563 | 0,0007 | 30,3158 | 0,0036 | 148,0784 | 0,1064 | 1,0000 |
| 202,0650 | $-18,3385$ | 3,5018 | $-18,3400$ | 3,5050 | 0,0005 | 18,6698 | 0,0039 | 159,5273 | 0,1064 | 1,0000 |

## 5. CONCLUSIONS

In this paper a RRP+PR structure had been described to use as digitizer of 3D-surface.
From its kinematics model, both the analysis of its workspace and the influence of the joint error in the position and the normal orientation can be concluded that the actuators must have excellent resolution, principally the second revolute joint and the prismatic one, in order to obtain all points of a 3D-surface.

Finally, a prototype is in construction based on the obtained results to read 3D-surfaces in association with a methodology for acquiring and processing the data of the surface.

## 5. ACKNOWLEDGEMENTS

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