

# DESIGN AND IMPLEMENTATION OF A PILOT TEST FOR AN INDUSTRIAL PROTOTYPE OF HARMONIC DRIVE USING A FPGA REAL-TIME SYSTEM TO IMPLEMENTS CONTROL AND INSTRUMENTATION.

**Javier Gamboa**, [hjgamboar@unal.edu.co](mailto:hjgamboar@unal.edu.co)  
**Ernesto Córdoba**, [ecordoban@unal.edu.co](mailto:ecordoban@unal.edu.co)  
**Juan Carlos Santamaría**, [juancarlos.santamariap@gmail.com](mailto:juancarlos.santamariap@gmail.com)  
**Arturo Perpiñán**, [aperpinanr@unal.edu.co](mailto:aperpinanr@unal.edu.co)

Department of Mechanical and Mechatronic Engineering, Universidad Nacional de Colombia, Bogotá. Avenida Carrera 30 No. 45 – 03 . Building 407 , Mechatronics laboratory.

***Abstract.** Harmonic transmission or Harmonic Drive (HD) is a gear based on the principle of transmissibility of mechanical waves, it begun to developing since 1955 by Walton Musse, he did the first patent on the subject. Actually there are four manufacturers of these transmissions in the world and there is little information about his design. This planetary mechanism has a flexible link in its kinematic chain. The principal attributes of this mechanism are an excellent positioning accuracy, high torque capacity, high single-stage reduction ratio and zero backlash. These characteristics had allowed betters applications in the aviation industry, aerospace and robotics industry.*

*The Universidad Nacional de Colombia build an industrial prototype. To validate the design assumptions we seek to estimate the transmission error. However, the instrumentation requires special techniques to make the measurements of output angular velocity of the transmission; this task was realized whit a PAC (Programmable Automation Controller), which includes an FPGA (Field Programmable Gate Array) as a I/O method and an operating system that allow make real time contro.*

*The article shows the research process followed on the instrumentation of the HD, its implementation its ,waveform of kinematic transmission error, analysis of results and a new model for predicting the error. This paper aims to characterize the industrial prototype, more specifically, to use this knowledge to design effective the next prototype.*

**Keywords:** Planetary gear, Harmonic Drive, Instrumentation, kinematic transmission error.

## 1. INTRODUCTION

Harmonic Drive (HD), also named strain-wave gearing, is a gear transmission based on the principle of transmissibility of mechanical waves; the concept was developed by Walton Musser during the mid-1950 (Musser, 1955). This planetary mechanism has a flexible link in its kinematic chain. The principal attributes of this mechanism are an excellent positioning accuracy, small volume, high torque capacity, high single-stage reduction ratio, and high efficiency and nearly zero backlash. It has been used widely in precision pointing and torque conversion purposes with space and weighs constraints. These characteristics had allowed betters applications in the aviation industry, aerospace and robotics industry.

The Universidad Nacional de Colombia built an industrial prototype (Santamaría and Herrera, 2001) with the idea of understanding this type of technology. Actually there are four manufacturers of these transmissions in the world and there is little information about his design. For the many favorable features they have, there are at least two unwanted characteristics, specifically soft torsional stiffness and gear error. The goal of this study is to characterize the behavior in the industrial prototype using techniques of real time processing.

## 2. MODEL OF TRANSMISSION

The components of HD are: the wave-generator, the circular spline and the flexible spline (Fig. 1). The wave-generator typically is an elliptically shaped steel core surrounded by a flexible race bearing. The circular spline is a rigid steel ring with teeth machined into the inner circumference. The flexible spline (flexspline) is a thin-walled flexible cup having less teeth (usually tow fewer) on its outer edge than on the inner edge of the circular spline. Upon assembly, the wave-generator is inserted into the flexspline which assumes an elliptical shape at that end. The other end, however, is circular in shape and is attached to the output shaft. The circular spline teeth then mesh with the flexspline teeth at the major axis of the ellipse defined by the wave-generator.

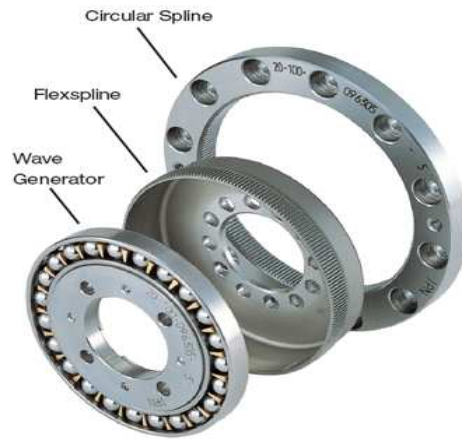


Figure 1. Components of a typical Harmonic Drive

It has been shown in the literature that nonlinear characteristics including transmission error, flexibility, and hysteresis are intrinsic in harmonic drives (Hsia, 1988). The strain-wave gearing is a special flexible transmission mechanism. HD employs a constant deflection wave along a non-rigid gear to let gradual engagement of gear teeth because of this unusual gear-tooth meshing action, harmonic drives can bring very high reduction ratios in a very small package. Their typical construction with meshing at two diametrically opposite ends gives them many useful properties. The transmission is designed such that several teeth are engaged at any given time making backlash virtually zero. But since a link in the kinematic chain is a non-rigid gear it reduces stiffness, and brings a transmission error into the system.

Every harmonic drive is distinguished by his transmission ratio,  $N$ , which describes its position, velocity and torque behavior. Specifically, given a know rotation on two of the three Harmonic-Drive ports as well as a value for  $N$ , the ideal rotation of the third harmonic-drive port can be predicted by the equation (Tuttle, 1992):

$$\theta_{wg} = (N + 1)\theta_{cs} - N\theta_{fs} \quad (1)$$

Where  $\theta_{wg}$  the rotation of the wave-generator is,  $\theta_{cs}$  is the rotation of the circular spline, and  $\theta_{fs}$  is the rotation of the flexspline. All tree rotations in this equation are defined in the same frame of reference, similarly, the derivative of this relationship yields a similar velocity constraint:

$$\omega_{wg} = (N + 1)\omega_{cs} - N\omega_{fs} , \quad (2)$$

Where  $\omega_{wg}$  ,  $\omega_{cs}$  and  $\omega_{fs}$  represent the angular velocities of the three harmonic-drive components relative to the same velocity reference.

### 3. MODEL OF KINEMATIC ERROR

In applications requiring high positional accuracy, the inherent kinematic errors manifested by harmonic drives expose the deficiencies of an ideal transmission model. The kinematic errors,  $\theta_{err}$  is typically measured by subtracting the rotation at the output of the harmonic drive from the input rotation scaled by the ideal ratio for the given transmission configuration (Tuttle and Seering, 1993):

$$\theta_{err} = \frac{\theta_{in}}{N} - \theta_{out} \quad (3)$$

Typically the input port is the wave generator, the output port is the flexspline and the circular spline is constrained. Making use of the kinematic model, Hsia in 1988 could consider how structural errors vary as functions of the geometric parameters of the mechanism. Specifically the relationship between the major axis and minor axis of the ellipse, from this analysis they shows that there are inherent positioning errors associated with the harmonic gear drives irrespective of manufacturing and assembly errors. These errors should be of particular concern when this mechanism is used in precision pointing applications.

Then in 1991 Nye and Kraml consider the gear error causes. They found that these errors are attributed, mainly, to: tooth placement errors on the flexspline, tooth placement errors on the circular spline and lack of concentricity between flexspline and circular spline. Nye and Kraml has demonstrated that typical kinematic-error signatures, vary periodically at once and twice the rotational frequency of the wave-generator and subsequent harmonics.

Based on this result, the harmonic-drive kinematic error  $\theta_{err}$  was modeled as sum of sinusoidal functions. For most cases, the first three sinusoidal terms were adequate to describe the kinematic-error profile (Hidaka et al, 1989):

$$\theta_{err} = A_1 \sin(\theta_{wg} + \phi_1) + A_2 \sin(2\theta_{wg} + \phi_2) + A_3 \sin(4\theta_{wg} + \phi_3) \quad (4)$$

Where  $\theta_{wg}$  is the wave-generator angle, and  $A_n$  and  $\phi_n$  are the measured amplitudes and phases of the kinematic error components at each frequency. A Fourier series is proposed for a model:

$$\theta_{err} = \sum A_k \sin(2k\theta + \phi_k) \quad (5)$$

Here  $\theta_{err}$  is transmission error,  $A_k$  is amplitude of respective harmonic function,  $\theta$  is input shaft position, and  $\phi_k$  is phase lag of respective harmonic function. When the gear is in operation, a transmission error produces a speed ripple as a derivative of the transmission error. Frequencies of the speed ripple become 2nd multiples of the input shaft turning frequency.

Kennedy and Desai (2003) say that kinematic error has a significant effect on the torque transmission characteristics of HD, they found that compensating for coulomb friction using the torque required to maintain slow velocity eliminated almost all the effects of kinematic error. Calvente in 2006 John gave a characterization of the kinematic error using FEA and found that the simulation performed without restrictions on the wave generator show how its center moves during operation of the HD resulting in the appearance of vibrations that are transmitted on the output shaft.

In order to reduce the speed ripple Godler et al (1994) proposed an active repetitive control and proved that the Fourier series is an appropriate mathematical model to represent the transmission error of the strain wave gearing. Later Taghirad and Belanger (1996) development a series of models with increasing complexity to describe the harmonic drive behavior. Their most complex model involved kinematic error, nonlinear stiffness, and gear-tooth interface with frictional losses. In 1997 Taghirad and Belanger proposed a robust torque control, his objective is well-suited to the general  $H_\infty$  problem. Miyazaki and Ohishi in 1999 treated the angular transmission error as harmonic function of motor angular position(6) and propose the model showed in figure 2.

$$\theta_{out} = \frac{1}{N} \theta_{in} + A \sin(2\theta_{in}) \quad (6)$$

The complete model of the transmission and the power unit is:

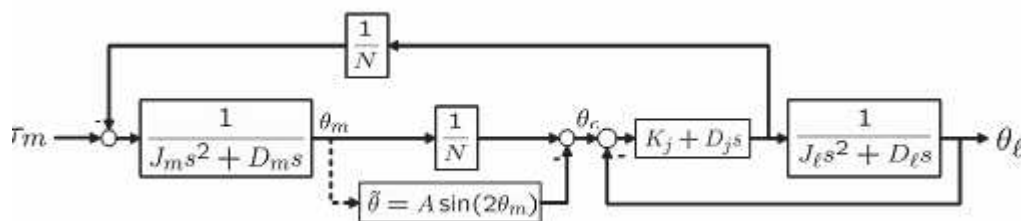


Figure 2. Non-linear model of HD into a motor model

Ghorbel et al (2001) decomposed the kinematics error. They present a more precise characterization and propose that the kinematic error is mostly dominated by two major components. The first component,  $\tilde{\theta}_p$ , is a basic component that is ‘‘pure’’ kinematic error resulting from the kinematic structure of the harmonic drive. The second component,  $\tilde{\theta}_x$ , is mostly due to the stiffness properties of the drive. Consequently, the expression of the kinematic error introduced in (3) could actually be decomposed into

$$\theta_{err} = \tilde{\theta}_p + \tilde{\theta}_s \quad (7)$$

The flexibility induced component of the kinematic error:.

$$\tilde{\theta}_p = \frac{a_0}{2} + \sum_{n=1}^k [a_n \cos(n\theta_{in}) + b_n \sin(n\theta_{in})] \quad (8)$$

Where

$$a_n = \frac{1}{\pi} \int_0^{2\pi} \tilde{\theta}(\theta_{in}) \cos(n\theta_{in}) d\theta_{in} \quad (9)$$

$$b_n = \frac{1}{\pi} \int_0^{2\pi} \tilde{\theta}(\theta_{in}) \sin(n\theta_{in}) d\theta_{in} \quad (10)$$

The flexibility-induced component in the kinematic error is mostly the source of the high-frequency components of kinematic error reported in the literature Ghorbel et al (2001).

Harmonic Drive should have a robust speed control algorithm to suppress vibration phenomena. When the frequency of the angular transmission error coincides with the resonant frequency of control system, the joint actuator having planetary gear may generate large torsional vibration torque. In this sense Gandhi and Ghorbel (1999) use a Closed-loop compensation of kinematic error. Miyazaki and Ohishi (2002) introduce a robust speed control. Lu and Lin propose a Disturbance-observer-based adaptive feedforward control when the control scheme consists of the internal model control (IMC) and an adaptive feedforward cancellation (AFC) based on a disturbance observer (DOB). Han et al (2008) reduce de velocity ripple using peak filter with acceleration feedback.

#### 4. DESCRIPTION OF THE EXPERIMENT

The goal of this study is to characterize the kinematic error presented in the industrial prototype. This harmonic drive has a wave generator planetary-type, tow rollers instead of an elliptical bearing, and the flexspline use a dynamic circular spline coupled to output shaft, instead of a wave generator cup-type (Santamaria and Herrera, 2001). It has 56.5 of transmission ratio. We can see the image of industrial prototype of harmonic drive (Fig. 3) whit his scheme (Fig. 4).



Figure 3. Industrial prototype of harmonic drive builds by Universidad Nacional de Colombia

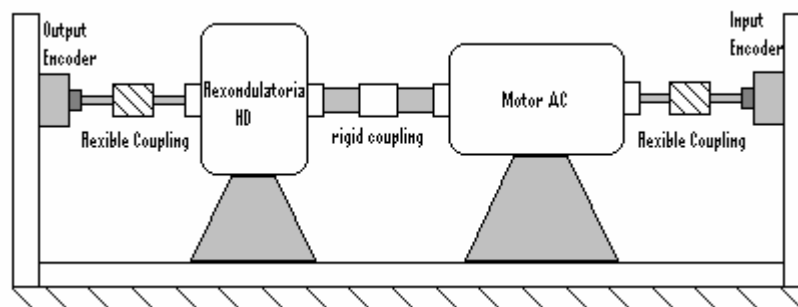


Figure 4. Scheme of Flexondulatoria (H. D.) testing apparatus

The input encoder has a resolution of 0.075 degrees and the output encoder has a resolution of 0.015 degrees. The motor is driven by a Variable Speed Drive communicated with a Programmable Automation Controller (PAC) its combines an embedded real-time processor and Field Programmable Gate Array (FPGA) it is a high-performance; reconfigurable chip programmed with LabVIEW. Then the labVIEW code is converted in VHDL code to configure the FPGA chip. In this test we implement encoder readings at high speed using the LabVIEW built-in functions for transferring data between the FPGA and the real-time processor. Now deterministically we calculate kinematic transmission error every millisecond using the equation (3). Then linearly interpolate the time-based position-error signal and finally take a Fast Fourier Transform (FFT) of the interpolated data vector to identify the frequency components of the error signature in terms of input revolutions.

### 5. MEASURE OF THE KINEMATIC TRANSMISSION ERROR

To measure the position error, the engine must have a constant speed, and then wait for the input shaft reaches a certain angular position to start the measurement. All samples are taken from the same angular position until complete a lap on the output shaft. The experiment was repeated several times to ensure repeatability of samples and reduce the effect of measurement errors. Figure 5 illustrates the position-error waveforms observed and Fig. 6 show the FFT result.

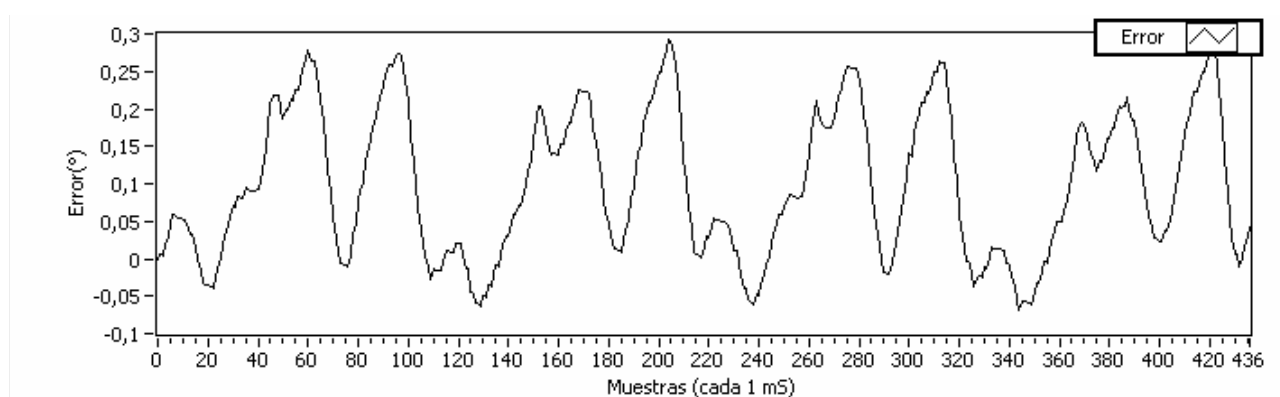


Figure 5. Position-error waveform at two input rotation

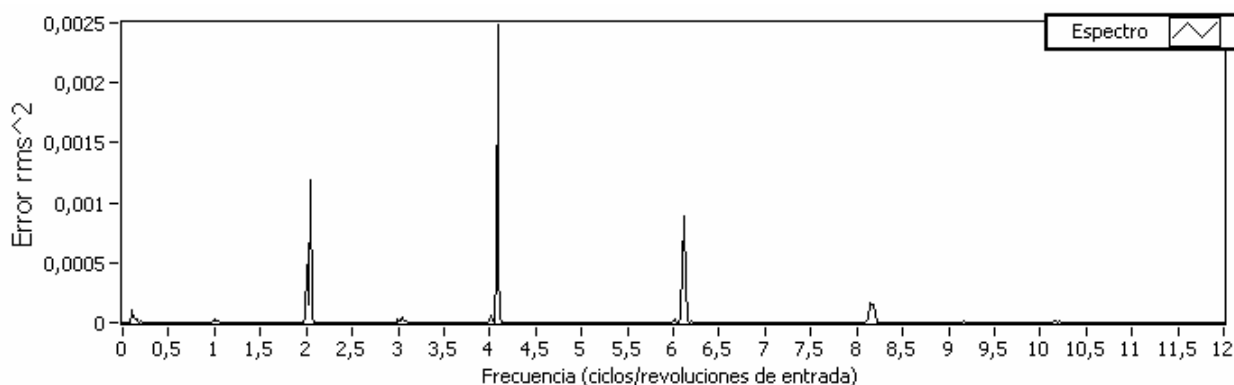


Figure 6. FFT of position error in cycles per input revolution

Figure 5 shows a signal with a defined period which is indicated by its frequency spectrum. In fig. 6 we can see how the frequency peaks are close to twice the speed of entry, but not exactly in that place also the highest peak is located near the fourth harmonic of the rotation speed at input port. This sample was taken at clockwise, figure 7 and Fig. 8 show the waveform of error at counterclockwise.

From figures 6 and 8 can be seen that the direction of rotation affects the harmonics present in the position error for the transmission tested. In the waveform position error there are more harmonics (7) at counterclockwise that harmonics (4) at clockwise. After performing a significant number of samples we found that in all cases the harmonic frequencies were higher than those predicted for (5). In this model the source of failure is the loss of concentricity between the circular spline and the flexspline. With these results one can characterize the behavior of the transmission.

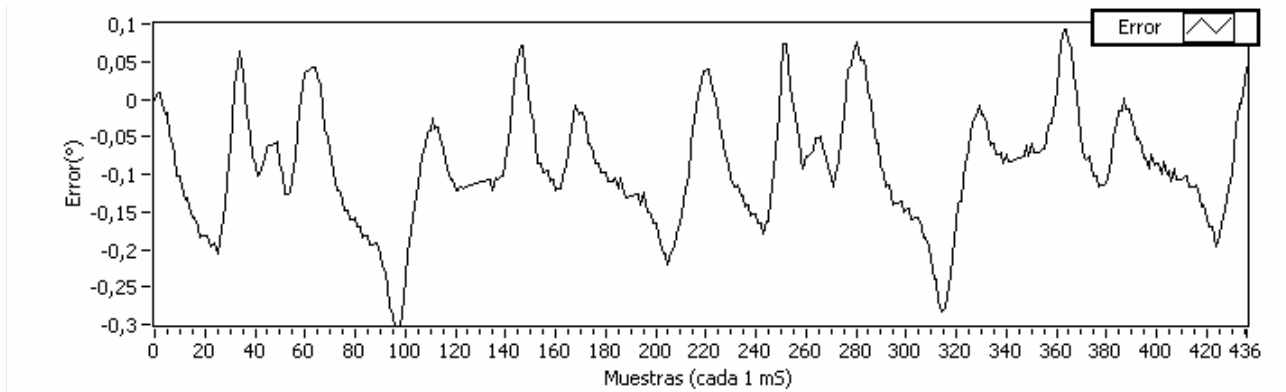


Figure 7. Position-error waveform at two input rotation at counterclockwise

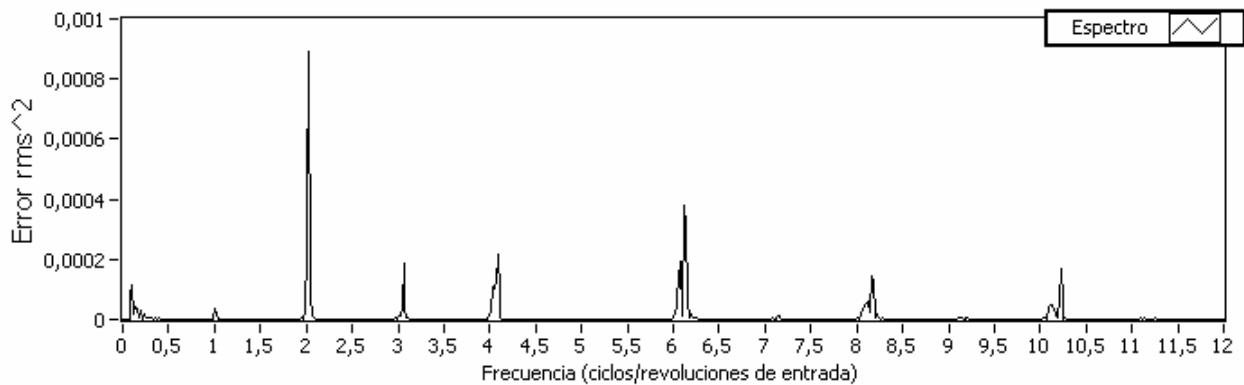


Figure 8. FFT of position error in cycles per input revolution at counterclockwise

Several experiments have lent further insight into the repeatability and variation of typical gear-error waveforms. The most important conclusion is that the harmonics are not presented in the frequency predicted by the equation 5. It is necessary to propose a new model to justify the observed behavior. One possibility being studied currently assessing whether the disruption of transmission error moves with the rotation of the wave generator and this would explain why the peaks in frequency are moved towards higher values. The proposed model for the error is:

$$\theta_{err} = \sum A_k \sin\left(2k\theta_{in}\left(\frac{N+1}{N}\right) + \varphi\right) \quad (11)$$

Are currently carrying out extensive testing to try to validate hypothesis is proposed. These tests vary the speed of entry and the direction of rotation for different types of wave generators. As a result of this investigation, we confirmed the expected position-error frequency distribution over characteristically frequencies. This is a starting point for analyzing the transmission designed by the group.

## 6. ACKNOWLEDGEMENTS

The work in this paper was supported by the Departamento Administrativo de Ciencia, Tecnología e Innovación. (Colciencias) in partnership with the Universidad Nacional de Colombia. Project “Novoluta y flexo ondulatoria, nuevos perfiles para engranajes de potencia.” Cod. 1101-08-17523 CT 507-2005.

## 7. REFERENCES

- Calvente, J., 2006. “Caracterización del Error Cinemático de Transmisiones de tipo Harmonic Drive por el Método de Elementos Finitos”, Universidad Carlos III de Madrid, Leganes, España, 116 p.
- Gandhi, P. S. and Ghorbel, F. H., 1999, “Closed-loop compensation of kinematic error in harmonic drives for precision control applications”, Proceedings of the IEEE 36th Conf. Decision and Control, Phoenix, U. S. A., pp. 5368–5370.
- Godler, I., Ohnishi, K. and Yamashita, T., 1994. “Repetitive control to reduce speed ripple caused by strain wave gearing” Proceedings of the IECON 1994, 20th International Conference on, volume 2, pp 1034–1038.
- Ghorbel, F., Gandhi, P. S. and Altpeter, F., 2001. “On the kinematic error in harmonic drive gears” Trans. ASME, J.

Mech. Design, vol. 123, pp. 90–97.

- Han, C. H., Wang, C. C. and Tomizuka, M., 2008. "Suppression of vibration due to transmission error of harmonic drives using peak filter with acceleration feedback" *Advanced Motion Control*, 2008. AMC '08. 10th IEEE International Workshop on, vol., no., pp.182-187.
- Hidaka, T., Zhang, Y., Sasahara, M. and Tanioka, Y. 1989. "Vibration of a Strain Wave Gearing in an Industrial Robot", in *Proceedings of the ASME International Power Transmission and Gearing Conference*, Vol. 11, pp. 789-794.
- Hsia, L. M., 1988. "The Analysis and Design of Harmonic Gear Drives" *Proceedings of the 1988 IEEE International Conference on Systems, Man, and Cybernetics*, Vol. 1, pp. 616-619.
- Kennedy, C.W. and Desai, J.P., 2003. "Estimation and Modeling of the Harmonic Drive Transmission in the Mitsubishi PA-10 Robot Arm", *IEEE/RSJ International Conference on Intelligent Robots and Systems (IROS)*, Las Vegas, U. S. A., pp. 3331–3336.
- Lu, Y. S. and Lin, S. M., 2007. "Disturbance-observer-based adaptive feedforward cancellation of torque ripples in harmonic drive systems". *Electr Eng (2007)* 90, pp. 95–106.
- Nye, T.W. and Kraml, R. P., 1991. "Harmonic Drive Gear Error: Characterization and Compensation for Precision Pointing and Tracking". *Proceedings of the 25th Aerospace Mechanisms Symposium*, volume 3113 of NASA Conference Publication.
- Miyazaki, T. and Ohishi, K., 1999. "Robust Speed Control Suppressing Vibration Caused by Angular Transmission Error of Planetary Gear", *Proceedings of the IEEE. IES. IECON'99*, pp.1051-1056.
- Miyazaki, T. and Ohishi, K., 2002. "Robust Speed Control System Considering Vibration Suppression Caused by Angular Transmission Error of Planetary Gear", *IEEE/ASME TRANSACTIONS ON MECHATRONICS*, VOL. 7, NO. 2.
- Musser, C., 1955. "Strain Wave Gearing," United States Patent Number 2,906,143.
- Santamaría, J. and Herrera, D., 2001 "Desarrollo de un prototipo experimental de transmisión planetaria ondulatoria sincrónica", thesis in Mechanical Engineering, Universidad Nacional de Colombia, 376p.
- Taghirad, H. D. and Belanger, P. R., 1996. "An experimental study on modeling and identification of harmonic drive systems" In *Proceedings of the 35th Conference on Decision and Control*, pp 4725-4730.
- Taghirad, H. D. and Belanger, P. R., 1997. "Robust torque control of harmonic drive under constrained-motion", In *Proceedings of the IEEE International Conference on Robotics and Automation*, 1, pp 248-253.
- Taghirad, H. D. and Belanger, P. R., 1998. "Modeling and parameter identification of harmonic drive systems", *Journal of Dynamic Systems, Measurements, and Control*, ASME Pub., Vol. 120, No. 4, pp 439-444.
- Tuttle, T. D., 1992. "Understanding and modeling the behavior of a harmonic drive gear transmission". Technical Report 1365, MIT Artificial Intelligence Laboratory, 314p.
- Tuttle, T. D. and Seering, W., 1993. "Modeling a harmonic drive gear transmission". In *Robotics and Automation, Proceedings, 1993 IEEE International Conference on*, volume 2, pp 624–629.

## 8. RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.