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NUMERICAL MODELING OF A CIRCULAR PIEZOELECTRIC ULTRASONIC TRANSDUCER RADIATING IN WATER

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Abstract. *This work presents a finite element analysis of a circular piezoelectric transducer operating in transmission mode in water. The model considers a transducer composed of the matching layer, piezoceramic, and backing layer, and the casings, and a water medium where the waves are radiated. Considering the finite element method, the model was implemented using the ANSYS software. The transient response of 2-MHz 12.7-mm-diam transducer is analysed using an excitation signal of 2-MHz sine-wave half cycle. The model results were compared to the ones obtained by the plane piston model and the experimental measurements using a hydrophone. These results show good agreement with the finite element ones. Furthermore, the waveform characteristics are related to the material properties of piezoceramic and layers for the edge and head waves.*

Keywords*: ultrasonic transducer, wave propagation, finite element method.*

1. Introduction

The vibrational modes of piezoelectric transducers have been analyzed theoretically and experimentally by many authors due to its importance in the understanding of pulse-echo in ultrasonic nondestructive evaluation applications. A good physical understanding of the radiated pressure waveform from circular transducer is interpreted as the superposition of the well-known plane and edge waves generated by a baffled piston (Weight, 1984). The plane wave propagates into the geometrical region straight ahead of the source, together with a spreading edge wave from the periphery of the source. The model considers a plane piston mounted within an infinite rigid baffle generating pressure waves into a fluid medium. Furthermore, the pressure wave is proportional to the particle velocity into the face of the piston. The edge wave radiated outside the geometrical region straight ahead of the source is in the same phase of the plane wave, and the edge wave radiated inside this region is in the opposite phase of the plane wave.

According to the experimental measurements of the ultrasonic pulse in liquids obtained by many authors, the measured edge wave appears to be distorted from the plane piston model. It shows that exist other effects not foreseen in the models based on the plane piston. The most important effect is related to the propagation of superficial radial waves of compression across the piezoelectric ceramic from its edge. The compression radial wave generates a conical wave called head wave that propagates into the liquid initially in front of the edge wave (Hayman and Weight, 1979). This head wave propagates just before the edge wave and interferes with it. The characteristics of the head wave are related to the properties of the piezoelectric material, the backing layer and the transducer case.

In models with fluid-structure interface, the application of pressure to simulate a plane piston produces spurious modes due to pressure rough variation in the edge of the piston (Stucky and Lord, 1998). Stucky and Lord show the necessity of the Fermi function to produce a radial variation of the pressure in the region next to the edge of the piston. It is verified the necessity of including more realistic models that simulate the acoustic wave propagation in liquids generated by the piezoelectric transducer.

In the past, the development of ultrasonic transducers was made by trial and error (Lerch, 1990), which consumed too much time and therefore was expensive. To improve the modeling and the simulation of ultrasonic transducers, some numerical tools can be used to evaluate its performance in specific situations. These tools also predict accurately the electroacoustic characteristics of the materials used in transducers (Piranda *et al.*, 2001). The purposes of studying the modeling and the simulation of transducers are to optimize its design without time-consuming experiments, to evaluate new materials in the device, and to investigate the acoustic wave generated by it.

Application of the finite element method to the problems mentioned above is one possible way to get a realistic transducer simulation and to visualize the real acoustic wave propagation into the liquid. The present work has an objective to improve the numerical modeling techniques of piezoelectric transducer responsible for the generation and reception of the ultrasonic waves, using the finite element method. The models are done with piezoelectric finite elements simulating a transducer radiating waves in a liquid medium. The transducer model was composed by the PZT-5A ceramic, backing layer, matching layer and casing. The vibrational behavior of the transducer is affected by its material properties, since that doesn't present the ideal behavior of a plane piston (Weight, 1984). The main properties of these materials are shown in this work. The models are developed in ANSYS and the discretization used two variables: time and space. It was verified the numerical convergence of the method for this type of model. To validate the model, experimental measurements were carried out using the piezoelectric transducers developed in laboratory with central frequency of 2 MHz.

2. Formulation of the Piezoelectric Finite Elements

Several authors have applied the finite element method to the analysis of piezoelectric media and to predict the performance of ultrasonic transducers (Allik and Hughes, 1970) (Boucher *et al.*, 1981) (Nailon *et al.*, 1983) (Lerch, 1990). In the piezoelectric structure, the constitutive matrix equations relating the mechanical and electrical quantities that are the basis for the derivation of the finite element method are given by:

$$T = c^E S + eE \tag{1}$$

$$D = eS + \varepsilon^S E \tag{2}$$

where

- T is the vector of mechanical stress,
- S is the vector of mechanical strain,
- E is the electric field,
- D is the vector of dielectric displacement,
- c^E is the stiffness matrix for constant electric field E,
- ε^S is the permittivity matrix for constant mechanical strain S,
- e is the piezoelectric matrix.

The finite element method used to solve problems with piezoelectric materials can provide some physical quantities of interest such as: mechanical displacements, mechanical stresses and electrical fields. The discretization of the materials in finite elements results in a mesh modeled by numerous single elements. For this analysis, it is necessary to determine at the nodes of theses elements the mechanical displacement and electric potential, which are used as degrees of freedom (Silva, 1993). Furthermore, it is necessary to describe these continuous mechanical and electrical quantities in terms of the nodal point values, using polynomial interpolation functions (Lerch, 1990). Hence, applying the variational principle to the Eqs. (1) and (2), one can obtain the system of discrete finite element expressions (3) and (4) for each element of the piezoelectric mesh, as following:

$$\left[M_{uu}^e\right]\ddot{u}^e + \left[C_{uu}^e\right]\dot{u}^e + \left[K_{uu}^e\right]u^e + \left[K_{u\phi}^e\right]\phi^e = F^e \tag{3}$$

$$\left[K_{u\phi}^e\right]u^e + \left[K_{\phi\phi}^e\right]\phi^e = Q^e \tag{4}$$

where u^e is the mechanical displacement vector, ϕ^e is the electric potential vector, F^e is the mechanical force vector, and Q^e is the electric charge vector for each element *e*. Moreover, $\left[K_{uu}^e\right]$ is the mechanical stiffness matrix, $\left[K_{u\phi}^e\right]$ is the piezoelectric coupling matrix, $\left[C_{uu}^e\right]$ is the mechanical damping matrix, $\left[K_{\phi\phi}^e\right]$ is the dielectric matrix, $\left[M_{uu}^e\right]$ is the mass matrix, and the superscript *e* indicates an element matrix. All these terms are completely defined in (Boucher *et al.*, 1981) and (Lerch, 1990).

Finally, the complete solution of the piezoelectric finite elements is mathematically described by a set of linear differential equations with symmetric band structure, and the values of u, ϕ, F and Q are the globally assembled field quantities. Therefore, the Eqs. (3) and (4) can be expressed by:

$$\begin{bmatrix} [M_{uu}] & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} \ddot{u} \\ \ddot{\phi} \end{Bmatrix} + \begin{bmatrix} [C_{uu}] & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} \dot{u} \\ \dot{\phi} \end{Bmatrix} + \begin{bmatrix} [K_{uu}] & [K_{u\phi}] \\ [K_{u\phi}]^T & [K_{\phi\phi}] \end{bmatrix}\begin{Bmatrix} u \\ \phi \end{Bmatrix} = \begin{Bmatrix} F \\ Q \end{Bmatrix} \tag{5}$$

3. Rayleigh damping

Mechanical damping is an important parameter in transducer modeling because it affects the dynamic response and the attenuation of the vibration, and consequently the radiated acoustic waveform. Several approaches can be found in the literature. The most common formulation is the Rayleigh damping that can be used in the time domain, with a reasonable approximation when the damping level is low (Liu and Gorman, 1995). This method assumes that the damping matrix $[C_{uu}]$ consists of a linear combination of the mass matrix $[M_{uu}]$ and the stiffness matrix $[K_{uu}]$, i.e.:

$$[C_{uu}] = \alpha[M_{uu}] + \beta[K_{uu}] \quad (6)$$

where α and β are the Rayleigh constants for the mass and stiffness, respectively. The first term, in Eq. (6), leads to a damping ration that is inversely proportional to the frequency, and the second to one that is linear in frequency. The simple relation between the damping ratio ζ_i and the natural angular frequency ω_i of the vibration mode *i* can be given by:

$$\zeta_i = \frac{\alpha}{2\omega_i} + \frac{\beta}{2}\omega_i \quad (7)$$

Four different cases for the physical damping exist: without damping (α=0, β=0), viscous damping (α=0, β>0), mass proportional damping (α>0, β=0) and Rayleigh damping (α>0, β>0). The determination of the constants α and β depends on the energy dissipation characteristic and generally are calculated from the damping ratios ζ_i for two values of ω_i.

In practice, many structural materials present only viscous damping (α = 0). In such case, the constant β can be calculated from known values of ζ_i and ω_i, as $\beta = 2\zeta_i / \omega_i$, or from the following expression:

$$\beta = \frac{1}{\omega_r Q_m} \quad (8)$$

where ω_r is the resonance angular frequency of the vibration mode being analyzed, and Q_m is the mechanical quality factor. The quality factor Q_m for piezoelectric material can be defined by the electrical impedance analysis, as following:

$$Q_m = \frac{f_r}{f_2 - f_1} \quad (9)$$

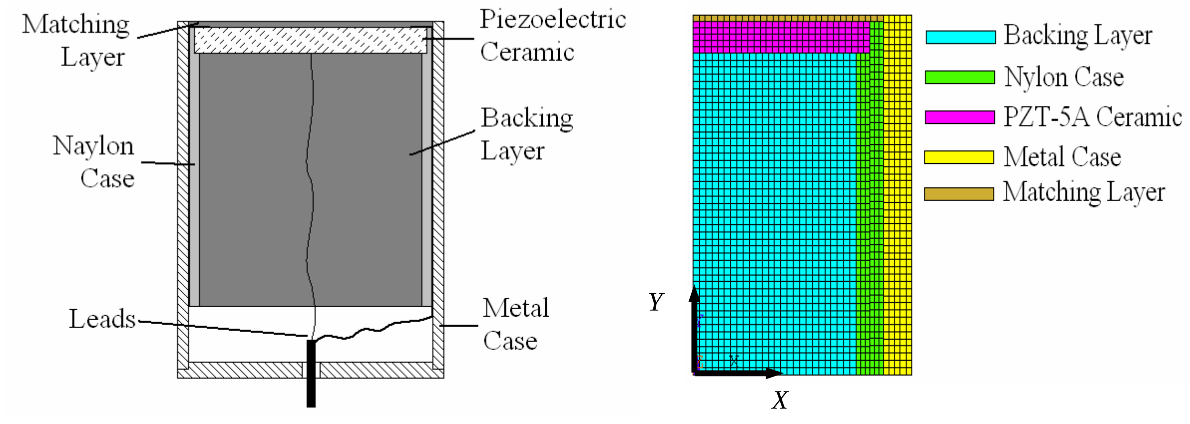
where f_r is the resonance frequency, and f_1 and f_2 are frequencies for which the amplitude of the resonance curve is 3dB below the resonance value, respectively.

In ANSYS program, the damping can be defined by β constant using Eq. (8) for the piezoelectric ceramic. The value of this constant must be adjusted to obtain the closest electrical impedance curves between the theoretical model and the experimental data. For non-piezoelectric materials, the damping can also be defined by β constant, which is related to the attenuation of the ultrasonic wave. In this work, an experimental measurement of the attenuation coefficient of these materials can be carried out in a water tank and then adjusting the β parameter in ANSYS to obtain the same attenuation measured experimentally.

4. Transducer Model

Figure 1(a) shows the schematic diagram of a piezoelectric ultrasonic transducer. The active element is a piezoelectric ceramic with electrodes in the two opposite faces. Applying an electric impulse, generally of less than 100ns duration, the ceramic vibrates at the resonance frequency. The backing layer is usually a highly attenuating material that is used to control the vibration of the ceramic. To minimize internal reflections, the acoustic impedance of the backing matches that of the piezoelectric material. The backing material absorbs the back-transmitted energy, and also attenuates the head waves. A typical backing material includes Tungsten powder in an epoxy matrix (Sayer and Tait, 1984). The matching layer has intermediate impedance to match impedances between the ceramic and the propagating medium, defined by the geometrical mean of these two impedances. Furthermore, a quarter wavelength (λ/4) matching layer is used to improve the transmitted wave into the propagating medium achieving a broad bandwidth (Wang *et al.* 2001). Finally, the nylon case acts as a mechanical insulator, and the metal case filters external noises. The piezoelectric ceramic thickness is related to the transducer central frequency, and its diameter is related to the directivity of the radiated acoustic field.

For the finite element modeling the piezoelectric material was modeled as orthotropic material. It's very important to define correctly the elastic, piezoelectric and dielectric constants observing the main axes of the material. The ceramic used in the construction of ultrasonic transducers has symmetry in the *XY* plane and is polarized on *Z* axis. Due to the direction of poling for the commercially available ceramic is different from that one adopted in ANSYS, it is necessary to transform the material constants into other coordinate system. In the case of circular transducer the model can be simplified considering the problem with an axial symmetry, and therefore the finite element mesh can be restricted to a half of the real configuration, as shown in Fig. 1(b). In ANSYS, the *Y* axis is adopted as the symmetry axis for axisymmetric model. The PZT-5A ceramic used in this work, with diameter of 12.7mm, thickness of 0.97mm, and central frequency of 2MHz, has symmetry around the *Z* axis and is considered an isotropic material in *X* and *Y* directions.



(a) (b)

Figure 1. Diagram of a single PZT-5A element transducer. (a) Physical design. (b) Axisymmetric model design.

As the model is axisymmetric, all components of the transducer have been modeled with axisymmetric plane elements, Fig. 1(b). To model the backing layer, a high attenuate composite material with 10-mm thickness was used. For this purpose, a composite material was modeled with Tungsten powder in epoxy matrix using a volume fraction of 25.7%. The matching layer was modeled with epoxy and quarter wavelength thickness. Finally, the cases are composed of two materials, nylon and steel, and were modeled as pipes with thickness of 1mm.

5. Materials and methods

For the simulations, a half-cycle sinusoidal signal at 2MHz was applied to excite the transducer. Convergence tests were performed to determine the best discretization of the models. It has been chosen 20 finite elements for each wavelength and therefore $\Delta t=1/20$ for the wave period.

In the model *X* axis is parallel to the transducer face, and *Y* axis is the direction of wave propagation. The boundary conditions for the transducer were simulated as null *X*-displacement in the symmetric axis. The boundary conditions for the water medium are selected to simulate an infinite medium. For this purpose, impedance value of 1 and null displacement in the *X* and *Y* directions are applied for the edges of the media, excepting the edge of contact with the transducer and the symmetric axis. These conditions ensure no reflections from the water medium edges.

The models were shaped with square 2D elements of four nodes with a total of 177153 elements and 0.0375mm element length. The total time of propagation was 9μs with 0.025μs for the time sampling. Table 1 summarizes all material parameters used in the finite element code.

The experimental setup was performed in water tank. The emitting transducer was developed in laboratory with the same characteristics as the FEM model. A Pulser/Receiver Panametrics 5072PR was used to generate the trigger of the transmitter with 104μJ of energy, damping of 15 Ω and gain 0dB with high pass frequency filter. The trigger signal was used in the Function Generator to generate one-cycle sinusoidal signal at 2MHz and 0.2V amplitude. Finally, the output signal of the generator was amplified and sends to the transducer. To receive the signal, a hydrophone of diameter 0.3mm was used. The signal obtained by the hydrophone was taken by the same Pulser/Receiver and sampled by a digital oscilloscope Hewllet-Packard 500MHz 2GSa/s, together with trigger generated by the pulser/receiver at 100 PRF Hz. Finally, the signal is acquired in the PC. The received signals had been 2505 points, with sampling period 4ns.

6. Results

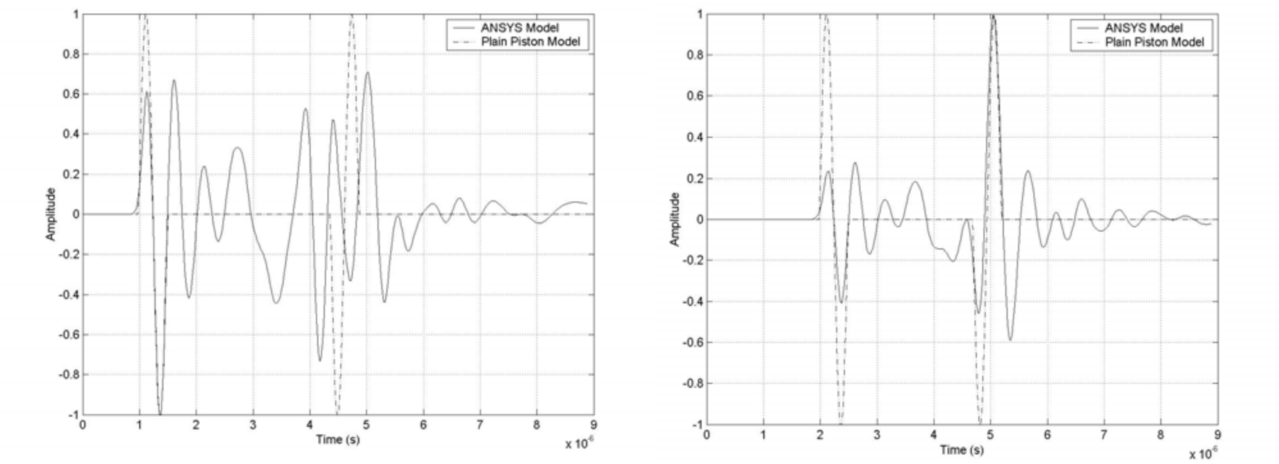
6.1 Comparison with the plane piston model

In the first part of the work, it was performed the FEM model of the transducer propagating in a square water medium with 12.7mm side length. The material used to modeling the backing layer was a composite material with 25.7% volumetric faction of Tungsten in epoxy matrix. This material was performed and characterized in laboratory obtaining its coefficient of attenuation and thus the β parameter. To evaluate the signal produced by the model, the MATLAB code was performed to simulate the theoretical plane piston behavior (Weight, 1984). The ANSYS simulation was performing with harmonic analysis.

Table 1. Summary of the relevant material parameters used in the models.

PZT-5A		Value	Units	**Araldite/Tungsten**	Value	Units
Piezoelectric Constants	d31	-175	10^{-12} m/V	Density	5766	kg/m^3
	d33	400		Poisson Ratio	0.34	---
	d15	590		Elasticity Module	1.05	10^{10} N/m^2
Elastic Constants	c11	12.1	10^{10} N/m^2	β Damping	1.5e-8	---
	c12	7.54		**Nylon**		
	c13	7.52		Density	1405	kg/m^3
	c33	11.1		Poisson Ratio	0.27	---
	c44	2.11		Elasticity Module	0.74	10^{10} N/m^2
	c66	2.26		β Damping	5e-9	---
Dielectric Constants	ε33/ε0	830	---	**Steel**		
	ε11/ε0	916		Density	7890	kg/m^3
Density		7650	kg/m^3	Poisson Ratio	0.29	---
β Damping		5.3e-10	---	Elasticity Module	26.5	10^{10} N/m^2
Araldite				β Damping	10e-8	---
Density		1096	kg/m^3	**Water**		
Poison Ratio		0.34	---	Density	998	kg/m^3
Elasticity Module		0.55	10^{10} N/m^2	Coefficient of Friction	1	---
β Damping		2.3e-8	---	Sound Velocity	1500	m/s

Figure 2 shows the waveforms on axis of the ultrasonic beam using the model developed in ANSYS at two distances: (a) 1.5mm and (b) 3mm from the transducer surface. In Fig. 2(b), the plane and edge waves are clearly defined; the head wave appears between these waves. But in Fig. 2(a) the edge wave is not clear. This behavior is due to the observed point is very close to the transducer, then the head wave appears to have bigger velocity than the edge wave, making interference in this wave. In Fig. 2(b) the edge wave is so clear but the amplitude is larger than the plane wave, this behavior can be due by the addition of the edge and head waves. For a best understand of the graphics all the curves are normalized.



(a) (b)

Figure 2. On-axis waveforms for a 12.7-mm diameter at 2MHz obtained by the FEM and analytical models at distances: (a) 1.5mm, and (b) 3mm from the transducer surface.

Figure 3 shows the waveform obtained by the transducer FEM model, off-axis 1.5mm and distance 3mm from the transducer surface. The plane wave and edge waves are clearly defined. In this result, two edge waves appear, the first produced by the closer edge and the second by the far edge of the transducer. Head wave appears closer the first edge wave, causing noise and large amplitude of this pulse. These results are so satisfactory and probe the necessity of realistic models to explain better the behavior of the ultrasonic wave propagating in water.

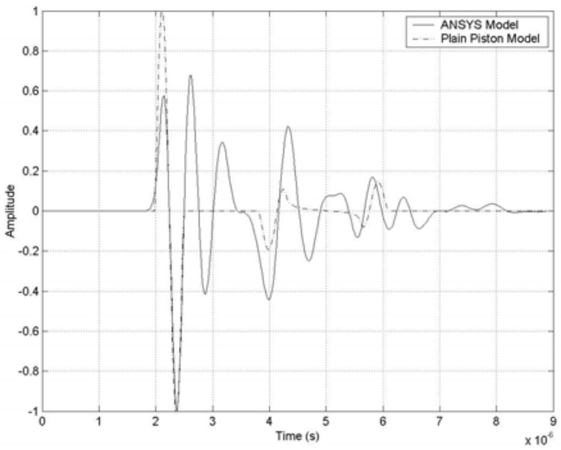


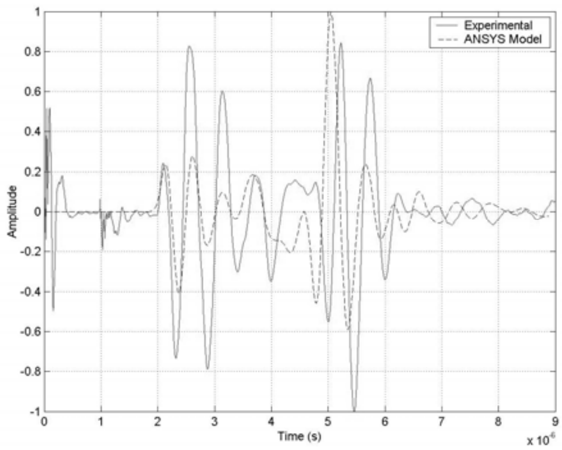
Figure 3. Measured and simulated waveform at 2MHz obtained a distance 3mm from de transducer face and 1.5mm off-axis.

6.2 Comparison with experimental results

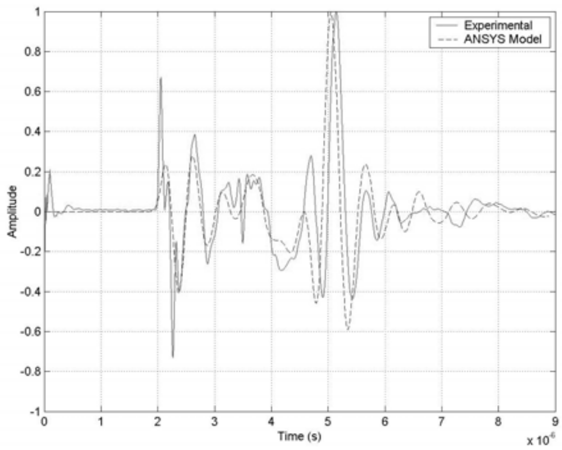
In the second part of the work, the FEM simulations were compared with the experimental data. The transducer excitation was making in two different forms: with one sine signal (generated by the function generator that have a long frequency band) and whit a short band pulse (generated by the Pulse/Receiver directly).

The Fig. 4 (a) shows the result of 3mm on axis ultrasonic waveform. In the experimental data the head wave has a different form and made interference in the edge wave decreasing the amplitude in compare whit the simulation. Also can be observed that the length of the experimental plane wave is longer than the simulated; these effects are caused by the different excitation.

In Fig. 4 (b) the second kind of exciting signal was applied and the 3mm on axis ultrasonic waveform is showed. The plane, head and edge wave have a good agreement with the simulation but appear some noises of high frequency. The amplitude of the edge wave was increased by the head wave interference. In the two figures, the amplitude of the edge wave can be affected by the directivity of the hydrophone. In this last experimental data the results are in better agreement with the simulation, this effect is caused by the kind of pulse applied. The necessity of the same excitation signal in the simulation and experimental test is verified.



(a)



(b)

Figure 4. 3mm on axis wave form for a 12.7mm diameter 2Mhz transducer. (a) One sine signal excited. (b) Short band pulse excited.

7. Conclusions

The modeling of ultrasonic transducer using the FEM is able to give a good description of physical reality since it is base on accuracy of the material properties. The elastic, piezoelectric and dielectric constants and the damping ratio are decisive parameters of piezoelectric materials that affect seriously the transducer response. The material used for the backing layer in the transducer has a big importance on the pressure waveform emitted into the propagating medium. A backing with a high degree of damping results in improved broadband transducer, and can also affects transducer sensitivity. In this work, it was verified that if the volume fraction of Tungsten in the backing layer decreases, the emitted pulse is longer and the head wave has major amplitude causing a bigger interference in the edge wave.

The comparison of the results obtained by the FEM and the plane piston model shows good agreement. However, finite element results present a more realistic model that can be describe better the behavior of the real ultrasonic transducer. In FEM simulation, the plane and the edge waves are shown like those obtained by the plane piston model, but another wave appears distorting the edge wave. This discrepancy between the models is caused by the head wave, which originates from a superficial radial wave propagating laterally across the ceramic. The form of this extra wave depends on the material of the backing layer and the transducer casing. The comparison between the experimental and FEM results also presents good agreement, and the presence of the head wave is confirmed.

These results demonstrate the necessity of realistic models to explain better the behavior of the ultrasonic wave propagating in water, and the FEM is a good tool to develop these models.

8. Acknowledgements

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