

# PREDICTIVE CONTROL OF A MAGNETIC LEVITATION SYSTEM WITH EXPLICIT TREATMENT OF OPERATIONAL CONSTRAINTS

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**Abstract.** *This paper concerns the application of a predictive control methodology to the stabilization and reference-following operation of a magnetic levitation process. From a control engineering point of view, the problem is challenging owing to the nonlinear and unstable nature of the plant, the required positioning accuracy and the operational restrictions on the manipulated and controlled variables during transients. The formulation employed in this work is based on a linear prediction model obtained by linearizing the plant dynamics around the center of the working range of the position sensor. Offset-free tracking is achieved by augmenting the cost function with a term associated to the integral of the tracking error. Operational constraints on the input (current in the electromagnet coil) and output (width of the air gap between the electromagnet core and the suspended object) of the process are enforced in the optimization process. The optimal control sequence is implemented in a receding-horizon strategy, in which the optimization is repeated at every sampling instant, by taking into account the new sensor readings. The design and validation of the predictive control loop are carried out by using physical parameters from a real magnetic levitation process. The results obtained by simulation show that the explicit treatment of operational constraints, especially those related to the input variation rate, is fundamental to an appropriate control of the system.*

**Keywords:** *magnetic levitation, predictive control, control with constraints, nonlinear systems.*

## 1. Introduction

Magnetic levitation technology has a wide range of applications, encompassing, for instance, high-speed transportation systems (Holmer, 2003), seismic attenuators for gravitational wave antennas (Varvella et al., 2004), self-bearing blood pumps (Masuzawa et al., 2003) for use in artificial hearts, haptic interfaces (Berkelman and Hollis, 2000), photolithography devices for semiconductor manufacturing (Kim and Trumper, 1998), and microrobots (Khamesee et al., 2002). For this reason, much research has been carried out on aspects of electromagnetic design (Hurley and Wölfle, 1997), power electronics (Li and Li, 1995), modelling (Agamennoni et al., 2004), and active control (Maggiore and Becerril, 2004) of magnetic levitation (maglev) systems.

From the point of view of control engineering, maglev systems are challenging because of the nonlinear nature of the plant dynamics, the very small degree of natural damping in the process, and the strict positioning specifications often required by the application. Such a challenge increases for attraction-based levitation (employed, for instance, in the suspension system of the Shanghai Transrapid Maglev Line serving the Pudong International airport in China (Holmer, 2003)), in which case the system dynamics are open-loop unstable (Galvão et al., 2003). Typical examples of design techniques that have been applied to the control of maglev systems include feedback linearization (Maggiore and Becerril, 2004), (Grimm, 2002), sliding mode (Shan and Menq, 2002), (Al-Muthairi and Zribi, 2004),  $H_\infty$  control (Kang et al., 2003), (Sinha and Pechev, 2004), and adaptive methods (Yang and Tateishi, 2001).

It can be argued that one of the main difficulties for the deployment of a maglev controller concerns the handling of operational constraints. In fact, the width of the air gap between the electromagnet core and the suspended object

must typically be kept within tight constraints in order to achieve an appropriate electromechanical conversion efficiency. Failure to enforce such output constraints may lead to a catastrophic fault, because the magnetic attraction force may not be strong enough to bring the object back to its operating point. Moreover, if the electromagnet current is adopted as the manipulated variable, the control may be subjected to a significant slew rate constraint if the coil inductance is large.

In this sense, it may be interesting to consider the potential use of model predictive control (MPC) strategies for maglev systems. In fact, the ability of handling operational constraints in an explicit manner is one of the main reasons for the popularity of predictive controllers in industrial applications (Maciejowski, 2002), (Rossiter, 2003). However, very few contributions on the use of MPC for maglev control are found in the literature. In a recent work, (Lepetic et al., 2003) predictive functional control was employed in the outer loop of a maglev control system, which was initially stabilized by a lead compensator. However, such a paper did not address constraint handling issues. A study regarding the design of a predictive maglev controller was described in (Miura and Galvão, 2003) and real-time implementation issues were also discussed in (Miura, 2003). Such studies presented a preliminary assessment of the utility of MPC for handling operational constraints in an attraction-type maglev system. However, the control law was not imbued with integral action, and thus steady-state error appeared as a result of external disturbances and model inaccuracies.

In the present paper, an MPC formulation with explicit integral control action is employed for the stabilization and reference-following operation of a simulated single-axis, attraction-based maglev system. In the case study under consideration, the simulation and prediction models are obtained on the basis of physical parameters of a real maglev system. The results, obtained for simulation scenarios considering modelling uncertainty and sensor noise, suggest that MPC may be a promising alternative to the control of maglev systems. Moreover, it is shown that appropriate constraint handling is fundamental for the successful operation of the system, especially for large-travel positioning tasks.

## 2. System description and modelling

This work was concerned with the dynamics of the Feedback Magnetic Levitation System<sup>©</sup>, which is depicted in Figure 1. The infrared photo-sensor is assumed to be linear in the required range of operation, yielding a voltage  $y$  that is related to distance  $h$  as  $y = \gamma h + y_0$ , where the gain  $\gamma > 0$  and the offset  $y_0$  are such that  $y \in (-2V, +2V)$ . Current  $i$  is regulated by an inner control loop, and is linearly related to the input voltage  $u$  as  $i = \rho u + i_0$  with  $\rho > 0$  and  $i_0 > 0$ . The working excursion of  $u$  is limited between  $-3V$  (corresponding to a null coil current) and  $+5V$  (saturation value). Rates of change larger than  $50V/s$  for  $u$  cannot be implemented by the current driver along its entire working range.

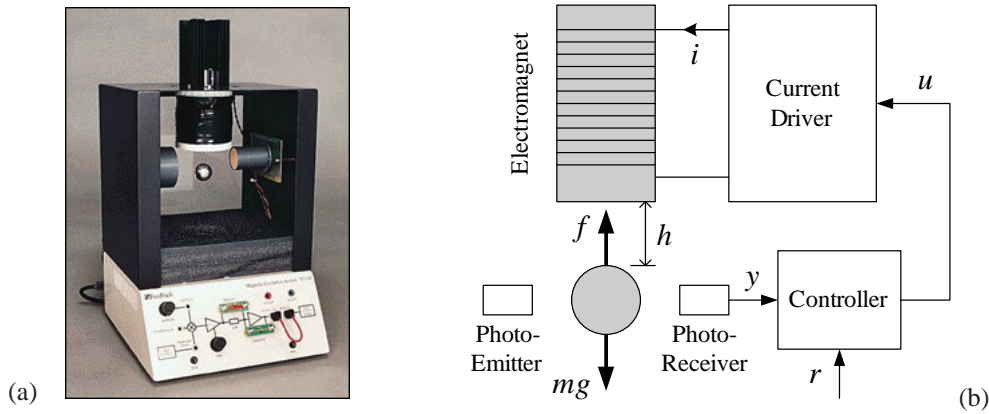


Figure 1. (a) Feedback Magnetic Levitation System<sup>©</sup>. (b) Main components of the control loop. The attraction force  $f$  is related to  $i$  and  $h$  in the form  $f = Ki^2/h^2$ , where  $K > 0$  is an electromechanical conversion gain.

The dynamics of the vertical movement of the suspended sphere can be modelled by the following equation:

$$m \frac{d^2 h}{dt^2} = mg - K \frac{i^2}{h^2} \quad (1)$$

where  $K > 0$  is an electromechanical conversion gain,  $m$  is the mass of the sphere,  $g$  is the acceleration of gravity, and  $i$  is the coil current. Alternatively, in view of the sensor and current driver characteristics, Eq. (1) can be re-written as

$$m \frac{d^2 y}{dt^2} = \gamma mg - \frac{K(\rho u + i_0)^2 \gamma^3}{(y - y_0)^2} \quad (2)$$

Finally, by taking  $x = [y, dy/dt]^T$  as state vector, the model can be realized in state-space form as

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = \gamma g - \frac{K(\rho u + i_0)^2 \gamma^3}{m(x_1 - y_0)^2} \quad (3)$$

The control value  $\bar{u}$  required to keep the system at an equilibrium point  $\bar{x} = [\bar{y}, 0]^T$  is given by

$$\bar{u} = \frac{1}{\rho} \left[ \frac{\bar{y} - y_0}{\gamma} \sqrt{\frac{mg}{K}} - i_0 \right] \quad (4)$$

By using a first-order Taylor expansion around such an equilibrium point, a linearized model can be written as

$$\dot{\hat{x}} = \begin{bmatrix} 0 & 1 \\ \alpha & 0 \end{bmatrix} \hat{x} + \begin{bmatrix} 0 \\ -\beta \end{bmatrix} \tilde{u} = A_c \hat{x} + B_c \tilde{u} \quad (5)$$

where  $\alpha = 2\gamma g(\bar{y} - y_0)^{-1}$ ,  $\beta = 2\rho\gamma^2(\bar{y} - y_0)^{-1}\sqrt{m^{-1}Kg}$  and the tilde denotes deviations from the equilibrium. It is worth noting that  $\bar{y}$  is larger than  $y_0$  because the distance  $h$  between the sphere and the electromagnet is strictly positive in the sensor equation ( $y = \gamma h + y_0$ ). It follows that  $\alpha > 0$  and thus matrix  $A_c$  has eigenvalues at  $\pm\sqrt{\alpha}$ , which shows that the equilibrium is open-loop unstable.

The following values for the physical model parameters were experimentally determined in (Grimm, 2002):  $m = 2.12 \times 10^{-2}$ kg,  $g = 9.8$ m/s<sup>2</sup>,  $y_0 = -7.47$ V,  $\gamma = 328$ V/m,  $i_0 = 0.514$ A,  $\rho = 0.166$ A/V,  $K = 1.2 \times 10^{-4}$ Nm<sup>2</sup>/A<sup>2</sup>.

### 3. Predictive control strategy

Figure 2 presents the main elements of the discrete-time predictive control formulation adopted in this work. The process model is employed to calculate output predictions up to  $N$  steps in the future, where  $N$  is termed "Prediction Horizon". Such predictions are determined on the basis of all information available up to the present time ( $k^{th}$  sampling instant), and are also dependent on the control sequence that will be applied. The optimization algorithm is aimed at determining the sequence  $\{u[k-1+i], i = 1, \dots, M\}$  that minimizes the cost function specified for the problem, subject to constraints on the input and output of the plant. The value of  $M$  ("Control Horizon") is smaller than  $N$ , and the optimization assumes that  $u[k-1+i] = u[k+M-1]$  for  $M < i \leq N$ . The control is implemented in a receding horizon manner, that is, only the first element of the optimized control sequence is applied to the plant and the optimization is repeated at the next sampling instant, on the basis of fresh state measurements.

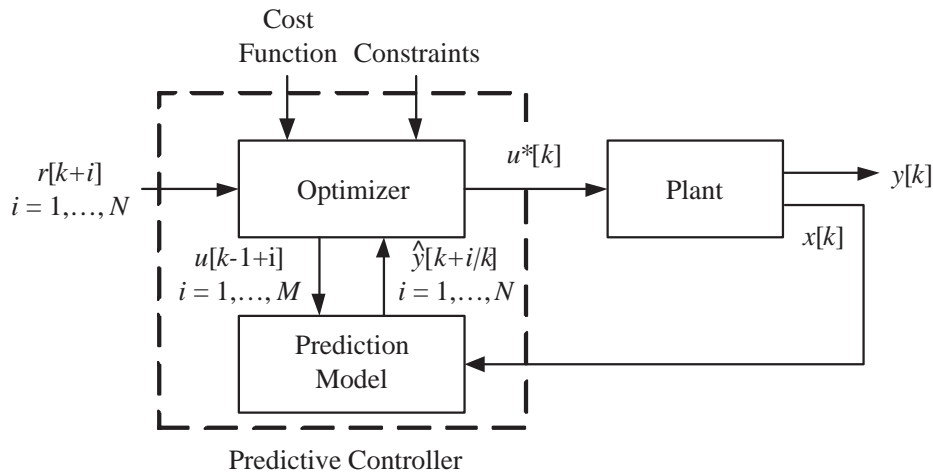


Figure 2. Predictive control loop employing state feedback. The plant input, the output of interest and the reference signal are denoted by  $u \in \mathbb{R}$ ,  $y \in \mathbb{R}$ , and  $r \in \mathbb{R}$ , respectively. In addition,  $\hat{y}[k+i|k]$  denotes the prediction of the output at instant  $k+i$  on the basis of the measured state  $x[k] \in \mathbb{R}^n$ . The optimal control at instant  $k$  is denoted by  $u^*[k]$ .

The following cost function, which penalizes tracking errors and control variations, was adopted:

$$J(\Delta U) = \sum_{i=1}^N (r[k+i] - \hat{y}[k+i|k])^2 + \rho \sum_{i=1}^M (\Delta u[k-1+i])^2 \quad (6)$$

where  $\Delta u[k] = u[k] - u[k-1]$  and  $\Delta U = [\Delta u[k], \Delta u[k+1], \dots, \Delta u[k+M-1]]^T$  is the vector of optimization variables. The design parameter  $\rho > 0$  may be adjusted to achieve a compromise between minimizing the output tracking error and minimizing variations on the control signal. Decreasing  $\rho$  tends to increase the speed of the closed-loop response at the cost of a larger control effort and a greater sensitivity to measurement noise.

By assuming a linear model for the plant dynamics of the form (the tilde notation was dropped for simplicity)

$$x[k+1] = A_d x[k] + B_d u[k], \quad y[k] = C_d x[k] \quad (7)$$

the relation between the control variations  $\Delta u$  and the state  $x$  can be expressed as

$$x[k+1] = A_d x[k] + B_d u[k] = A_d x[k] + B_d (u[k-1] + \Delta u[k]) = \begin{bmatrix} A_d & B_d \end{bmatrix} \begin{bmatrix} x[k] \\ u[k-1] \end{bmatrix} + B_d \Delta u[k] \quad (8)$$

By defining an augmented state vector  $\xi[k] = [x^T[k], u[k-1]]^T$ , it follows that

$$\xi[k+1] = \begin{bmatrix} x[k+1] \\ u[k] \end{bmatrix} = \begin{bmatrix} A_d & B_d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x[k] \\ u[k-1] \end{bmatrix} + \begin{bmatrix} B_d \\ 1 \end{bmatrix} \Delta u[k] = A \xi[k] + B \Delta u[k] \quad (9)$$

In a similar manner, the output equation can be re-written as

$$y[k] = C_d x[k] = \begin{bmatrix} C_d & 0 \end{bmatrix} \begin{bmatrix} x[k] \\ u[k-1] \end{bmatrix} = C \xi[k] \quad (10)$$

It can then be shown (Rossiter, 2003) that the output predictions can be related to the future control variations as  $\hat{Y} = P \Delta U + Q \xi[k]$ , where

$$\hat{Y} = \begin{bmatrix} \hat{y}[k+1] \\ \hat{y}[k+2] \\ \vdots \\ \hat{y}[k+N] \end{bmatrix}, \quad P = \begin{bmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{N-1}B & CA^{N-2}B & \cdots & CA^{N-M}B \end{bmatrix}, \quad Q = \begin{bmatrix} CA \\ CA^2 \\ \vdots \\ CA^N \end{bmatrix} \quad (11)$$

Therefore, the cost in Eq. (6) can be re-written as

$$J(\Delta U) = (R - \hat{Y})^T (R - \hat{Y}) + \rho \Delta U^T \Delta U = (R - P \Delta U - Q \xi[k])^T (R - P \Delta U - Q \xi[k]) + \rho \Delta U^T \Delta U \quad (12)$$

where  $R = [r[k+1], r[k+2], \dots, r[k+N]]^T$ . It can thus be seen that the cost is a quadratic function of the optimization variables  $\Delta U$ . In the absence of constraints, the control sequence  $\Delta U^*$  that minimizes the cost is given by  $\Delta U^* = (P^T P + \rho I_M)^{-1} P^T (R - Q \xi[k])$ , where  $I_M$  is an  $M \times M$  identity matrix.

If restrictions on the manipulated and controlled variables of the form  $\Delta u_{min} \leq \Delta u[k-1+i] \leq \Delta u_{max}$ ,  $i = 1, \dots, M$ ;  $u_{min} \leq u[k-1+i] \leq u_{max}$ ,  $i = 1, \dots, M$ ;  $y_{min} \leq \hat{y}[k+i|k] \leq y_{max}$ ,  $i = 1, \dots, N$  are to be satisfied, the minimization of the cost is subject to the following linear constraints on  $\Delta U$ :

$$\begin{bmatrix} I_M \\ -I_M \\ T_M \\ -T_M \\ P \\ -P \end{bmatrix} \Delta U \leq \begin{bmatrix} \Gamma_M \Delta u_{max} \\ -\Gamma_M \Delta u_{min} \\ \Gamma_M (u_{max} - u[k-1]) \\ -\Gamma_M (u_{min} - u[k-1]) \\ \Gamma_N y_{max} - Q \xi[k] \\ -\Gamma_N y_{min} + Q \xi[k] \end{bmatrix} \quad (13)$$

where  $T_M$  is a lower triangular matrix of ones ( $T_M(i, j) = 1$  for  $i \geq j$  and zero otherwise) and  $\Gamma_M, \Gamma_N$  are  $M \times 1$  and  $N \times 1$  column vectors of ones, respectively (Maciejowski, 2002). In this case, the unconstrained solution may not be a feasible point. The optimization problem then becomes one of Quadratic Programming (Maciejowski, 2002).

### 3.1 Integral control action

The use of control variations in the formulation presented above leads to offset-free tracking if there are no external disturbances and if the prediction model matches the steady-state gain of the plant (Rossiter, 2003). However, if such assumptions do not hold, additional procedures must be used to eliminate the steady-state error. A simple approach consists of assuming that the plant output is affected by a constant additive disturbance, which can be estimated as the difference between the measured value  $y[k]$  and the model prediction  $\hat{y}[k|k-1]$ . However, such a procedure requires the use of an independent model (Maciejowski, 2002), that is, a model that is not realigned with the plant state at each new sampling instant. In the case of an unstable plant, the predictions of an independent model would diverge, thus preventing its use for disturbance correction.

In the present work, an explicit integral control action is achieved by including a term associated to the integral of the tracking error in the cost function. For this purpose, the plant model is augmented with an additional state  $z$  with the following dynamics:

$$z[k+1] = z[k] + r[k] - y[k] = z[k] + r[k] - C \xi[k] \quad (14)$$

By defining a new augmented state vector as  $\xi_a[k] = [\xi^T[k], z[k]]^T$ , the state equation becomes

$$\xi_a[k+1] = \begin{bmatrix} A\xi[k] + B\Delta u[k] \\ -C\xi[k] + z[k] + r[k] \end{bmatrix} = \begin{bmatrix} A & 0 \\ -C & 1 \end{bmatrix} \xi_a[k] + \begin{bmatrix} B \\ 0 \end{bmatrix} \Delta u[k] + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r[k] = A_a \xi_a[k] + B_a \Delta u[k] + H r[k] \quad (15)$$

Moreover, if the model output is augmented with the new state  $z[k]$  as  $y_a[k] = [y[k], \eta z[k]]^T$  for a fixed  $\eta > 0$ , the output equation becomes

$$y_a[k] = \begin{bmatrix} C & 0 \\ \Theta & \eta \end{bmatrix} \xi_a[k] = C_a \xi_a[k] \quad (16)$$

where  $\Theta$  is a row vector of  $(n + 1)$  zeros. A prediction equation for  $\hat{y}_a[k + i|k]$ ,  $i = 1, \dots, N$ , can be written as  $\hat{Y}_a = P_a \Delta U + Q_a \xi_a[k] + V R$ , where  $P_a, Q_a$  are obtained by using  $A_a, B_a, C_a$  instead of  $A, B, C$  in Eq. (11). Matrix  $V$  is obtained as  $P_a$  by using  $H$  instead of  $B_a$  e  $N$  instead of  $M$ .

Therefore, the integral of the tracking error can be incorporated into a new cost function  $J_a$  as

$$J_a(\Delta U) = (R_a - \hat{Y}_a)^T (R_a - \hat{Y}_a) + \rho \Delta U^T \Delta U = \\ = (R_a - P_a \Delta U - Q_a \xi_a[k] - V R)^T (R_a - P_a \Delta U - Q_a \xi_a[k] - V R) + \rho \Delta U^T \Delta U \quad (17)$$

where  $R_a = [r[k + 1], 0, r[k + 2], 0, \dots, r[k + N], 0]^T$ . It is worth noting that the zeros inserted in  $R_a$  correspond to the desired value for  $\eta z[k]$ , where  $z[k]$  is a cumulative sum of the tracking errors. The design parameter  $\eta$  can be used to adjust the weight of integral action in the resulting control law.

#### 4. Methodology

In this study, the nonlinear equations of the maglev system, presented in section 2, were employed in the simulation model. The prediction model was obtained by linearizing the plant dynamics around the center of the working range of the position sensor ( $\bar{y} = 0$ ). The control tasks consisted of tracking steps in the reference from an initial resting position at  $y = 0$ .

On the basis of a previous study (Galvão et al., 2003) concerning the digital control of this system, a 5ms sampling period was adopted. Assuming that a zero-order-hold will keep the control signal constant between sampling instants (Hemerly, 2000), the model matrices resulting after linearization and discretization are as follows:

$$A_d = \begin{bmatrix} 1.0108 & 0.0050 \\ 4.3185 & 1.0108 \end{bmatrix} \quad B_d = \begin{bmatrix} -0.0142 \\ -5.6779 \end{bmatrix} \quad C_d = [1 \quad 0] \quad (18)$$

All simulations were carried out by using the Matlab 6.5 software in the Simulink environment. A specific Matlab S-function was written to implement the predictive control law. The Quadratic Programming problem was solved by using the `quadprog` function of the Matlab Optimization Toolbox.

The initial part of the study consisted of determining appropriate values for the prediction and control horizons. The importance of handling input variation constraints, specially for large steps in the reference, was then investigated. The use of integral control action was studied to achieve offset-free tracking in the presence of a mismatch between the gains of the prediction model and the actual process. Finally, the effect of estimating the speed from noisy measurements of the position was analyzed with a view on the future real-time deployment of the controller.

### 5. Results and Discussion

#### 5.1 Adjusting the prediction and control horizons

In order to investigate the effect of varying the prediction horizon  $N$ , the cost parameter was fixed at  $\rho = 1$ , the control horizon was fixed at  $M = 5$  and four values of  $N$  were tested ( $N = 6, 10, 20, 50$ ). Figure 3a presents the resulting responses for a +0.2V step in the reference. It is worth noting that positive reference values correspond to downward movements of the suspended sphere (increase in the signal of the infrared photo-sensor). As can be seen, decreasing the prediction horizon tends to decrease the damping of the control loop. In fact,  $N = 5$  actually results in an unstable behaviour (not shown in this graph). On the basis of the results presented in Fig. 3a, it was deemed that  $N = 20$  provides a good compromise between speed of response and damping. Therefore, such a value was adopted for the prediction horizon.

The effect of adjusting the control horizon  $M$  was investigated by fixing  $N = 20$  and testing four values of  $M$  ( $M = 1, 2, 4, 5$ ). The results presented in Fig. 3b show that increasing the control horizon above  $M = 4$  does not bring performance improvements. For this reason, the value  $M = 4$  was adopted.

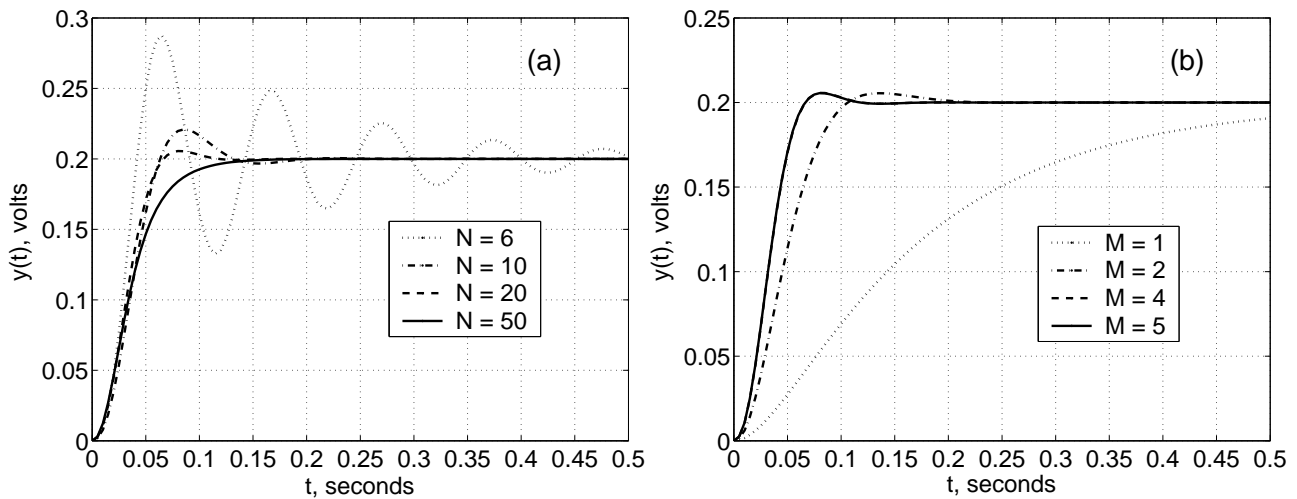


Figure 3. (a) Effect of varying the prediction horizon  $N$  (values indicated in the legend) for a fixed control horizon ( $M = 5$ ). (b) Effect of varying the control horizon  $M$  (values indicated in the legend) for a fixed prediction horizon ( $N = 20$ ). The responses for  $M = 4$  and  $M = 5$  are indistinguishable in the graph.

## 5.2 Importance of constraint handling

Previous studies with this magnetic levitation process indicated that the rate limitation on the input is the constraint that presents the greatest challenge to the design of an effective controller (Miura, 2003). In order to illustrate the importance of handling this constraint, a simulation was carried out by using a step reference of  $0.85V$ . As shown in Fig. 4a, if the rate constraint is not taken into account in the optimization process, the controller generates a control signal with variations that cannot be implemented by the current driver. As a result, the closed-loop performance is poor, as indicated in Fig. 4b. Actually, by using a slightly larger step reference ( $0.9V$ ), the response diverges because the current driver is not able to act fast enough to avoid the fall of the suspended sphere. This problem is solved by including such a rate constraint as a restriction in the Quadratic Programming algorithm. As shown in Fig. 4b, the resulting performance is considerably improved by this procedure.

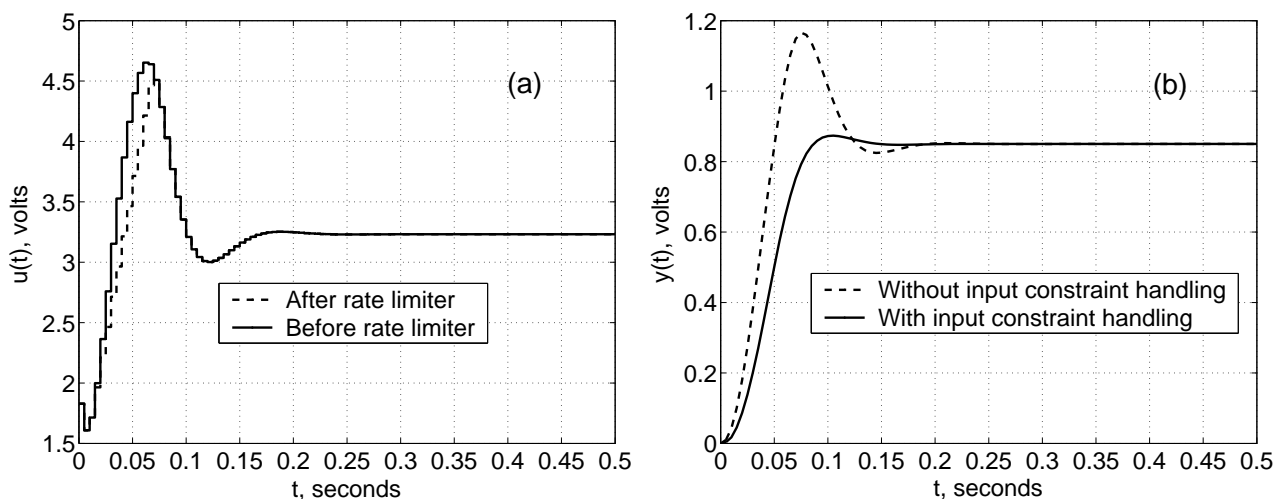


Figure 4. Results for a  $+0.85V$  step in the reference: (a) output of the unconstrained MPC controller (solid line) and control signal that can be implemented by the actuator (dashed line); (b) plant output using an unconstrained MPC controller (dashed line) and an MPC controller that considers the limitation on the control variation rate (solid line).

## 5.3 Integral control action

It is worth noting that, in the nominal case illustrated above, offset-free tracking is obtained even without the use of the integral term in the cost function. To illustrate the need for the additional integral control action, the simulation for a  $+0.2V$  step in the reference was repeated after decreasing the magnetic levitation gain  $K$  by 10% with respect to the nominal value. As can be seen in Fig. 5, the mismatch between the plant gain and the prediction model results in a

negative steady-state error. Such an error arises because the prediction model over-estimates the strength of the magnetic field for a given current and, as a result, a control signal smaller than necessary is applied. However, Fig. 5 also shows that the inclusion of the integral term in the cost function compensates such problem and eliminates the steady-state error. As can be seen, the error correction becomes faster as the weight  $\eta$  of the integral term is increased.

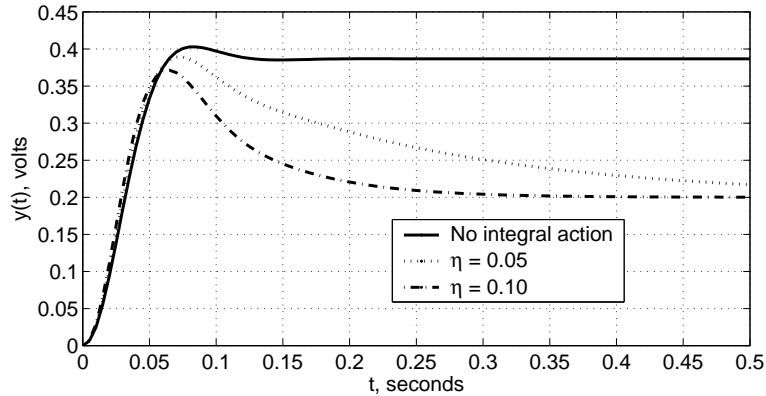


Figure 5. Response to a +0.2V step in the reference for different settings of the integral control action. The magnetic levitation gain was decreased by 10% with respect to the nominal value.

#### 5.4 Sensitivity to noise

In a real application, the speed information would need to be estimated from the readings of the position sensor. In order to investigate such an issue, the simulation with the nominal parameters was repeated by using the numerical derivative of the position signal as an estimate of the speed. Moreover, a zero-mean, white, gaussian noise with a standard deviation of 0.01V was added to the output of the position sensor. Figure 6 presents the resulting response to a +0.2V step in the reference. As can be seen, the measurement noise leads to a deterioration of the closed-loop performance. Such an effect can be partially compensated by increasing the weight  $\rho$  of the control variations in the cost function, which leads to a less vigorous control action and a decrease in the closed-loop bandwidth. If the controller were to be deployed in a real setting, the weight  $\rho$  would need to be adjusted according to the level of noise in the position readings.

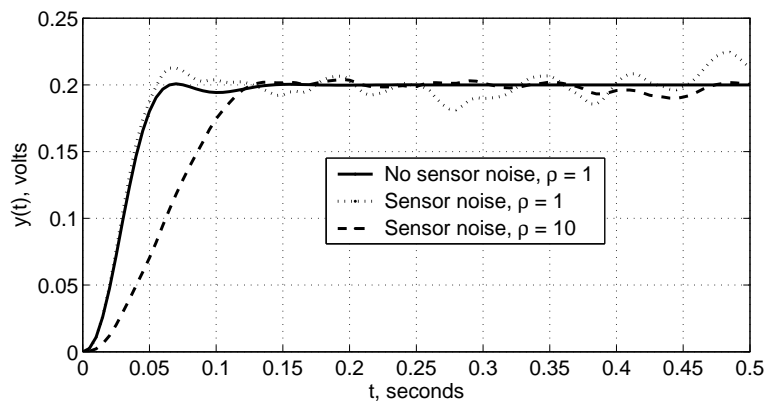


Figure 6. Effect of adding measurement noise to the readings of the output sensor and estimating the velocity by numerical differentiation of the position. The effect of increasing the weight  $\rho$  of the control variations in the cost is also illustrated.

## 6. Conclusion

The results obtained in this case study suggest that MPC methodologies may be a promising alternative to the control of unstable maglev systems. In this context, the constraint-handling features of MPC may be particularly valuable, especially to ensure an appropriate tracking of large steps in the setpoint.

As regards the real-time deployment of the MPC controller, the use of a linear prediction model, as the one adopted in this work, may be more advisable than resorting to more elaborate nonlinear models. In fact, the quadratic programming problem stemming from the combination of a linear model, a quadratic cost function, and linear constraints, can be solved by very efficient numerical algorithms (Maciejowski, 2002). Such a feature would be essential to cope with the fast

dynamics of the levitation process by using moderate computational resources.

Further work concerning the application of MPC formulations for maglev systems could address the issue of robustness with respect to modelling uncertainty. In fact, the problem of ensuring the robust stability of predictive control loops in the presence of constraints has been a matter of intense research (Kerrigan and Maciejowski, 2004).

## 7. Acknowledgements

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