EFFECTS OF STATIC CONDENSATION ON ELECTRICAL IMPEDANCE TOMOGRAPHY

Iara Kazuyo Ike
Escola Politécnica da USP - Engenharia Mecânica - Av. Prof. Luciano Gualberto, trav. 3, n^o 380 - São Paulo
iara.ike@poli.usp.br

Raul Gonzalez Lima
Escola Politécnica da USP - Engenharia Mecânica - Av. Prof. Luciano Gualberto, trav. 3, n^o 380 - São Paulo
lima.raul@gmail.com

Abstract.
Frequently, to determine resistivity distribution on Electrical Impedance Tomography (EIT), a sequence of direct Finite Elements problems must be solved. This is what happens on Newton-Raphson based algorithms and Kalman Filter based algorithms for EIT. Only a small fraction of the nodal potentials are possible to be measured and only this small number of potentials are taken into account in the error index that is to be minimized to estimate the resistivity distribution. This work investigates the effects of static condensation, iterative refinement, and routines for sparse matrices, to solve the direct problem in EIT from the point of view of computational time and numerical error propagation. Results indicate that computational time and numerical error propagation can be diminished under certain conditions.

Keywords: *static condensation, electrical impedance tomography, numerical error, sparse matrix*

1. Introduction

The knowledge of the electrical properties and its variation in time, such as the electric conductivity inside the body, is useful in several medical problems, for example, in the detection of blood clots in the lungs (CHENEY et al., 1999), detection of fluids in the lungs, detection of collapsed regions in the lungs, in the noninvasive monitoring of heart function and blood flow, monitoring of internal bleeding, in the studies to determine the boundary between dead and living tissue and detection of the breast cancer.

There are several systems that use electrical measures for image reconstruction, representations of the conductivity and the electric permittivity inside the body. These systems are called electrical impedance tomographs.

The electrical impedance tomograph applies currents through the electrodes attached on the surface of the skin and then the resulting voltages are measured. In that way, it is possible to estimate the conductivity or the electric permittivity inside the body.

To improve the resolution of the Electrical Impedance Tomography images, there were developed many studies about the behavior of the current density distribution near the electrode (CIULLI et al., 1996), the ideal amount of electrodes (CHENEY et al., 1999), the contact impedance (VILHUNEN et al., 2002), current patterns (POLYDORIDES and MC-CANN, 2002) and the development of current sources (ROSS et al., 2003).

The first commercially available system for electrical impedance tomography was the system developed by David Barber and Brian Brown (BARBER and BROWN (1984) apud CHENEY et al. (1999)). The system uses a single current source and 16 electrodes to generate low resolution images of the conductivity variations inside the body.

The electrical impedance tomography images don't have resolution comparable to the Computerized Tomography scans nor to the image of magnetic resonance but the electrical impedance tomography has low cost, high speed of data acquisition and through EIT it is possible to characterize tissues (BROWN, 2001).

1.1 EIT Image Reconstruction

The reconstruction problem consists of an estimation of the conductivity distribution inside the body, according to measured voltages and injected currents in its boundary. This problem is challenging because it is nonlinear and ill posed. It means that great changes in the resistivity correspond to small changes in the measurements and, consequently, they should be made accurately. Regularization techniques are necessary to stabilize the inversion, such as SMORR (" SPECTRAL MODELLING REGULARIZED RECONSTRUCTOR ") (BRANDSTATTER ET al., 2003), Gaussian anisotropic regularization filters (BORSIC et al., 2002), the weighted regularization (CLAY and FERREE, 2002) and the regularizations based on a priori knowledge (ADLER, 1996). Among these regularizations, the Tikhonov regularization is used frequently (HOFMANN, 1998; KAIPIO et al., 1999; KAIPIO et al., 2000; KOLEHMAINEN et al., 2001; BORCEA et al., 2003).

Among the different reconstruction algorithms stand out:

- the non-iterative linear methods, which assume that the conductivity does not differ very much from a constant, Barber-Brown Backprojection Method (BARBER and BROWN (1984) apud CHENEY (1999)), Calderón's approach (CALDERóN (1980), ISAACSON AND CHENEY (1991), ISAACSON AND ISAACSON (1989), CHENEY ET al. (1990), AND ISAACSON AND CHENEY (1990) APUD CHENEY, (1999)), moment methods (BERNTSEN et al. (PREPRINT), CONNOLLY AND WALL (1988), ALLERS and SANTOSA (1991) apud CHENEY, (1999)) and one-step Newton Method (CHENEY et al. (1990), BLUE (1997), EGGLESTON et al. (1989), FUKS et al. (1991), GOBLE (1990) AND SIMSKE (1987) APUD CHENEY (1999));

- the iterative methods (EGGLESTON et al. (1989), KOHN and MCKENNEY (1990), WEXLER et al. (1985), YORKEY et al. (1987), JAIN et al. (1997), HOLDER (1993), WOO et al. (1990), BRECKON AND PIDCOCK (1988), JIANG (PREPRINT), DORSON (1992), KLIBANOV (preprint), SANTOSA and VOGELIUS (1990) apud CHENEY (1999));
- the adaptive methods that adjust the applied current patterns to obtain the best reconstruction (GISSER et al. (1990), GISSER et al. (1987), NEWELL et al. (1988), SIMSKE (1987), BRECKON AND PIDCOCK (1988) AND ISAACSON AND CHENEY (1996) APUD CHENEY (1999));
- methods based on the "layer-stripping" algorithm (SYLVESTER (1994) and SOMERSALO et al. (1991) APUD CHENEY (1999));
- Fuzzy methods and Genetic algorithms (CHO et al., 1999; OLMI et al.);
- Neural Networks has been used in the work of TAKTAK et al. (1996) and it increased the speed of image reconstruction in real-time;
- the Topological Optimization methods (BYUN et al.; LIMA and SILVA, 2004; LIMA and LIMA, 2004);
- Simulated Annealing (YANG et al., 1997);
- Statistical Inversion and Monte Carlo Method (KAIPIO et al., 2000);
- "Generalized Vector Sample Pattern Matching (GVSPM)" (DONG et al., 2003);
- Kalman's Filter: the method is used in reconstruction algorithms of EIT images to detect variations in the impedance (VAUHKONEN et al., 1998) or in the resistivity (KIM et al., 2001; KIM et al., 2002; TRIGO, 2001; TRIGO et al., 2004).

The main objective of this paper is to investigate the effects of static condensation in the conductivity matrix, iterative refinement and routines for sparse matrices. The advantage consists of working only with small number of potentials, that is, those potentials measured by the electrodes. In this manner, it is possible to obtain a decrease of dimension of the system to be solved in the EIT direct problem. Most of the EIT methods benefit from a faster and more accurate direct problem solver.

2. The Direct Problem Formulation

The solution of an EIT direct problem consists of determining the potentials in the surface of the body, given the distribution of conductivities in a section of the body and the current injected in its boundary.

A Finite Elements model is developed to represent the domain, for instance a section of the human thorax. Each node on the boundary represents an electrode (punctual model of electrode) and this is positioned to the same distance among the adjacent ones. It is possible to assemble the global conductivity matrix $[Y]$. The programs FElt and EasyMesh were used to generate the 2D-meshes.

It is possible to obtain the following relationship among the voltage vector $\{V\}$, the global conductivity matrix $[Y]$ and the current vector $\{C\}$ (MOLINA, 2002) from the Finite Element model:

$$[Y]\{V\} = \{C\}. \quad (1)$$

The boundary conditions are used to turn $[Y]$ nonsingular and the voltage vector $\{V\}$ can be written as

$$\{V\} = [Y]^{-1}\{C\}. \quad (2)$$

In the next section, two static condensation methods are presented. The objective of these methods is to reduce the dimension of the linear system to be solved and so improve accuracy of the solution.

3. Algorithms for Static Condensation

In the static condensation method a square matrix whose order is equal to the number of electrodes is obtained by means of two methods. In the first method, elementary operations are applied on the linear system (Equation 1). In the second method, the system matrix is partitioned in submatrices. These methods are explained in the next subsections. In that way, the dimension of new matrix is smaller than the dimension of $[Y]$. Besides, it is expected that the static condensation is capable to reduce the numerical error propagation during the solution of the direct problem.

The results obtained through simulations using static condensation, using submatrices or using pivotation, just represent a stage of the estimation process. In this stage, it is assumed that the conductivities have been calculated by an estimation method. The simulations represent the solution of the direct problem of EIT, in other words, the voltage vector $\{V\}$ is determined given the conductivity distribution inside the body's section and the injected current in its boundary.

3.1 Static Condensation using Pivoting

This method is based on the partial pivoting (GOLUB et al., 1996; PRESS et al., 1992), the conductivity matrix $[Y]$ is modified by linear combination on just the lines. The condensation using pivoting was applied to the global conductivity matrix $[Y]$.

Renumerating the mesh it is possible to modify the system described by Equation (1), so that the first elements of $\{V\}$ and $\{C\}$ represent the electrodes (Figure 1).

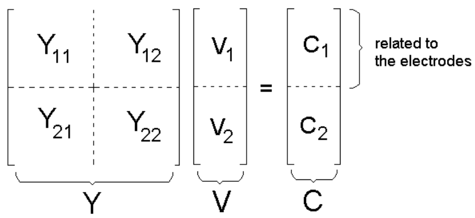

Figure 1: The linear system $[Y]\{V\} = \{C\}$

The vector $\{V_1\}$ represents the vector of electrical potential that are being looked for.

The linear combination on the lines of $[Y]$ was performed to obtain $[\widetilde{Y}]$ such that,

- $\left[\widetilde{Y}_{12}\right]$ is null,
- $\left[\widetilde{Y}_{22}\right]$ is a lower-diagonal matrix,
- $\left[\widetilde{Y}_{11}\right]$ and $\left[\widetilde{Y}_{21}\right]$ are non null matrix.

The resulting matrix $[\widetilde{Y}]$ is presented in the Figure 2.

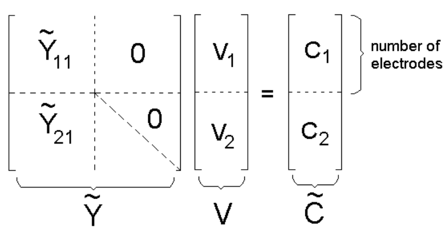

Figure 2: System obtained by static condensation using pivoting in the global conductivity matrix [Y]

The system above can be written in the following way:

$$\begin{cases} \left[\widetilde{Y}_{11}\right]\{V_1\} + \left[\widetilde{Y}_{12}\right]\{V_2\} = \{\widetilde{C}_1\} \\ \left[\widetilde{Y}_{21}\right]\{V_1\} + \left[\widetilde{Y}_{22}\right]\{V_2\} = \{\widetilde{C}_2\} \end{cases} \tag{3}$$

The submatrix $\left[\widetilde{Y}_{12}\right]$ is null, therefore, the voltage vector $\{V_1\}$ can be obtained by the first equation only

$$\{V_1\} = \left[\widetilde{Y}_{11}\right]^{-1}\{\widetilde{C}_1\} \tag{4}$$

Thus the inversion of whole $[Y]$ is unnecessary.

The vector $\{V_2\}$ can be obtained substituting $\{V_1\}$ in the second equation of the system. The vector $\{C\}$ possesses only one nonzero element that corresponds to the electrode where the current is injected. The vector $\{C_1\}$ is null except one element and $\{C_2\}$ is null. Thus the vector $\{V_2\}$ can be calculated by the expression:

$$\{V_2\} = -\left[\widetilde{Y}_{22}\right]^{-1}\left[\widetilde{Y}_{21}\right]\{V_1\}. \tag{5}$$

In the next section the second static condensation algorithm is described.

3.2 Static Condensation using Submatrices

The system matrix is partitioned into submatrices (LOGAN, 1986). Let the system presented in the Figure 1. It can be written as

$$\begin{cases} [Y_{11}]\{V_1\} + [Y_{12}]\{V_2\} = \{C_1\} \\ [Y_{21}]\{V_1\} + [Y_{22}]\{V_2\} = \{C_2\} \end{cases} \tag{6}$$

where $\{V_1\}$ represents the vector of electrical potential that are being looked for.

The current vector $\{C\}$ is known, static condensation using submatrices becomes advantageous since when the system can be rewritten in a compact way.

The current pattern is represented by a vector $\{C\}$ where just one or two elements are nonzero. In this study the vector $\{C_1\}$ has only one nonzero element and $\{C_2\}$ is null.

The vector $\{V_2\}$ can be calculated by the second equation of the linear system:

$$\{V_2\} = [Y_{22}]^{-1}(\{C_2\} - [Y_{21}]\{V_1\}). \tag{7}$$

Substituting $\{V_2\}$ in the first equation

$$([Y_{11}] - [Y_{12}][Y_{22}]^{-1}[Y_{21}])\{V_1\} = \{C_1\} - [Y_{12}][Y_{22}]^{-1}\{C_2\}. \tag{8}$$

Since vector $\{C_2\}$ is null, then the new system is given by:

$$([Y_{11}] - [Y_{12}][Y_{22}]^{-1}[Y_{21}])\{V_1\} = \{C_1\}. \tag{9}$$

The calculation of $\{V_1\}$ still involves the inversion of the matrix $[Y_{22}]$, however, its dimension is smaller than the dimension of matrix $[Y]$.

4. Matrix Sparsity

MOLINA (2002) achieved a significant improvement in the estimation method of electrical conductivity distribution using the conductivity matrix in the band diagonal form which is a particular representation of sparse matrices.

In an attempt to reduce the numerical error propagation, the present work uses sparse matrix algorithms (DUFF et al., 1986; PRESS et al., 1992).

The sparse matrix is represented by two vectors, one stores just nonzero elements of the matrix and the other one stores the element index (PRESS et al., 1992). The sparse storage mode is useful to reduce the processing time and to optimize the memory space for data storage.

5. Iterative Refinement

In most of linear system solutions, it is not easy to obtain precision comparable to the machine precision. The roundoff error can accumulate, promoting the numerical error propagation, specially when the system matrix is close to singular (GOLUB et al., 1996; PRESS et al., 1992; WATKINS, 1991) .

Furthermore, it was also obtained an improvement in the precision of the impeditivity and an increase in the algorithm convergence rate using iterative refinement in the calculation of the conductivity matrix inverse which is used in the Kalman's filter (MOLINA, 2002). In this article, the iterative refinement was used to obtain a more accurate voltage vector.

6. Sensitivity Analysis of Linear Systems

WATKINS (1991) affirmed that given the linear system $[A]\{x\} = \{b\}$, where $[A]$ is a nonsingular matrix and $\{b\}$ is a non null vector, the inequality follows

$$\frac{\|\delta x\|}{\|x\|} \leq \kappa(A)\frac{\|\delta b\|}{\|b\|} \tag{10}$$

Taking into account Equation (1) and Equation (10) follows

$$\frac{\|\delta V\|}{\|V\|} \leq \kappa(Y)\frac{\|\delta C\|}{\|C\|} \tag{11}$$

where $[A]$, $\{x\}$ and $\{b\}$ were replaced by the conductivity matrix $[Y]$, the voltage vector $\{V\}$ and the applied current vector $\{C\}$, respectively.

The Equation (11) relates how much the perturbation in the vector $\{C\}$ may reach the vector $\{V\}$. The factor $\kappa(Y)$ represents the condition number of $[Y]$. The product $\kappa(Y)\frac{\|\delta C\|}{\|C\|}$ represents an upper bound of $\frac{\|\delta V\|}{\|V\|}$. It is worth to highlight that the 2-norm is used in this case, where the condition number is given by the ratio (WATKINS, 1991; POOLE, 2004)

$$\kappa_2(Y) = \frac{\sigma_M}{\sigma_m} \tag{12}$$

where σ_m and σ_M are the smallest and the largest singular value, respectively.

7. Simulation

In this study, a circular domain was discretized in 189 triangular elements, with a total of 111 nodes which 32 of them are located on the boundary, representing the electrodes (punctual electrode model). It was considered that a current of $85\ mA$ was injected in one of the electrodes and the diametrically opposite electrode was considered a ground. It was supposed a homogeneous conductivity distribution of $0.6\ (\Omega m)^{-1}$.

The simulations followed the items:

- the conductivity matrix was calculated;
- the mesh generated by EasyMesh program was re-numerated to put the positions relative to the 32 electrodes at beginning of the voltage vector and the applied current vector;
- rows and columns of the conductivity matrix were interchanged due to the mesh renumeration;
- the conductivity matrix without static condensation is inverted to determine the voltage vector, with and without iterative refinement ;
- the two static condensation methods were applied to the conductivity matrix, with and without iterative refinement;
- the same procedure of the previous items was adopted, however, considering the sparsity of the conductivity matrix.

The results are presented in the next section.

8. Results

First, to better present the results the log-scale is used in y-axis for all graphics.

The condition number for the conductivity matrix obtained by each method is presented in Figure 3.

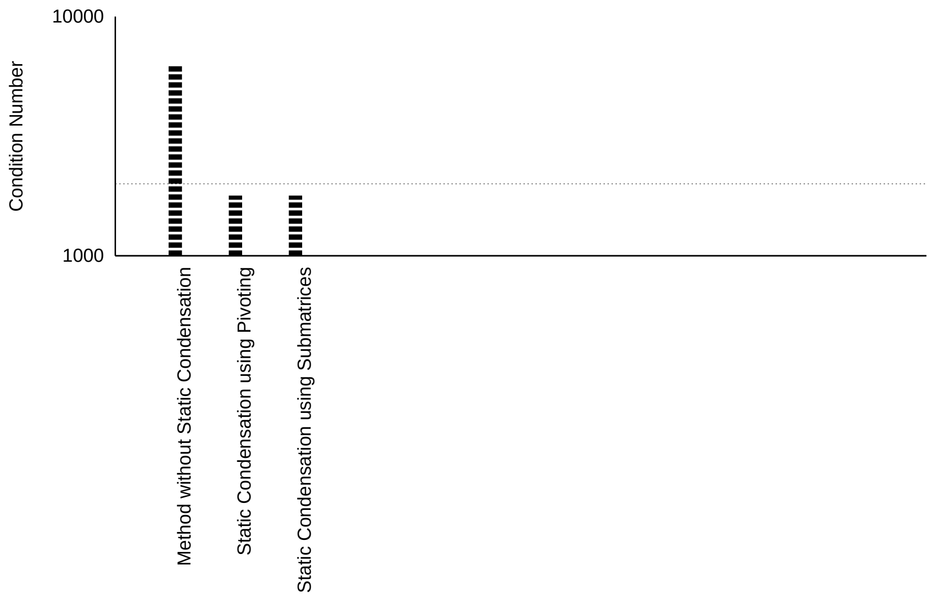

Figure 3: Condition Number

It is possible to verify on Figure 3 that the condition number for the matrix with static condensation was smaller than for the system without condensation. However, the condition number for the matrices using the condensation methods were approximately the same. The reduction of the condition number represents a better behavior of the linear system to perturbations.

Figure 4 shows the 2-norm of the residual vector, $\|[Y].\{V\} - \{C\}\|_2$, where $\{V\}$ represents the voltage vector calculated by the three methods and $\{C\}$ the applied current vector.

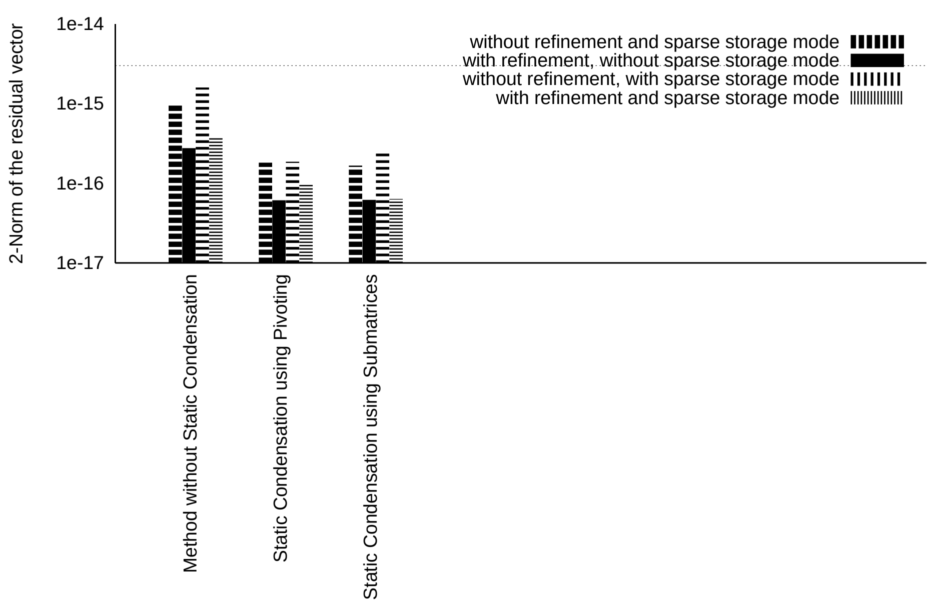

Figure 4: 2-Norm of residual vector

It is possible to notice that there was a significant reduction in the 2-norm of residual vector when the iterative refinement was applied to the linear system solution.

The methods using static condensation presented ten times smaller upper bounds for $\frac{\|\delta V\|_2}{\|V\|_2}$ than the method without condensation, as shown in Figure 5. The results reflect the reduction of the condition number (Figure 3) and have as consequence a reduction of the 2-norm of the residual vector (Figure 4).

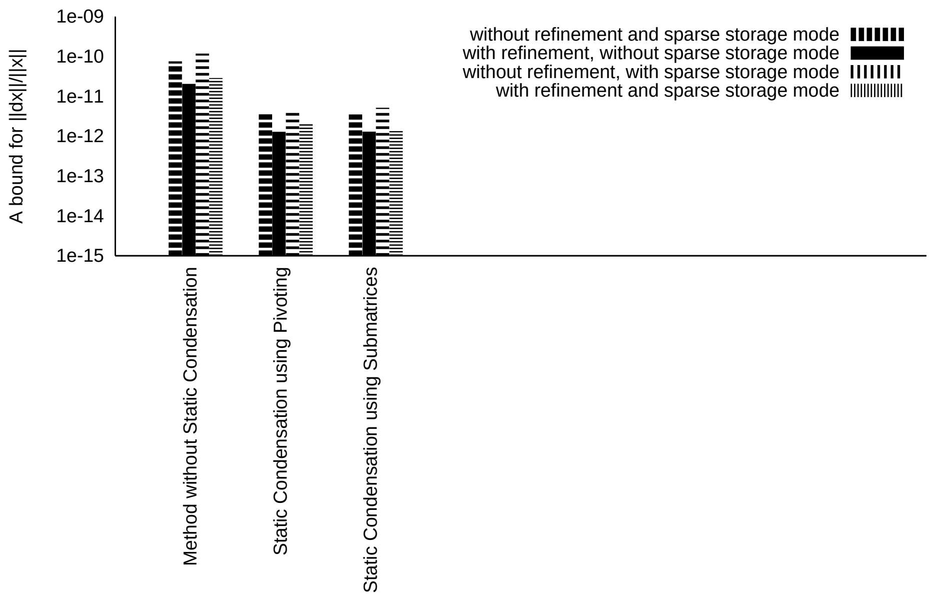

Figure 5: A bound for $\|\delta x\|_2/\|x\|_2$

The processing time of each method is shown in Figure 6. The processing time of the methods with iterative refinement was calculated by the sum of the time spent to solve the linear system and to apply the iterative refinement in its solution.

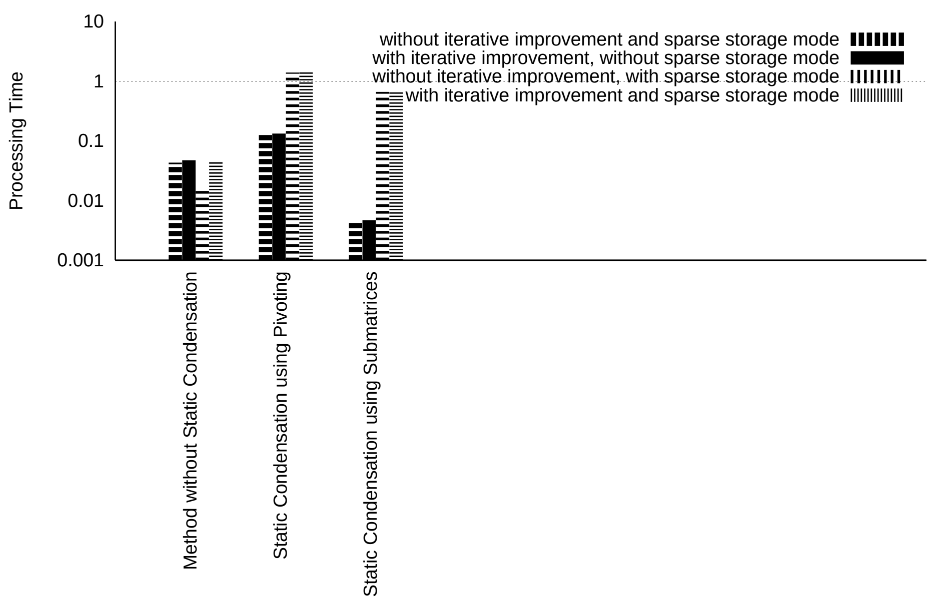

Figure 6: Processing Time

The static condensation method using submatrices without sparse storage mode had the smallest processing time, following the method without static condensation but with sparse storage mode.

The sparse storage mode provided a processing time decrease when the static condensation wasn't applied. The conclusions are presented in the next section.

9. Conclusions

The application of static condensation in the system matrix yields the reduction of:

- the condition number,

- the 2-norm of the residual vector,
- the bound for $\frac{\|\delta x\|_2}{\|x\|_2}$,
- the processing time in the static condensation using submatrices.

In this study, the results for sparse storage mode were not better than those obtained without it. Besides, there was an increase in the processing time.

The application of iterative refinement improves the accuracy of the solution.

Based on the presented results and considering the proposed techniques to be used in EIT problems, it is possible to conclude that the best performance was reached by static condensation method using submatrices without the sparse storage mode.

It is worthwhile to point out that the presented techniques were applied to a small order matrix. The same methodology will be applied to matrices of larger order, different patterns of sparsity and condition number, to test the preliminary conclusion of the present work.

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