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COMPLEX NEAR-WALL TURBULENT FLOWS: A PERFORMANCE ANALYSIS OF VELOCITY AND TEMPERATURE LAWS OF THE WALL

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Abstract. *The main goal of this work is to provide a functional performance analysis of laws of the wall, for velocity and temperature, applied in numerical modelling of turbulent wall flows, which may suffer density variations due to the presence of temperature variations, and boundary layer separation due to low intensity pressure gradients or to sudden expansion. The analysis is based on comparing the experimental data and results from other numerical simulations with those obtained from the numerical simulations of the test cases of Ng (1981), Tsuji and Nagano (1988), Buice and Eaton (1996) and Vogel and Eaton (1984) using the velocity wall functions of Cruz and Silva Freire (1998), Nakayama and Koyama (1984), Mellor (1966) and the classic logarithmic law, associated with the temperature laws of the wall of Cruz and Silva Freire (1998) and Cheng and Ng (1982). The mean equations of conservation of mass, momentum and energy are obtained using the* $\kappa-\varepsilon$ *model of Jones and Launder (1972), based on Favre's averaging (1965). Spacial discretization is done by P1/isoP2 finite element method and temporal discretization is implemented using a semi-implicit sequential scheme of finite difference. The coupling pressure-velocity is numerically solved by a variation of Uzawa's algorithm. To filter the numerical noises, originated by the symmetric treatment used by Galerkin method to the convective fluxes, it is adopted the balance dissipation method proposed by Huges and Brooks (1979) and Kelly et al. (1980). The remaining non-linearities, due to laws of wall, are treated by minimal residual method proposed by Fontoura Rodrigues (1991).*

keywords: *turbulence, variable density,* $\kappa-\varepsilon$ *model, laws of the wall, finite-elements method*

1. Introduction

In the simulation of parietal turbulent flows in which the density is a function of of temperature gradients, it is necessary the simultaneous resolution of the conservative equations of mass, momentum and energy, as well as equations of state capable of representing the variation of some thermodynamic properties of the fluid with temperature gradients. When the use of wall functions is the choice to model the near wall region, it is also needed the simultaneous employment of wall functions for velocity and temperature in the system of equations to be solved.

The objective of this work is to analyze the numerical performance of wall laws for velocity and for temperature in cases of confined flows under adverse pressure gradient, with thermal variation due to the presence of a heated wall and in combined heat transfer and detaching boundary layer flows. The analyzed wall functions were the velocity laws of the wall of Cruz and Silva Freire (1998), Nakayama and Koyama (1984), Mellor (1966) and the classic logarithmic law, and the temperature laws of the wall of Cruz and Silva Freire (1998) and Cheng and Ng (1982).

The algorithm to be tested, Turbo 2D, is a combination of the numerical simulation methodology using finite elements of strongly heated wall flows, proposed by Brun (1988), with an error minimization method adapted to a finite elements, for the simulation of turbulent wall flows with non-linear boundary conditions, proposed by Fontoura Rodrigues (1990) e (1991), using the classic $\kappa-\varepsilon$ turbulence model of Jones and Launder (1972). By applying Galerkin's method for finite elements to the calculation of convection dominant flows, numerical oscillations without physical meaning can appear. This fact occurs due to the usage of Galerkin's method, that gives a symmetric treatment to the flow modelling, which is a non symmetric physical phenomenon. To lower the tendency of numerical oscillation, a balancing dissipation method, proposed by Huges and Brooks (1979) and Kelly et al. (1980) and implemented by Brun (1988), is used in Turbo 2D.

To accomplish the test of performance of laws of the wall, four test cases were selected. To evaluate the performance of the temperature wall laws, the turbulent flow over a heated wall of Ng (1981) and the turbulent natural convective boundary layer flow along a vertical heated plate of Tsuji and Nagano (1988) were chosen, to evaluate the performance of the velocity laws of the wall, the test case of the turbulent flow in the asymmetric plane diffuser of Buice and Eaton (1996) was selected, and finally, to evaluate both effects simultaneously, the turbulent flow of the thermal backward-facing step of Vogel and Eaton (1984), which combines the effects of the simultaneous presence of an adverse pressure gradient and a thermal field, was chosen. Results were compared with the experimental data available of each selected test case and with some results from other numerical simulations.

2. Analytic formulation

Following the procedures of obtaining the governing equations of the turbulent flows developed in Soares and Fontoura Rodrigues (2003), some considerations are taken to obtain the governing equations for the one-phase, homogeneous, bidimensional and at low Mach number turbulent flows, with density variations performed exclusively by the presence of thermal fields.

In the present analysis, the flows are considered to suffer variations of density and viscosity due to temperature variations of the fluid, according to the ideal gas equation and to the empiric relation for the viscosity as a function of temperature proposed by Ng (1981), which are, respectively:

$$\rho = \frac{p}{RT} \quad and \quad \mu = \mu(T) = aT^n \ , \tag{1}$$

where R, a and n are material constants for the air, and they are, respectively, 287 $\frac{J}{kg\,K}$, 3,68x10-7 $\frac{m^2}{s\,K}$ and 0,685. The implication of the low Mach number of the turbulent flows studied is that some terms of the energy equation can be neglected. In dimensionless form, the energy equation is given by:

$$\rho\frac{DT}{Dt} = \frac{1}{Re\ Pr}\frac{\partial}{\partial x_i}\left(\frac{\partial T}{\partial x_i}\right) + (\gamma - 1)M_a^2\frac{Dp}{Dt} + M_a^2\frac{1}{Re}\Phi \ , \tag{2}$$

with γ being the relation between the specific heat coefficients, at constant pressure,C_p, and volume, C_v, Φ represents the mechanical energy dissipation rate per fluid volume, due to shear viscosity effects, andM_a is the Mach number, defined as a function of the reference values of velocity and temperature. Considering that the flows of the test cases considered in this work are at low Mach numbers ($M_0 \leq 0,3$), the terms multiplied by a second order Mach number can be neglected.

To proceed with the obtainment of the equations of turbulent flows, a mean formulation is adopted considering the solution given by Favre (1965), for flows with considerable variation of density, that uses the Reynolds averaging only for density and pressure, while for velocity and temperature, a mass-weighted averaging is adopted, called the Favre averaging (1965). It is important to note that the Favre averaging is equivalent to the Reynolds averaging in flows that does not have variations in density. The closure of the mean equations is based on Boussinesq's (1877) hypothesis of eddy viscosity, adapted to variable density flows, by Jones and McGuirk (1979) for the velocity fluctuation correlation tensor, the Reynolds Stress Tensor, and on the turbulent diffusivity hypothesis for the velocity and temperature fluctuations correlation vector, interpreted as the turbulent flux of temperature, respectively:

$$\bar{\rho}\overline{u_i''u''_j} = \frac{2}{3}\left(\bar{\rho}\kappa + \mu_t\frac{\partial\widetilde{u}_l}{\partial x_l}\right)\delta_{ij} - \mu_t\left(\frac{\partial\widetilde{u}_i}{\partial x_j} + \frac{\partial\widetilde{u}_j}{\partial x_i}\right) \tag{3}$$

and

$$\bar{\rho}\overline{u_i''\theta} = -\alpha_t\frac{\partial\widetilde{T}}{\partial x_i} = -\frac{\mu_t}{Pr_t}\frac{\partial\widetilde{T}}{\partial x_i} \ , \tag{4}$$

where μ_t is the eddy viscosity, κ is the turbulence kinetic energy, $\widetilde{T}$ is the mean temperature from Favre's averaging, θ represents the temperature fluctuations, α_t is the turbulent thermal diffusivity and Pr_t is the turbulent Prandtl number, considered as a constant of value equal to 0.9 in the present work. In order to equations (3) and (4) provide a solution to the closure problem of the system of mean equations, it is necessary to determine the value of the eddy viscosity μ_t, adopted in this work as a function of the turbulence kinetic energy κ and the dissipation rate of turbulence kinetic energyε , is using the Prandtl - Kolmogorov relation

$$\mu_t = C_\mu\bar{\rho}\frac{\kappa^2}{\varepsilon} = \frac{1}{Re_t} \ , \tag{5}$$

where C_μ is a constant of value 0,09. With the adoption of relation (5), the$\kappa - \varepsilon$ turbulence model relation imposes the necessity of two supplementary transport equations to the system of mean equations, destined to evaluation of variables κ and ε. Once defined the closure of the system of mean equations, the direction proposed by Brun (1988) produces the following system of equations:

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\partial \bar{\rho} \widetilde{u}_i}{\partial x_i} = 0 \ , \tag{6}$$

$$\bar{\rho}\left(\frac{\partial \widetilde{u}_i}{\partial t} + \widetilde{u}_j \frac{\partial \widetilde{u}_i}{\partial x_j}\right) = -\frac{\partial \bar{p}^*}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\left(\frac{1}{Re} + \frac{1}{Re_t}\right)\left(\frac{\partial \widetilde{u}_i}{\partial x_j} + \frac{\partial \widetilde{u}_j}{\partial x_i}\right)\right] + \frac{1}{Fr}\bar{\rho} g_i \ , \tag{7}$$

$$\bar{\rho}\left(\frac{\partial \widetilde{T}}{\partial t} + \widetilde{u}_j \frac{\partial \widetilde{T}}{\partial x_j}\right) = \frac{\partial}{\partial x_j}\left[\left(\frac{1}{Re\,Pr} + \frac{1}{Re_t\,Pr_t}\right)\frac{\partial \widetilde{T}}{\partial x_j}\right] \ , \tag{8}$$

$$\bar{\rho}\left(\frac{\partial \kappa}{\partial t} + \widetilde{u}_i \frac{\partial \kappa}{\partial x_i}\right) = \frac{\partial}{\partial x_i}\left[\left(\frac{1}{Re} + \frac{1}{Re_t\,\sigma_\kappa}\right)\frac{\partial \kappa}{\partial x_i}\right] + \Pi - \bar{\rho}\varepsilon + \frac{\bar{\rho}\beta g_i}{Re_t\ Pr_t}\frac{\partial \widetilde{T}}{\partial x_i} \ , \tag{9}$$

$$\bar{\rho}\left(\frac{\partial \varepsilon}{\partial t} + \widetilde{u}_i \frac{\partial \varepsilon}{\partial x_i}\right) = \frac{\partial}{\partial x_i}\left[\left(\frac{1}{Re} + \frac{1}{Re_t\,\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_i}\right] + \frac{\varepsilon}{\kappa}\left(C_{\varepsilon 1}\Pi - C_{\varepsilon 2}\bar{\rho}\varepsilon + C_{\varepsilon 3}\frac{\bar{\rho}\beta g_i}{Re_t\ Pr_t}\frac{\partial \widetilde{T}}{\partial x_i}\right) \ , \tag{10}$$

$$\bar{\rho} = \frac{1}{1+\widetilde{T}} \ , \tag{11}$$

where:

$$p^* = \bar{p} + \frac{2}{3}\left[\left(\frac{1}{Re} + \frac{1}{Re_t}\right)\frac{\partial \widetilde{u}_l}{\partial x_l} + \bar{\rho}\kappa\right] \ , \tag{12}$$

$$\Pi = \left[\left(\frac{1}{Re_t}\right)\left(\frac{\partial \widetilde{u}_i}{\partial x_j} + \frac{\partial \widetilde{u}_j}{\partial x_i}\right) - \frac{2}{3}\left(\bar{\rho}\kappa + \frac{1}{Re_t}\frac{\partial \widetilde{u}_l}{\partial x_l}\right)\delta_{ij}\right]\frac{\partial \widetilde{u}_i}{\partial x_j} \ , \tag{13}$$

with the numbers of Reynolds, Prandtl and Froude, represented by Re, Pr and Fr respectively, defined as functions of the reference values as follows:

$$Re = \frac{\rho_0 U_0 L_0}{\mu_0} \quad , \quad Pr = \frac{C_p\ \mu_0}{k} = \frac{\nu_0}{\alpha} \quad \text{and} \quad Fr = \frac{U_0^2}{g_0\ L_0} \ . \tag{14}$$

and the constants of the model are given:

$$C_\mu = 0,09 \ , \ C_{\varepsilon 1} = 1,44 \ , \ C_{\varepsilon 2} = 1,92 \ , \ C_{\varepsilon 3} = 0,288 \ , \ \sigma_\kappa = 1 \ , \ \sigma_\varepsilon = 1,3 \ , \ Pr_t = 0,9 \ . \tag{15}$$

3. Law of the wall formulation

The $\kappa - \varepsilon$ turbulence model is incapable of properly representing the laminar sub-layer and the transition regions of the turbulent boundary layer. To solve this inconvenience, the solution adopted in this work is the use of laws of the wall for temperature and for velocity, capable of properly representing the flow in the inner region of the turbulent boundary layer.

The following laws of the wall are the focus of the present analysis, and are implemented on Turbo-2d, the code that uses the proposed methodology. It is important to note that the log-law of the wall for velocity is implemented and used in the analysis, but not discussed here once it is of common knowledge.

3.1. Velocity law of the wall of Mellor(1966)

Deduced from the equation of Prandtl for the boundary layer flow, considering the pressure gradient term for integration, this wall function is a primary approach to flows that suffer influence of adverse pressure gradients. Its equations are, respectively, for the laminar and turbulent region

$$u^* = y^* + \frac{1}{2}p^* y^{*2} \ , \tag{16}$$

$$u^* = \frac{2}{K}\left(\sqrt{1+p^*y^*} - 1\right) + \frac{1}{K}\left(\frac{4y^*}{2+p^*y^*+2\sqrt{1+p^*y^*}}\right) + \xi_{p^*} \ , \tag{17}$$

where the asterisk super-index indicates dimensionless quantities of velocityu^*, pressure gradient p^* and distance to the wall y^*, as functions of scaling parameters to the near wall region, K is the Von Karman constant, and ξ_{p^*} is Mellor's integration constant, function of the near-wall dimensionless pressure gradient, determined in his work of (1966).

The intersection of both regions is considered to be the same as the log law expressions, with$y^* = 11,64$. The relations between the dimensionless near wall properties and the friction velocityu_f are:

$$y^* = \frac{y\,u_f}{\nu} \quad , \quad u^* = \frac{\widetilde{u}_x}{u_f} \quad \text{and} \quad p^* = \frac{1}{\bar{\rho}}\frac{\partial \bar{p}}{\partial x}\frac{\nu}{{u_f}^3} \ . \tag{18}$$

In equation (17) the term ξ_{p^*} is a value obtained from the integration process proposed by Mellor (1966) and is a function of the dimensionless pressure gradient. Its values are obtained through interpolation of those obtained experimentally by Mellor, shown in table (1).

Table 1: Mellor's integration constant (1966)

p^*	$-0,01$	$0,00$	$0,02$	$0,05$	$0,10$	$0,20$	$0,25$	$0,33$	$0,50$	$1,00$	$2,00$	$10,00$
ξ_{p^*}	$4,92$	$4,90$	$4,94$	$5,06$	$5,26$	$5,63$	$5,78$	$6,03$	$6,44$	$7,34$	$8,49$	$12,13$

3.2. Velocity law of the wall of Nakayama and Koyama (1984)

In their work, Nakayama and Koyama (1984) proposed a derivation of the mean turbulent kinetic energy equation, that resulted in an expression to evaluate the velocity near solid boundaries. Using experimental results and those obtained by Strattford (1959), the derived equation is

$$u^* = \frac{1}{K^*}\left[3(t-t_s) + ln\left(\frac{t_s+1}{t_s-1}\frac{t-1}{t+1}\right)\right] \quad , \tag{19}$$

with

$$t = \sqrt{\frac{1+2\tau^*}{3}} \quad , \quad \tau^* = 1 + p^* y^* \quad , \quad K^* = \frac{0,419 + 0,539p^*}{1+p^*} \quad and \quad {y^*}_s = \frac{e^{K\ C}}{1+p^{*0,34}} \ , \tag{20}$$

where K^* is the expression for the Von Karman constant modified by the presence of adverse pressure gradients, τ^* is a dimensionless shear stress, $C = 5,445$ is the log-law constant and t, ${y^*}_s$ and t_s, a value of t at position ${y^*}_s$, are parameters of the function.

3.3. Velocity law of the wall of Cruz and Silva Freire (1998)

Analyzing the asymptotic behavior of the boundary layer flow under adverse pressure gradients, Cruz and Silva Freire (1998) derived an expression for the velocity. The solution of the asymptotic approach is

$$u = \frac{\tau_w}{|\tau_w|}\frac{2}{K}\sqrt{\frac{\tau_w}{\rho} + \frac{1}{\rho}\frac{dp_w}{dx}y} + \frac{\tau_w}{|\tau_w|}\frac{u_f}{K} ln\left(\frac{y}{L_c}\right) \quad with \quad L_c = \frac{\sqrt{\left(\frac{\tau_w}{\rho}\right)^2 + 2\frac{\nu}{\rho}\frac{dp_w}{dx}u_R} - \frac{\tau_w}{\rho}}{\frac{1}{\rho}\frac{dp_w}{dx}} \ , \tag{21}$$

where the sub-index $_w$ indicates the properties at the wall, K is the Von Karman constant, L_c is a length scale parameter and u_R is a reference velocity, evaluated from the highest positive root of the asymptotic relation:

$${u_R}^3 - \frac{\tau_w}{\rho}u_R - \frac{\nu}{\rho}\frac{\partial p}{\partial x} = 0 \ . \tag{22}$$

The proposed equation for the velocity (21) has a behavior similar to the log law far from the separation and reattachment points, but close to the adverse pressure gradient, it gradually tends to Stratford's equation (1959). The same process was used to derive the temperature law of the wall by Cruz and Silva Freire (1998).

3.4. Temperature log law of the wall of Cheng and Ng (1982)

For the calculation of the temperature, Cheng and Ng (1982) derived an expression for the near wall temperature similar to the log law of the wall for velocity. For the laminar and turbulent regions, the equations are respectively

$$\frac{(T_0 - T)_y}{T_f} = y^*\ Pr \quad \text{and} \quad \frac{(T_0 - T)_y}{T_f} = \frac{1}{K_{Ng}} ln(y^*) + C_{Ng} \ , \tag{23}$$

where T_0 is the environmental temperature and T_f is the friction temperature, as defined by Brun (1988). The intersection of these regions are at $y^* = 15,96$, and the constants K_{Ng} and C_{Ng} are, respectively, 0,8 and 12,5.

3.5. Temperature law of the wall of Cruz and Silva Freire (1998)

Using the same arguments of the velocity law of the wall, Cruz and Silva Freire (1998) derived an expression for the friction temperature for thermal flows under adverse pressure gradients. Considering Q_w as the wall heat flux and E as a constant of value equal to 9.8, the expressions of the wall law are:

$$\frac{T_w - T}{Q_w} = \frac{Pr_t}{K\rho C_p u_f}\ ln\ \frac{\sqrt{\frac{\tau_w}{\rho} + \frac{1}{\rho}\frac{dp_w}{dx}y} - \sqrt{\frac{\tau_w}{\rho}}}{\sqrt{\frac{\tau_w}{\rho} + \frac{1}{\rho}\frac{dp_w}{dx}y} + \sqrt{\frac{\tau_w}{\rho}}} + \frac{Pr_t}{K\rho C_p u_R}\ ln\ \frac{4Eu_R^3}{\nu|\frac{dp_w}{dx}|} + AJ \ , \tag{24}$$

with the parameters AJ and AX and the friction temperature defined as:

$$AJ = 1.11 Pr_t \sqrt{\frac{AX}{K}} \left(\frac{Pr}{Pr_t} - 1 \right) \left(\frac{Pr}{Pr_t} \right)^{0,25} \quad , \quad AX = 26 \frac{\left| \frac{\tau_w}{\rho} \right|^{\frac{1}{2}}}{u_R} \quad and \quad T^* = \frac{Q_w}{\rho C_p u_f} \ . \tag{25}$$

4. Numerical methodology

The numerical solution of the proposed system of governing equations, of a dilatable turbulent flow, has as main difficulties: the coupling between all equations; the non-linear behavior resulting of the simultaneous action of convective and eddy viscosity terms; the explicit calculations of boundary conditions in the solid boundary; the methodology of use the continuity equation as a manner to link the coupling fields of velocity and pressure.

The solution proposed in the present work suggests a temporal discretization of the system of governing equations with a sequential semi-implicit finite difference algorithm proposed by Brun (1988) and a spatial discretization using finite elements of the type P1-isoP2. The temporal and spatial discretization implemented in Turbo 2D is presented in Soares and Fontoura Rodrigues (2003).

5. Numerical results

Four test cases were simulated to test the performances of the velocity and thermal laws of the wall, being one turbulent isothermal flow under adverse pressure gradients, the asymmetric plane diffuser of Buice and Eaton (1996), two turbulent thermal boundary layer flows, the turbulent flow over a heated wall of Ng (1981) as a convective dominant turbulent flow and the vertical turbulent natural convective boundary layer flow of Tsuji and Nagano (1988) as a buoyant driven flow, and a turbulent flow with combined effects of thermal fields and adverse pressure gradients, the thermal backward facing step flow of Vogel and Eaton (1984). The results were compared with available experimental and numerical data available via the ERCOFTAC community.

5.1. Isothermal confined turbulent flow under adverse pressure gradient

In order to verify the performance of the velocity laws of the wall implemented on Turbo-2D in solving isothermal separating flows due to adverse pressure gradients, the test case chosen was the turbulent flow in the asymmetric plane diffuser of Buice and Eaton (1996), in which the boundary layer separation is not imposed by geometry, it occurs due to the presence of a smooth adverse pressure gradient.

The asymmetric plane diffuser geometry has an expansion rate equal to 4.7:1 with Reynolds number equal to 20000 based on the height of the inlet channel. The boundary conditions imposed consist of a fully developed turbulent channel flow in the inlet, experimental data for κ and ε, respectively, the turbulent kinetic energy and its dissipation rate, and ambient pressure at the outlet. The scheme of the case and the meshes used in the simulations are shown in figure (1).

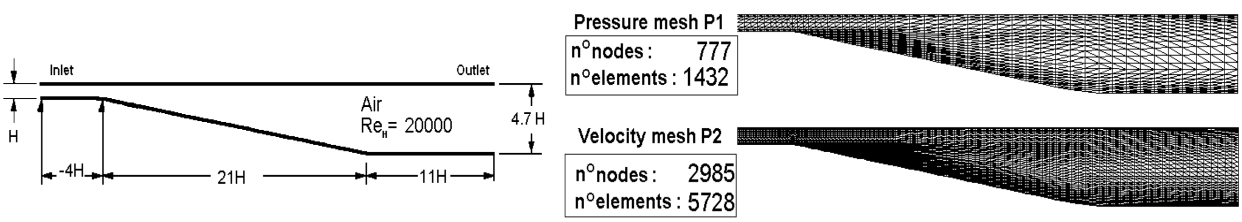

Figure 1: Isothermal test case: Buice & Eaton (1996) asymmetric plane diffuser

The results are compared with the experimental data and the results from the simulations of NASA's code WIND, that uses various methodologies and turbulent models. Some velocity and pressure coefficient profiles are shown in figures (2) and (3).

It is noted that the agreement of the results with experimental data increase with the use of more complex laws of the wall. This fact is proven by noting that only the wall laws of Nakayama and Koyama (1984) and Cruz and Silva Freire (1998) could predict the detachment of the boundary layer. Comparing these two, a slight advantage from Cruz and Silva Freire (1998) can be seen, for it reaches better values at the near wall and at the maximum velocity regions.

The profiles of the pressure coefficient C_p, figure (3), show that the wall laws of Nakayama and Koyama (1984) and Cruz and Silva Freire (1998) could predict more precisely the pressure field, and as a consequence, the detachment of the boundary layer.

The use of the laws of the wall of Nakayama and Koyama (1984) and Cruz and Silva Freire (1998) doubled the computational time of the simulations when compared to the use of the log law of the wall, although only with the use of these laws one could predict the boundary layer separation and more accurate results of the pressure distribution on the walls in the case of the diffuser.

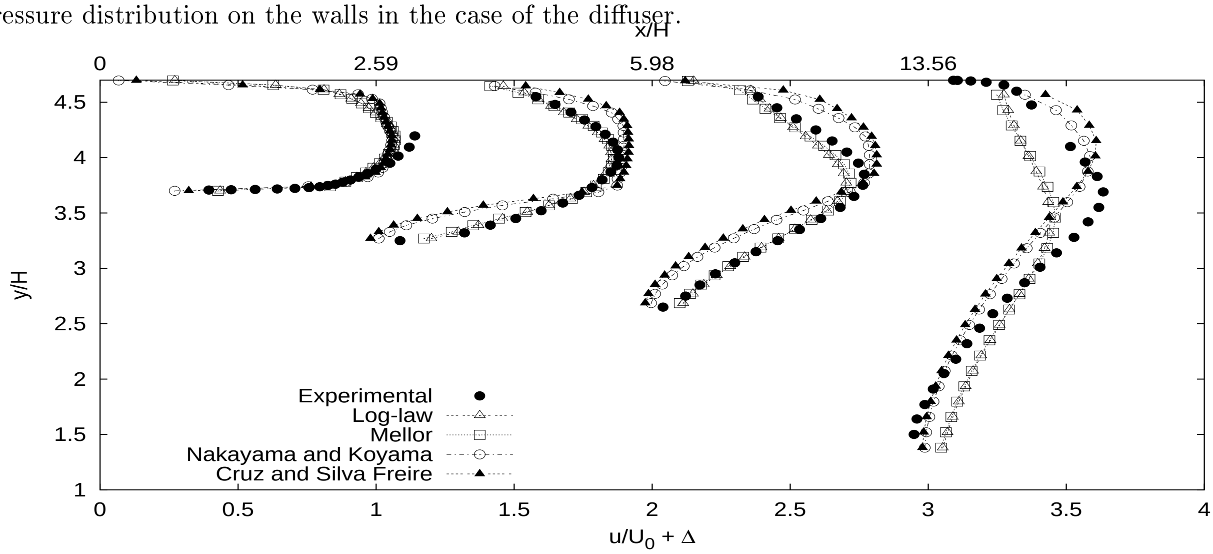

Figure 2: Velocity profiles in the asymmetric plane diffuser (Δ is a displacement factor of the graphic, all velocity values vary in a range of 0 to 1)

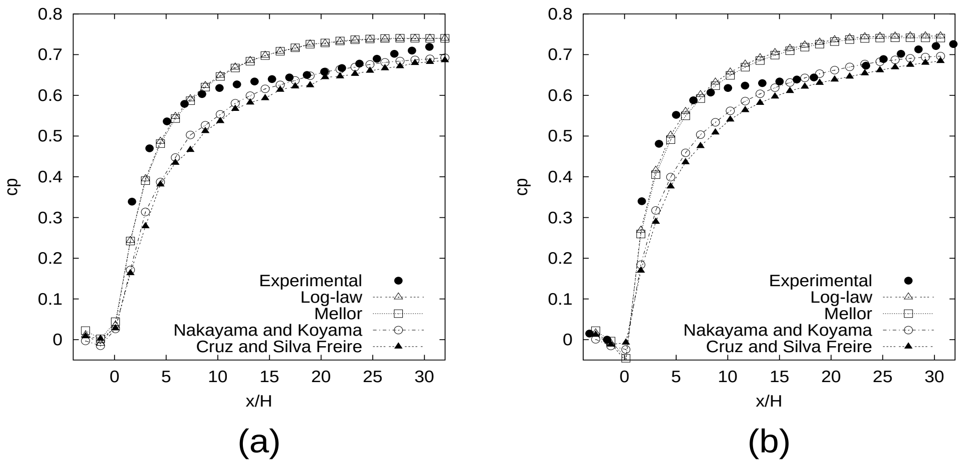

Figure 3: Dimensionless pressure coefficient at (a) upper wall and (b) lower wall

A comparison with results achieved by the code WIND of NASA's NPARC Alliance CFD Group can be seen at table (2), where x_d and x_r are respectively the location where the detachment and the reattachment of the recirculating zone occur, and H is the height of the inlet. The best results of Turbo 2D were achieved using the law of the wall of Cruz and Silva Freire (1998), and comparing to other low Reynolds models, the results from Turbo 2D are closer to the experimental value of the recirculating zone, although is presents early separation, which is a characteristic of the $\kappa - \varepsilon$ model.

Table 2: Results of recirculating zones length and of boundary layer detachment and reattachment locations

Code and turbulence model	**Detachment point ($\mathbf{x_d/H}$)**	**Reattachment point ($\mathbf{x_r/H}$)**	**Recirculating zone ($\mathbf{(x_r - x_d)/H}$)**
WIND Chien $\kappa - \varepsilon$	0,7583	20,943	20,185
WIND SST	1,9168	30,6793	28,76
WIND Spalart-Allmaras	4,0377	32,680	28,64
TURBO 2D - Nakayama and Koyama	2,04	22,2	20,16
TURBO 2D - Cruz and Silva Freire	1,06	24,1	23,04
Experimental	**6,00**	**29,5**	**23,5**

5.2. Thermal semi-confined turbulent flows

To verify the performance of the thermal laws of the wall implemented on Turbo 2D, two test cases were selected, the turbulent flow over a heated wall of Ng (1981), and the test case of the vertical turbulent natural convective boundary layer of Tsuji and Nagano (1988). The case of Ng (1981) has Reynolds number equal to 66460 based on the distance H showed in figure (4a), and the wall is heated at constant temperature of 1250K. The case of Tsuji and Nagano (1988) has a range of global Grashof number, based on the temperature difference between the wall and environmental ones, from $1.553 \text{x} 10^{10}$ to $1.797 \text{x} 10^{11}$, the wall and environmental temperatures are, respectively, $60^0 C$ and $15^0 C$, resulting in a Reynolds number of 7564 based on the height of the wall.

The boundary conditions for the test case of Ng (1981) were based on the experimental data, a fully developed isothermal turbulent flow in the inlet and environmental conditions at the other free boundaries. The boundary conditions for the test case of Tsuji and Nagano were based on experimental data, a experimental profile at the inlet of velocity, temperature, turbulent kinetic energy and its dissipation rate, velocity and temperature profiles at the lateral free boundary calculated from the conservation equations, and environmental properties at the outlet. The scheme of each case and the meshes used in the simulations are shown in figure (4).

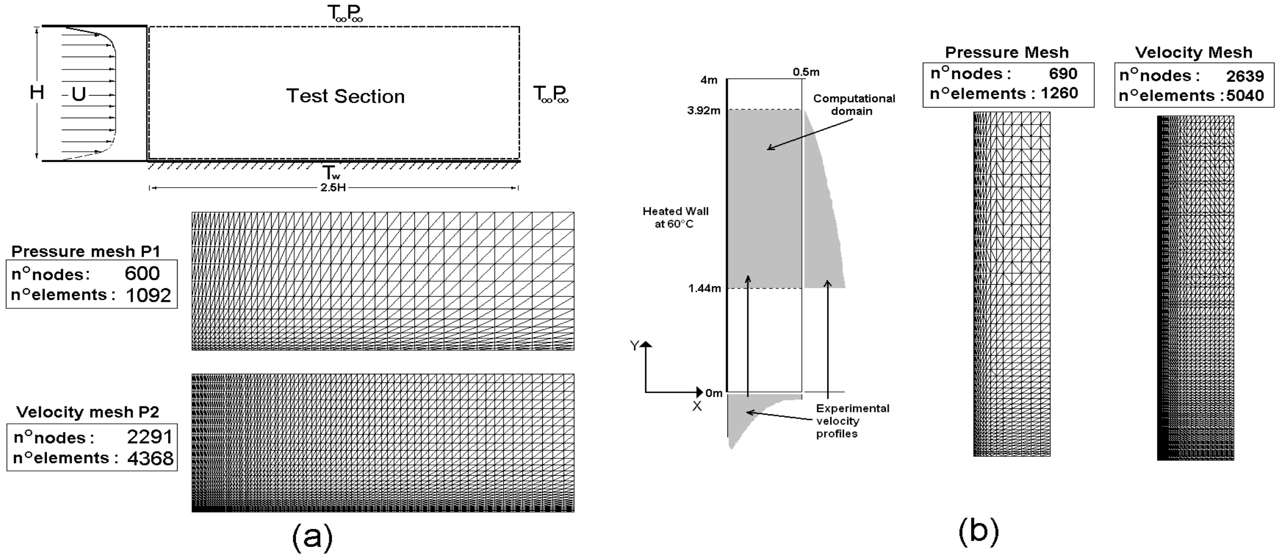

Figure 4: Thermal test cases: (a) Ng (1981) turbulent flow over a heated wall (b) Tusuji and Nagano (1988) vertical turbulent natural convective flow

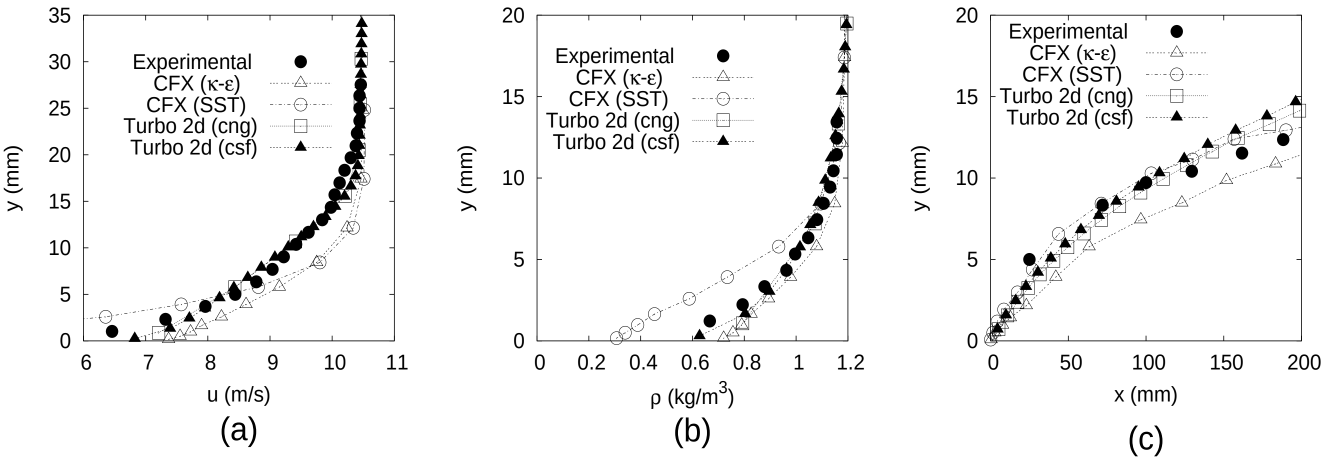

Figure 5: Turbulent heated flow of Ng (1981): (a) velocity and (b) density profiles at x=125mm and (c) thermal boundary layer thickness along the wall - (cng) means the Cheng and Ng (1982) temperature law of the wall, and (csf) means the Cruz and Silva Freire temperature law of the wall

Figures (5a) to (5c) show the results obtained from the simulations with Turbo 2D compared to the experimental data available for the case of Ng (1981), with the results obtained by Soares and Fontoura Rodrigues

(2003) with the industrial code CFX 5.0 using the $\kappa - \varepsilon$ and SST turbulence models, using the same mesh geometry for both codes.

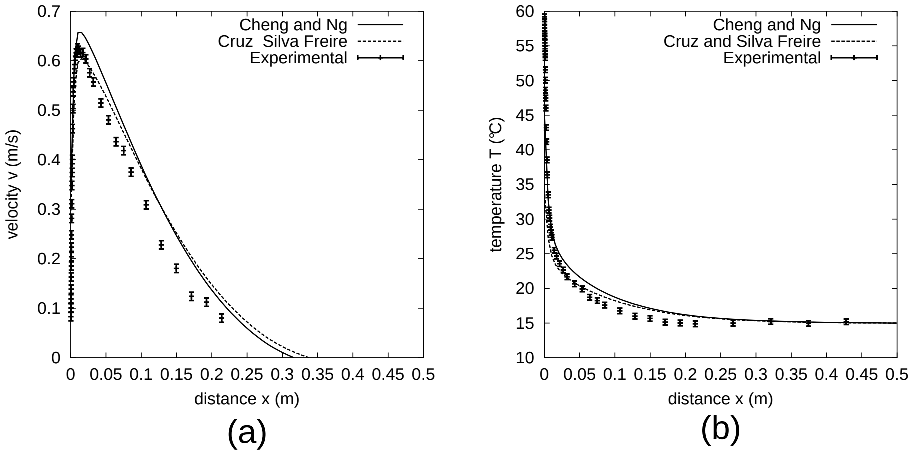

Figure 6: Turbulent natural convective flow of Tsuji and Nagano (1988): (a) velocity and (b) temperature profiles at y=2.535mm

The results of the case of Ng (1981) show that the results obtained with Turbo 2D are better than those obtained with CFX, and that the best density profile is obtained with the temperature law of the wall of Cruz and Silva Freire (1998). The prediction made by Cruz and Silva Freire (1998) that their proposed expression of the temperature law of the wall would tend to the log law approach is true, but a small difference can still be seen. Despite of this small difference, both showed good agreement with the experimental data. The CFX results were taken with the same mesh, which is under refined using the SST model, explaining the disagreement of this result from the experimental data. Also good agreement is seen at the Stanton number plot along the wall.

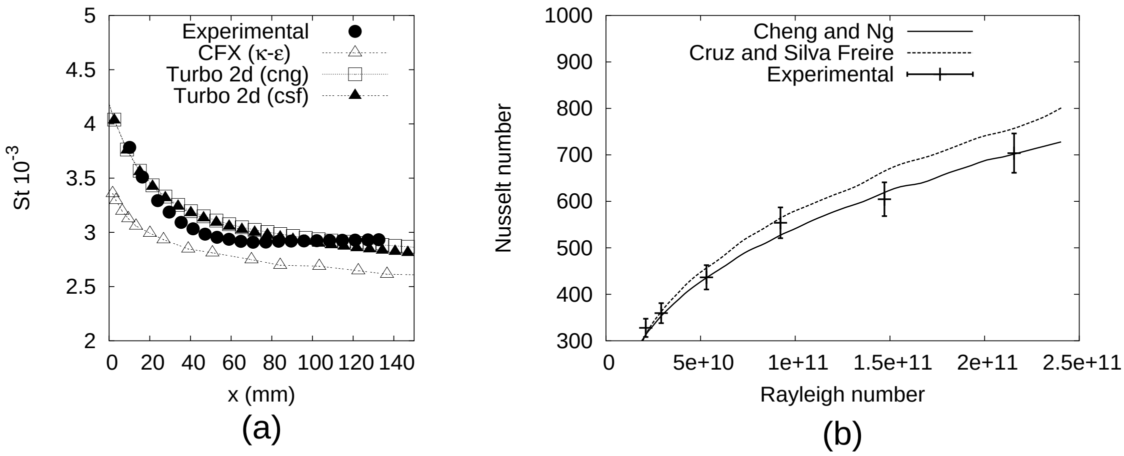

Figure 7: (a) Stanton number profile for the case of Ng (1981) and (b) Nusselt - Rayleigh number relation for the case of Tsuji and Nagano (1988)

The global Grashof number and the Stanton number are calculated with relations (26), where β is the thermal expansion coeficient, α is the thermal diffusivity coeficient and H is the wall height:

$$G_r = \frac{g\beta\Delta T H^3}{\nu^2} \quad and \quad S_t = \frac{\left(\frac{\Delta T}{\Delta y}\right)_{y=y^*}\left(\frac{\delta_T}{T_w - T_\infty}\right)}{\frac{u_f x}{\alpha}} . \tag{26}$$

The results of velocity and temperature profiles of the case of Tsuji and Nagano (1988), in figures (6a) and (6b) show good agreement with the experimental data, with advantage with the use of the thermal law of the wall of Cruz and Silva Freire (1998) when comparing these profiles in the near wall region. But when comparing the Nusselt number relation with the Rayleigh number, better agreement is obtained using the thermal log law of Cheng and Ng (1982).

5.3. Combined effects of temperature and adverse pressure gradients

This case was selected with the intention to test the influence of the presence of the thermal field in a recirculating zone, and how the thermal law of the wall is capable of predicting the temperature profiles inside the recirculating zone.

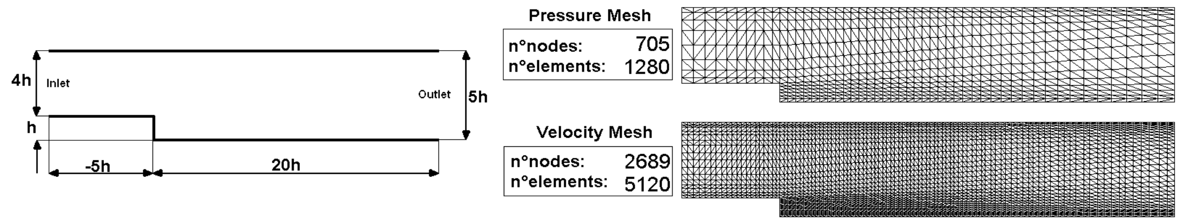

Figure 8: The thermal backward facing step of Vogel and Eaton (1984)

The Reynolds number of this case is 27.600, based on the reference velocity equal to 11.3 m/s and on the backward step height of 0,0381 m, both at the inlet. Figure (8) shows a diagram of the case and the finite element meshes used in the simulation. The heat flux is inserted is the domain through the bottom horizontal wall right after the step, and its value is 270 W/m^2. The boundary conditions used are of fully developed isothermal turbulent flow in the inlet, wall heating of $270 W/m^2$ on the heated wall and environmental pressure at the outlet.

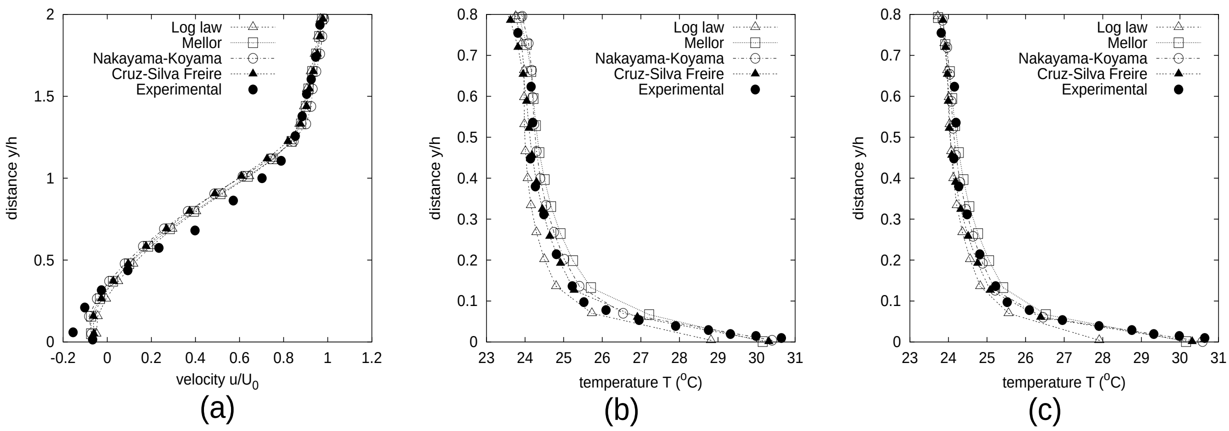

Figure 9: (a) Velocity profiles at x=4,66h; (b) Temperature profiles with each velocity wall function and the thermal log-law of Cheng and Ng (1982)(c) Temperature profiles with each velocity wall function and the thermal law by Cruz and Silva Freire (1998)

Table 3: Reattachment length for the thermal backward facing step of Vogel and Eaton (1984)

Model	Reattachment location **x/h**
TURBO 2D - Log	5,15
TURBO 2D - Mellor	5,65
TURBO 2D - Nakayama e Koyama	5,95
TURBO 2D - Cruz e Silva Freire	6,05
Experimental	**6,66**

In order to perform the test of the thermal law of the wall proposed by Cruz and Silva Freire (1998), several profiles were taken, as shown in figure (9). Figure (9a) shows the velocity profile at 4,66 step heights from the location of the backward facing step. These velocity profiles show a better agreement with the experimental data from the simulation with Cruz and Silva Freire (1998) wall function. It must be marked that once the convective forces are predominant over the buoyancy effects, there was no noticeable difference in the velocity field by changing the thermal wall function.

In figures (9b) and (9c) it is made clear the effects of the velocity field over the temperature profile, and it is noticeable that for the same velocity wall function used, there are two different temperature profiles, clearly not proportional one to the other. In other words, this fact can lead to a preliminary conclusion that the temperature profile can assume behaviors that differ from that one would estimate by using the Reynolds analogy. But this topic will be better investigated in further researches. Table (3) concludes the analysis presented in this work by showing that the reattachment length obtained wall function proposed by Cruz and Silva Freire (1998) is a little closer to the experimental data than the length obtained with the one proposed by Nakayama and Koyama (1984).

6. Conclusions

This analysis has shown that the velocity law of the wall proposed by Cruz and Silva Freire (1998) is very robust, as it provided the closer results from the experimental data of all cases analyzed in this work. It was able not only to predict the separation in the diffuser, but also to get the best result for the length of the recirculating zone of the diffuser, accompanied by the results obtained with the velocity law of the wall of Nakayama and Koyama (1984).

The expression proposed by Cruz and Silva Freire (1998) for the thermal law of the wall not only shows good results, but instigate further research on thermal boundary layers subjected to adverse pressure gradients and on turbulent flows driven by or affected by buoyancy forces. A more detailed investigation will be the topic of further works, associated with studies of models for the transport of temperature fluctuations and cases that can make evident weather this law of the wall can predict a different behavior to the thermal boundary layer. These results show that the use of wall laws is a reliable way of getting good results while avoiding the use of very refined meshes and heavy calculation.

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