Analytic and finite element models of a human long bone

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Introduction

A simple analytic model of stress analysis of external surface of a human long bone, like a femur, is proposed based in former works of authors, Kenedi and Riagusoff (2008), (2007). An elliptic cross section, with constant thickness is used to model a cross section of a medial human femur. Only cortical bone is recognized by the model that is loaded by a static force. Also, a finite element (F. E.) model is developed, using a well known F.E. commercial program, which is used as a reference to compare results of the simple analytic model.

Analytic model

A force acting at femur's head is shown as a red arrow at fig. 4.a. By equilibrium requirements loading components N, V_x, V_y, M_x, M_y and T can be determined at a cross section, as at Kenedi and Riagusoff (2008).

The axial stress σ_N is modeled as:

$$\sigma_N = \frac{N}{A}, \text{ where } A = \pi t(a+b-t)$$

Where, N is the force vertical component, t is the constant thickness bone wall, $2a$ and, $2b$, are respectively, the long and the short axis and A is the cross section area. Figures 1 and 2 shows x and y axis, which are respectively, coincident with 2a and 2b. The z axis is obtained by the application of the *right-hand rule*. Each section has its own local axis configuration, always maintaining x axis coincident with 2a.

The bending stresses components, σ_{F_x} and σ_{F_y}, are:

$$\sigma_{F_x} = \frac{M_x y_f}{I_x} \quad \text{and} \quad \sigma_{F_y} = \frac{M_y(-x_f)}{I_y}$$

$$x_f = r_e(\gamma)\cos(\gamma) \text{ and } y_f = r_e(\gamma)\sin(\gamma)$$

$$I_x = \frac{\pi}{4}\left(ab^3 - (a-t)(b-t)^3\right)$$

$$I_y = \frac{\pi}{4}\left(a^3 b - (a-t)^3(b-t)\right)$$

$$r_e(\gamma) = \sqrt{a^2\cos^2(\gamma) + b^2\sin^2(\gamma)}$$

Where, M_x and M_y are bending moments components, x_f and y_f are, perpendicular distances from neutral axis to external bone surface, I_x and I_y are second moment of area, r_e is the distance from the centre of cross section to the point of interest and γ is the angle between positive x axis and the point of interest.

Fig. 1 shows the bending variables:

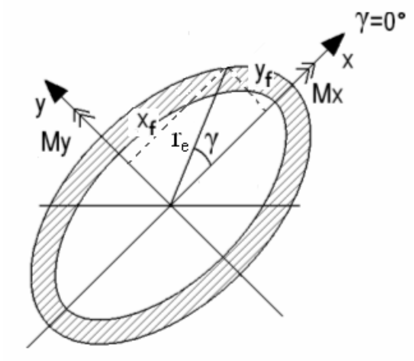

Figure 1: Bending variables.

The torsional stress τ_T is:

$$\tau_T = \frac{T}{2t\mathcal{a}}, \text{ where } \mathcal{a} = \pi\left(a - \frac{t}{2}\right)\left(b - \frac{t}{2}\right)$$

Where, T is the torsional moment and $\mathcal{a}$ is the area inside a line which passes in middle thickness of bone cross section. The transverse shear stress components τ_{v_x} and τ_{v_x}, are:

$$\tau_{v_x} = \frac{V_x Q_y}{I_y t_y} \quad \text{and} \quad \tau_{v_y} = \frac{V_y Q_x}{I_x t_x}$$

where,

$$t_x(\gamma) = 2a\sqrt{1 - \left(\frac{y_f(\gamma)}{b}\right)^2} - \left[2(a-t)\sqrt{1 - \left(\frac{y_f(\gamma)}{b-t}\right)^2}\right]k_x$$

$$t_y(\gamma) = 2b\sqrt{1 - \left(\frac{x_f(\gamma)}{a}\right)^2} - \left[2(b-t)\sqrt{1 - \left(\frac{x_f(\gamma)}{a-t}\right)^2}\right]k_y$$

$$Q_x = \int_{y_f}^{b} 2ay\sqrt{1 - \left(\frac{y_f(\gamma)}{b}\right)^2}\,dy - \int_{r_i(\gamma)\sin(\gamma)}^{b-t} 2(a-t)y\sqrt{1 - \left(\frac{r_i(\gamma)\sin(\gamma)}{b-t}\right)^2}\,dy$$

$$Q_y = \int_{x_f}^{a} 2bx\sqrt{1 - \left(\frac{x_f(\gamma)}{a}\right)^2}\,dx - \int_{r_i(\gamma)\cos(\gamma)}^{a-t} 2(b-t)x\sqrt{1 - \left(\frac{r_i(\gamma)\cos(\gamma)}{a-t}\right)^2}\,dx$$

$$r_i(\gamma) = \sqrt{(a-t)^2\cos^2(\gamma) + (b-t)^2\sin^2(\gamma)}$$

where, V_x and V_y are shear force components, k_x =0 for $\left|y_f(\gamma)\right| \geq (b-t)$, k_x = 1 otherwise, and k_y = 0 for $\left|x_f(\gamma)\right| \geq (a-t)$, k_y = 1 otherwise.

Fig. 2 shows the transverse shear variables:

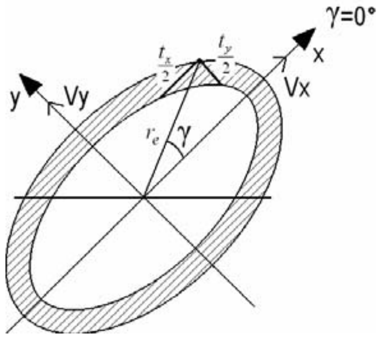

Figure 2: Transverse shear variables.

The total normal and shear stresses components are:

$\sigma_y = \sigma_N + \sigma_F$ and $\tau_{xy} = \tau_T + \tau_v$

where, $\sigma_F = \sigma_{F_x} + \sigma_{F_y}$ and $\tau_v = \tau_{v_x} + \tau_{v_y}$.

Finally, using Mohr circle approach is possible to determine the principal stresses:

$$\sigma_1, \sigma_3 = \frac{\sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Finite element model

A commercial package of F.E., the ANSYS Workbench 11.0, was used to do an elastic analysis of a human femur. A simple model was implemented with Solid 186/187 structural elements types with tetrahedron, hexahedron, wedge and pyramid shapes. The geometry and loading data was provided by Bergmann et al. (2001). Figure 3 shows details of finite element mesh model of a human femur.

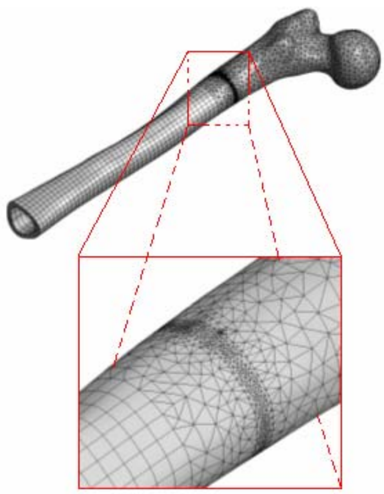

Figure 3: Details of finite element mesh model of a human femur.

Results

Figure 4 shows the concentrated load at femur's head and an example of F.E. output results at the cross section of the femur. Note at fig.4b the results are shown in a circular pattern, were the mesh is refined, to generate enough nodes at external surface of the cross section to provide sufficient output data.

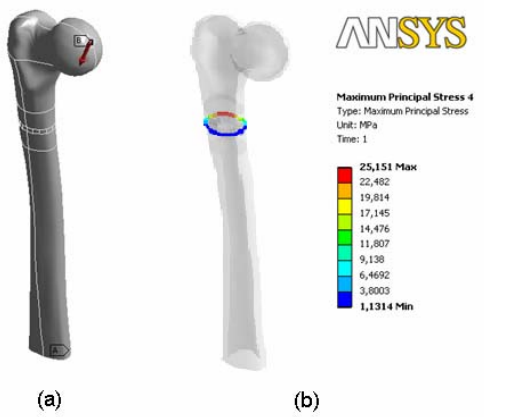

Figure 4: (a) Loading and (b) F.E. results – maximum principal stress.

Figure 5 shows good agreement between analytic and F.E. models.

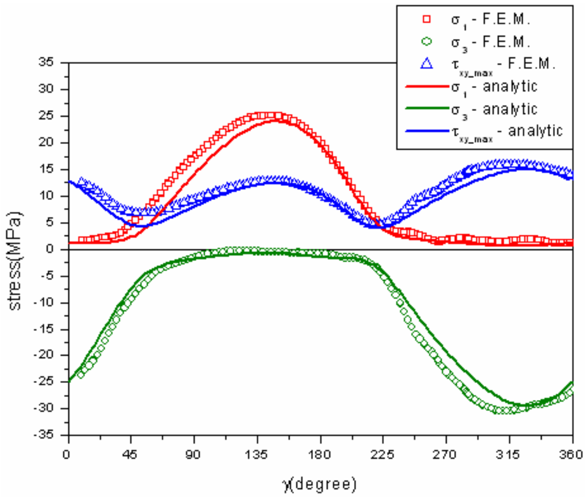

Figure 5: Comparative diagram, between analytic and F.E. approaches.

Conclusions

The analytic and F.E. models show good agreement, the little differences between results of models are mainly caused by real bone thickness be only fairly constant and cross section shape be only rough elliptic. Nevertheless, the major goal of this model is to provide an explicit way of estimating principal and maximum shear stresses at external surfaces of long bones, which can be used as input variables to a failure criterion.

References

Kenedi, P.P., Riagusoff, I.I.T. (2008), "A Model of Stress Analysis of a Human Long Bone", In: *Anais do 21° Congresso Brasileiro de Engenharia Biomédica*, Salvador, 16-20 Nov.

Kenedi, P.P., Riagusoff, I.I.T. (2007), "Modeling initial fracture patterns in human long bones", In: *Anais do 1° Encontro Nacional de Engenharia Biomecânica*, Petrópolis, Rio de Janeiro, 23-25

Bergmann, G., Deuretzbacher, G. Heller, M., Graichen, F., Rohlmann, A., Strauss, J. and Duda, G. N., (2001), "Hip contact forces and gait patterns from routine activities", J. Biomech., Vol. 34, pp.859-871.