DYNAMIC STABILITY OF ROTOR-BEARING SYSTEMS

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Abstract-A rotating shaft supported by two hydrodynamic bearings is composed by two basic sub-system: (1) A flexible axis acted by an unbalanced rotating force and (2) Spring-damper supports which interacts with the shaft motion, represented by a hydrodynamic bearing. The forces produced by the dynamic pressure of the lubricant are obtained by the solution of the Reynolds equation for the fluid-film. Parameters associated to the speed of the axis and to the physical and geometric characteristics of the bearing, such as viscosity of the fluid, radial clearance, length and diameter of the bearing, are considered in the evaluation of the dynamic behavior of the axis. The matrix equations of motion for the axis are characterized by the occurrence of a mass and a stiffness matrix. The Method of the Finite Elements can be applied for derivation of those equations. The movement of the axis is described by four degrees of freedom for each node. For obtaining the time response of the vibrating axis, Newmark procedure has been employed for integrating the motion equations. The dynamic pressure produced by the fluid motion is integrated along the circumferential direction of the bearing and the resulting forces are introduced in the force vector of the matrix equation of the axis. An iterative procedure has to be implemented, for the fluid forces have a dependency on the displacement of the axis at the supports position. A preliminary model considers elastic and damping forces acting on the shaft at the bearings positions. Numeric solutions are obtained to describe the conditions of dynamic stability of the rotor-bearing system.

Keywords- Numerical Methods, Mechanical Vibrations, Hydrodynamic Bearings

Nomenclature

c	radial clearance (housing radius - journal radius)
$[C]$	damping matrix
h	oil film thickness
L	bearing land length
t	time
g	gravitational acceleration
m	rotor mass (per bearing land)
$[M]$	mass matrix
$[K]$	stiffness matrix
r_a	bearing radius
r_b	journal radius
F_r	fluid film force in radial direction ($F_r = w_z' L$)
F_t	fluid film force in tangential direction ($F_t = w_y' L$
F_u	unbalance force

B	bearing parameter $(12\eta L r_b\, r_a^2/c^2)$
W	rotor weight (per bearing land)
Q	force vector
w'_y	load component per unit width perpendicular to line of centers
w'_z	load component per unit width along line of centers
(')	denotes differentiation with respect to ωt
(.)	denotes differentiation with respect to t

Greek Symbols

e	eccentricity between journal center and housing center
η	absolute viscosity
ρ	material density
ε	eccentricity ratio (e/c)
Φ	attitude angle
ω	rotational velocity of journal about sleeve center when eccentricity ratio is constant (rad/s)
ω_a	bearing angular speed of surface bearing (rad/s)
ω_b	rotor angular speed of surface journal (rad/s)
ϕ	angular distance from the positive y-axis in the fixed y-z coordinates set
ϕ_m	upper limit of the positive pressure

Subscripts

a	relative to bearing
b	relative to rotor
m	relative to upper limit of the positive pressure
r	relative to fluid film force in radial direction
t	relative to fluid film force in tangential direction
u	relative to unbalance force
y	relative to perpendicular to line of centers
z	relative to along line of centers

1. INTRODUCTION

Vibration considerations about parts revolving in a machine are not complete if one does not consider the elastic and damping effects of a present bearing. Most comonly a rotating shaft is analysed under vibration criterions disregarding gyroscopic effects, rotating inertia and the bearings contributions to the system. A full model considering all real actions is troblesome and requires a adequate analysis of the pertinent sub-systems. Hydrodynamic bearings based on the solution of Reynods's equation have been studied extensively in order to furnish accurate discription of the forces produced by the fluid motion in the bearing (Cookson-Kossa, 1979). The main purpose of the present study is to carry out indroductory considerations about the effect of damping and elastic forces at the support positions of a rotor.

Rotor vibrations are strongly dependent on the rotor geometry, on the bearings type and on the excitation forces type. Usually the performances of mechanical systems are improved by methods, which lead to an increase of the dynamic stiffness. Rotating shafts perform a motion with a frequency different than the natural frequency of the non-rotating shaft, and the first critical speed is different from the first natural frequency. The subject is most interesting in the design of aeronautic turbines shafts.

In this work the Finite Element Method is used, adopting beam elements with two nodal points and four degree of freedom for each node, two displacements and two rotations. The motion equations are obtained from Lagrange's equations, and describe the motion in two transverse planes. As a result, for each element, eight differential equations have to be accounted for. To obtain the mass and stiffness matrix, the software MATHEMATICA® was used for the integrations of the kinetic and potential energy, and for the differentiation required by Lagrange's equations. A MATLAB® computer code was written, to assembly the shaft element matrixes into global matrixes, and simulates the shaft response, for several dimensions, materials, angular velocities and external unbalanced loads. To obtain the time domain response, the integration procedure of Newmark was adopted (Sacramento-Menezes, 2003).

2. MODEL OF A ROTOR-BEARING SYSTEM

2.1 Formulation of the Equations of Motion

The model consists of a rotor, treated as elastic and continuous, to which a rigid disk is coupled. Additionally, at support positions, hydrodynamic bearing can be considered. Presently, gyroscopic has not been taken into account. The equations of the hydrodynamic forces produced by the oil film, due to pressure increase promoted by the fluid motion, may be formulated and solved numerically.

Coordinate systems adopted are depicted in Fig 1b, where XYZ is the fixed frame, and *xyz* the rotating frame. The rotor revolves about the x-axis with an angular velocity ω counter-clockwise. The oil film forces in the x and y direction, F_r and F_t, are obtained from the oil pressure distribution, which is obtained from the Reynolds equation. The general Reynolds equation governing the flow of the squeeze film damper is well known as (Hamrock, 1994 and Bode-Menezes, 2002):

$$\frac{\partial}{\partial x}\left(-\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(-\frac{\rho h^3}{12\eta}\frac{\partial p}{\partial y}\right)+\frac{\partial}{\partial x}\left[\frac{\rho h(u_a+u_b)}{2}\right]+\frac{\partial}{\partial y}\left[\frac{\rho h(v_a+v_b)}{2}\right]+$$
$$+\rho(w_a+w_b)-\rho u_a\frac{\partial h}{\partial x}-\rho v_a\frac{\partial h}{\partial y}+h\frac{\partial p}{\partial t}=0 \tag{1}$$

In Eq. 1, the following assumptions were made: (i) the fluid inertia terms in Navier-Stokes equations have been neglected due to their small magnitude; (ii) the flow of lubricant is laminar; (iii) the fluid is Newtonian; (iv) no slip exists at the fluid-solid interface; (v) the flow in the radial direction has been neglected. Thus the flow of lubricant is two-dimensional; (vi) the inclination of one surface relative to the other is so small that the sine of the angle of inclination can be set equal to the angle and the cosine can be set equal to unity. The general Reynolds equation given in Eq. (1) can be applied to any section of the oil film and in this paper only the dynamically loaded infinitely wide-journal-bearing solution will be presented. The film thickness can be described as (Hamrock, 1994, Dubois-Ocvick, 1953):

$$h=c(1+\varepsilon\cos\phi) \tag{2}$$

If the side-leakage term is neglected, Eq. (1) can be rewritten and integrated while making use of Eq. (2) which gives:

$$\frac{\partial p}{\partial \phi}=\frac{12\eta\left(\frac{r_a}{c}\right)^2\left[\frac{\partial\varepsilon}{\partial t}\operatorname{sen}\phi-\varepsilon\cos\phi\left(\omega-\frac{\omega_a+\omega_b}{2}\right)-\frac{3\varepsilon^2}{2+\varepsilon^2}\left(\omega-\frac{\omega_a+\omega_b}{2}\right)\right]}{(1+\varepsilon\cos\phi)^3} \tag{3}$$

Once the pressure is known, the load components can be evaluated. One may determine the components of the resultant load along and perpendicular to the line of centers, as:

$$w'_y = r_b \int_0^{\phi_m} \cos\phi \frac{dp}{d\phi} d\phi \tag{4}$$

$$w'_z = r_b \int_0^{\phi_m} \operatorname{sen}\phi \frac{dp}{d\phi} d\phi \tag{5}$$

Replacing Eqs. (3), (4) and (5) for to $\overline{F_r}$ and $\overline{F_t}$ gives the oil film forces of the squeeze film damper about the damper centre in the radial and tangential directions, which are obtained by integrating the oil film pressure distribution along and normal to the line of the centers of the journal over the positive pressure values region as (Bode-Menezes, 2003)

$$F_r = B\int_0^{\phi_m} \frac{\left[\frac{\partial\varepsilon}{\partial t}\sin^2\phi - \varepsilon\sin\phi\cos\phi\left(\omega - \frac{\omega_a - \omega_b}{2}\right) - \frac{3\varepsilon^2}{2+\varepsilon^2}\left(\omega - \frac{\omega_a - \omega_b}{2}\right)\sin\phi\right]d\phi}{\left(1+\varepsilon\cos\phi\right)^3} \tag{6}$$

$$F_t = B\int_0^{\phi_m} \frac{\left[\frac{\partial\varepsilon}{\partial t}\sin\phi\cos\phi - \varepsilon\cos^2\phi\left(\omega - \frac{\omega_a - \omega_b}{2}\right) - \frac{3\varepsilon^2}{2+\varepsilon^2}\left(\omega - \frac{\omega_a - \omega_b}{2}\right)\cos\phi\right]d\phi}{\left(1+\varepsilon\cos\phi\right)^3} \tag{7}$$

To obtain the matrix equation of motion for the shaft, Lagrange equation was employed according to (Meirovitch, 1997):

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = \left\{Q_j\right\} \qquad j=1,\ 2,\ n \tag{8}$$

where n is the number of degrees of freedom of the system, q_j are the generalized coordinates, $\left\{Q_j\right\}$ are the non-conservative forces, and L is the Lagrangian defined by:

$$L = T - V \tag{9}$$

where T is the element's kinetic energy and V is the element's potential energy.

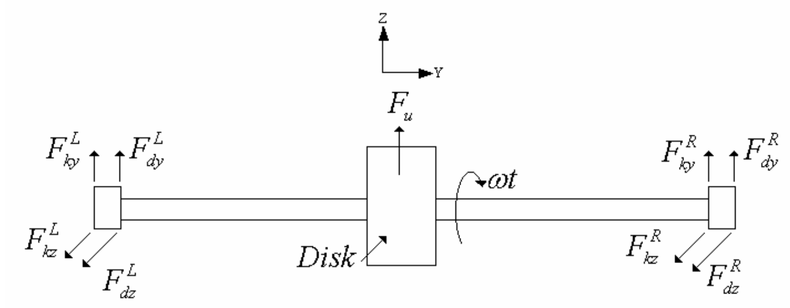


Figure 1 - Coordinate systems of the rotor-bearing system

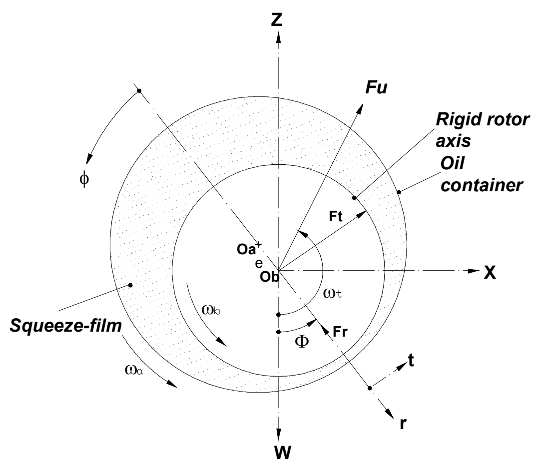


Figure 2 - Squeeze-film damper with the dynamic forces and coordinates defined.

Employing the Finite Element Method one may obtain the solution for the proposed problem. This method considers the structure, in this case a rotating shaft, as an assembly of finite elements. Any displacement component along the element can be depicted as nodal values interpolations, according to the following expression (Bathe, 1996):

$$q(x,t) = \sum_{i=1}^{n} N_i(x) q_i(t) \tag{13}$$

For the proposed problem, the vector of displacement components, illustrated in Fig. 2, is expressed as

$$\{q\} = \left[u_y^1 \gamma^1 u_z^1 \beta^1 u_y^2 \gamma^2 u_z^2 \beta^2 \right]^T \tag{14}$$

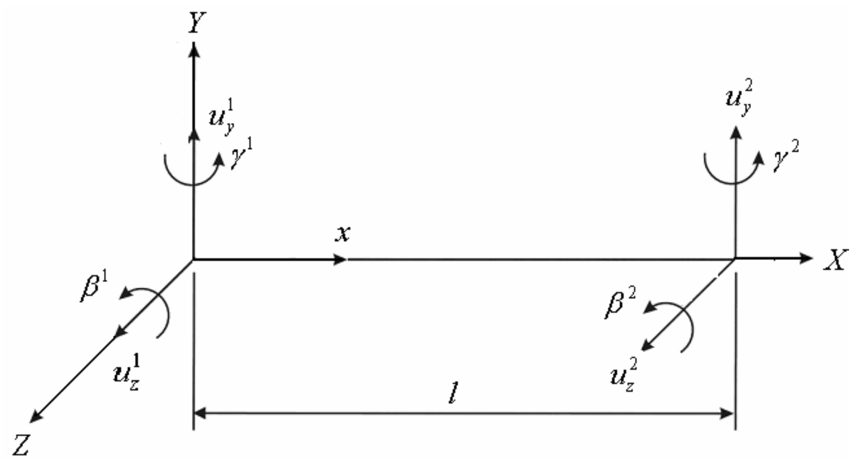


Figure 3. Nodal displacements for a two nodded beam finite element

According to Eq. (13), the displacement components of Eq. (14) can be expanded as:

$$u_z(x,t) = N_z^1 u_z^i(t) + N_z^2 \beta^i(t) + N_z^3 u_z^j(t) + N_z^4 \beta^j(t) \tag{15}$$

$$\beta(x,t) = N_\beta^1 u_z^i(t) + N_\beta^2 \beta^i(t) + N_\beta^3 u_z^j(t) + N_\beta^4 \beta^j(t) \tag{16}$$

$$u_y(x,t) = N_y^1 u_y^i(t) + N_y^2 \gamma^i(t) + N_y^3 u_y^j(t) + N_y^4 \gamma^j(t) \tag{17}$$

$$\gamma(x,t) = N_\gamma^1 u_y^i(t) + N_\gamma^2 \beta^i(t) + N_\gamma^3 u_y^j(t) + N_\gamma^4 \gamma^j(t) \tag{18}$$

A MATLAB® computer code was written, to assembly the shaft element matrixes into global matrixes, and simulates the shaft response, for several dimensions, materials angular frequency and loads. To obtain the time domain response, the integration procedure of Newmark method was adopted (Rao, 1995).

For the use of Lagrange's equations, one has to express the kinetic and potential energies of the element. The elemental kinetic energy consists of two translational terms related to y and z motion, and accordingly, the potential energy of the element takes into account the bending in both planes. Then:

$$T = T_t^y + T_t^z \tag{19}$$

$$T_t^y = \frac{1}{2}\int_0^l m(x)\left(\frac{\partial u_y(x,t)}{\partial t}\right)^2 dx \tag{20}$$

$$T_t^z = \frac{1}{2}\int_0^l m(x)\left(\frac{\partial u_z(x,t)}{\partial t}\right)^2 dx \tag{21}$$

$$V = V_t^y + V_t^z \tag{22}$$

$$V_t^y = \frac{1}{2}\int_0^l EI(x)\left(\frac{\partial u_y(x,t)}{\partial x}\right)^2 dx \tag{23}$$

$$V_t^z = \frac{1}{2}\int_0^l EI(x)\left(\frac{\partial u_z(x,t)}{\partial x}\right)^2 dx \tag{24}$$

where E is the elasticity modulus and I is the moment of inertia of area.

Replacing Eqs. (15), (16), (17) and (18) into Eqs. (20), (21), (23) and (24), one may obtain the total kinetic and potential energies for the element, according to Eqs. (19) and (22) expressed as a function of the nodal displacements $q_i\ (t)$. According to Lagrange's equations, the motion equations are obtained by the direct integration of the kinetic and potential energy equations, followed by differentiations with respect to each nodal displacement. The set of eight equations for each element can be written in matrix form as:

$$[M]\cdot\{\ddot{q}\} + [C]\cdot\{\dot{q}\} + [K]\cdot\{q\} = \{Q\} \tag{25}$$

2.2. Integration of the Equations of Motion

In 1959 Newmark presented a family of single-step integration methods for the solution of structural dynamic problems. During the past 40 years Newmark´s method has been applied to the dynamic analysis of many practical engineering structures. In addition, it has been modified and improved by many other researches. In order to illustrate the use of this family of numerical integration methods one may consider the solution of the linear dynamic equilibrium equations written in the following form:

$$[M]\{\ddot{q}_{n+1}\}+[C]\{\dot{q}_{n+1}\}+[K]\{q_{n+1}\}=\{Q_{n+1}\} \tag{27}$$

The most basic self-starting method is simply a Taylor Series Expansion truncated after some arbitrary number of terms. By truncating the series, which is known as Newmark's Method (Rao, 1995) one may obtain:

$$\{q_{n+1}\}=\{q_n\}+\Delta t\{\dot{q}_n\}+\frac{(\Delta t)^2}{4}\{\ddot{q}_n\}+\frac{(\Delta t)^2}{4}\{\ddot{q}_{n+1}\} \tag{28}$$

$$\{\dot{q}_{n+1}\}=\{\dot{q}_n\}+\frac{\Delta t}{2}\{\ddot{q}_n\}+\frac{\Delta t}{2}\{\ddot{q}_{n+1}\} \tag{29}$$

Equation (28) can be used to express $\{\ddot{q}_{n+1}\}$ in terms of $\{q_{n+1}\}$, and the resulting expression can be substituted into Eq. (29) to express $\{\dot{q}_{n+1}\}$ in terms of $\{q_{n+1}\}$. By substituting these expressions for $\{\dot{q}_{n+1}\}$ and $\{\ddot{q}_{n+1}\}$ into Eq. (27), we can obtain an expression for obtaining $\{q_{n+1}\}$:

$$\left(\frac{4}{(\Delta t)^2}[M]+\frac{2}{\Delta t}[C]+[K]\right)\{q_{n+1}\}=$$

$$=\{F_{n+1}\}+[M]\left(\frac{4}{(\Delta t)^2}\{q_n\}+\frac{4}{\Delta t}\{\dot{q}_n\}+\{\ddot{q}_n\}\right)+[C]\left(\frac{2}{\Delta t}\{q_n\}+\{\dot{q}_n\}\right) \tag{30}$$

The previous equation can be solved for $\{q_{n+1}\}$, as a linear system of equations, and the nodal displacements of the vibrating problem can be obtained for any time. Employing Eqs. (28) and (29), nodal velocities and accelerations are obtained for the present time as a function of the present displacement, and previous velocities and accelerations

3. RESULTS

A computer code, based on Newmark approach, according to Eq. (30), has been written. An interactive routine had to be created to get convergence at each time step. For numerical analysis purposes, the following parameters of a simply supported rotating shaft were adopted:

D_s	Diameter of shaft	0.02m
D_d	Diameter of disk	0.2m
I_s	Shaft area moment of inertia	7.854e-9m^4
I_d	Disk area moment of inertia	7.854e-5m^4
L_s	Length of the shaft	1m
ρ	Material density	7800kg/m^3
E	Elastic Young's modulus of the rotor shaft material	2.07e11Pa
Number of finite elements of the shaft		35
Number of the element where a stiff disk may be included		18
Number of the element where for an unbalanced force		18

One may conclude that for the values and boundary conditions adopted, the first natural frequency of the non-rotating beam is 40.4602(Hz). For the results presented, the computational time for the estimator Δt=3e-3s was considered. At the support positions, elastic and damping concentrated forces were considered, as illustrated in Fig. 1. For this purpose the elastic and damping forces, which simulates the presence of a bearing, were considered proportional do the node displacement and velocity, according to the following general equation:

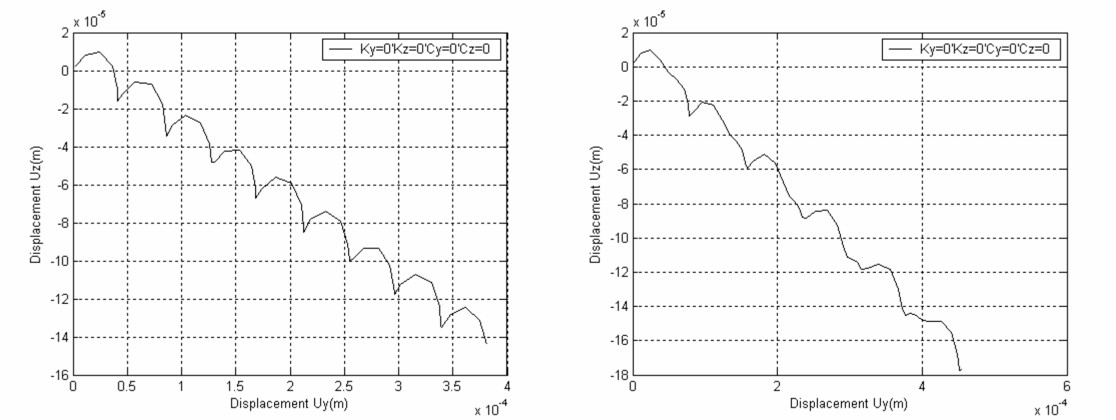
$$F_k^y = k_y u_y \tag{31}$$

$$F_k^z = K_z u_z \tag{32}$$

$$F_d^y = C_y \dot{u}_y \tag{33}$$

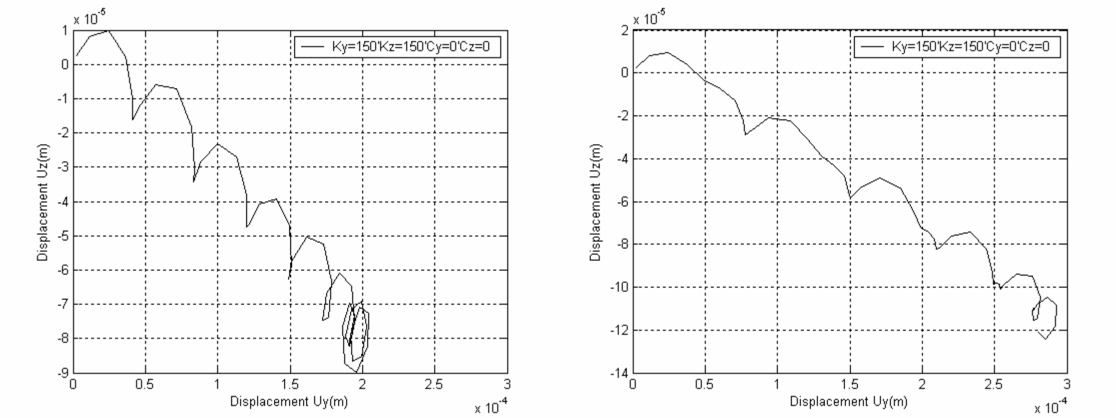
$$F_d^z = C_z \dot{u}_z \tag{34}$$

Computational time for the estimator Δt=3e-3s



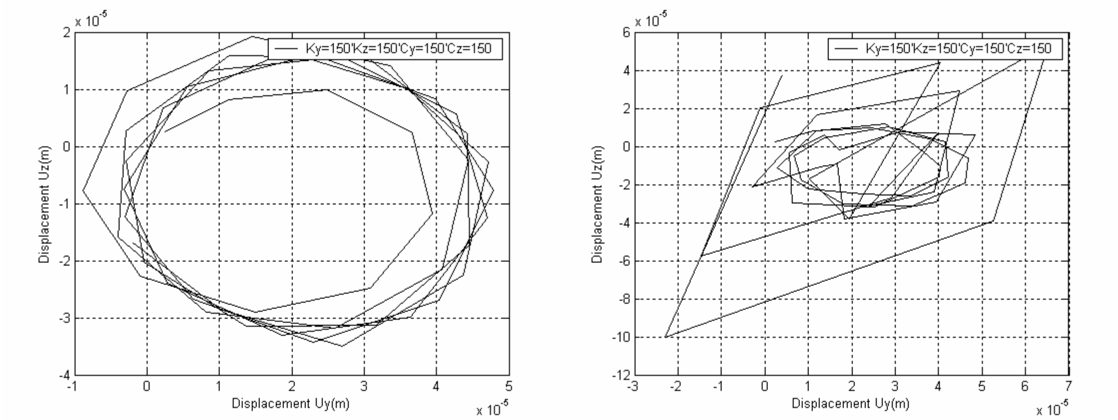
(a) (b)

Figure 4. Effect of bearing parameters (K_y=0, K_z=0, C_y=0, C_z=0) on the rotor response for unbalance force (F_u=1N); (a) shaft having a disk attached and (b) uniform shaft.



(a) (b)

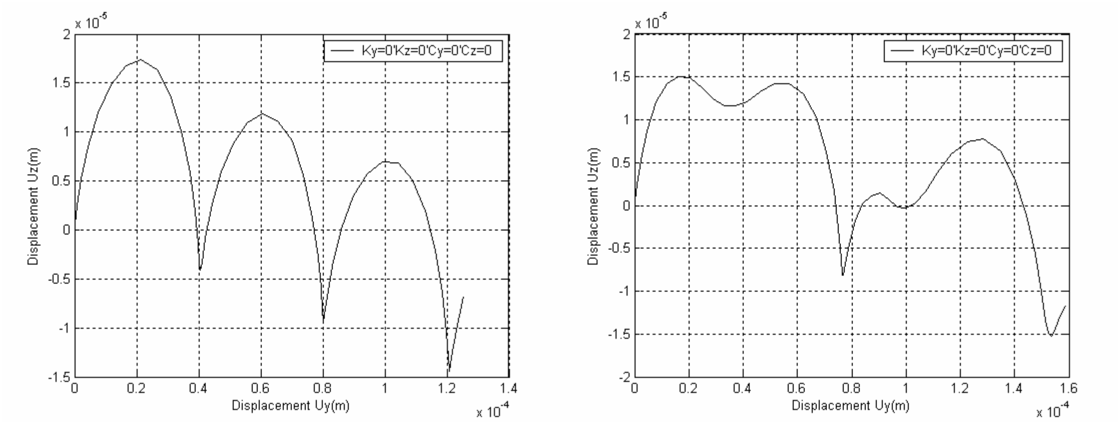
Figure 5. Effect of bearing parameters (K_y=150, K_z=150, C_y=0, C_z=0) on the rotor response for unbalance force (F_u=1N); (a) shaft having a disk attached and (b) uniform shaft.



(a) (b)

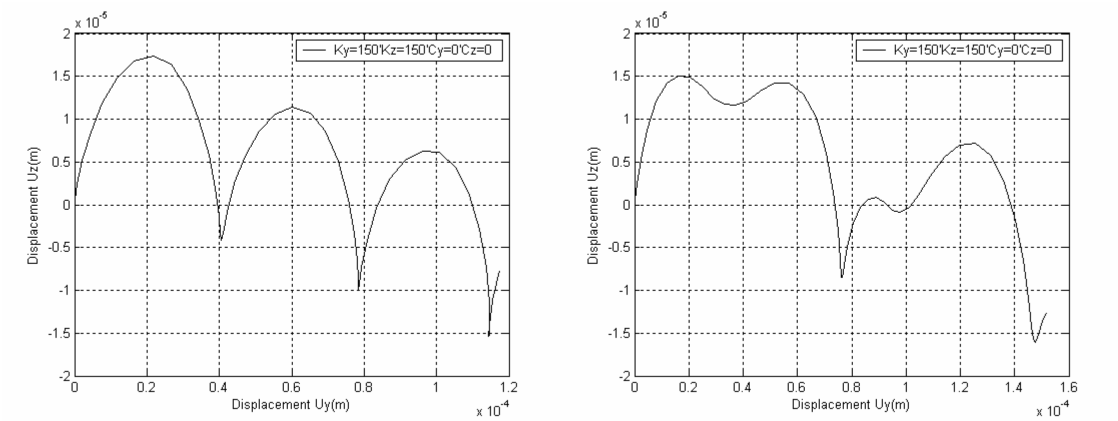
Figure 6. Effect of bearing parameters (K_y=150, K_z=150, C_y=150, C_z=150) on the rotor response for unbalance force (F_u=1N); (a) shaft having a disk attached and (b) uniform shaft.

Computational time for the estimator Δt=1e-3s



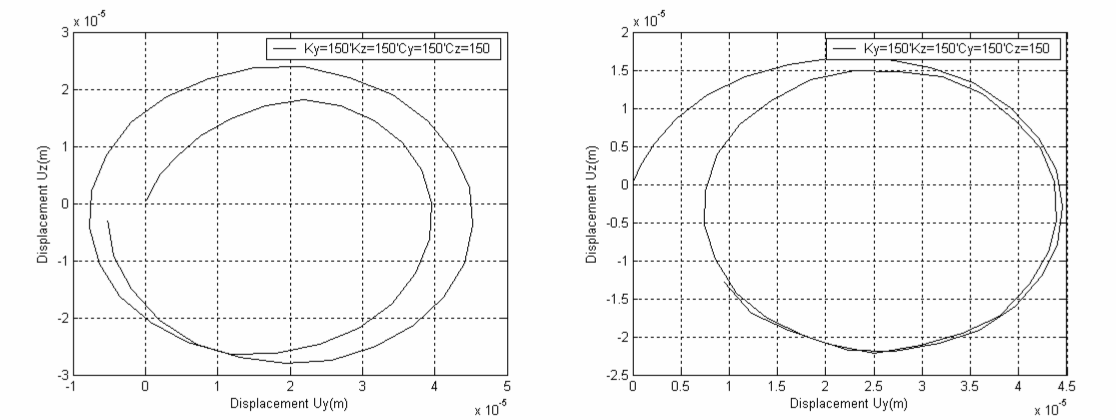
(a) (b)

Figure 7. Effect of bearing parameters (K_y=0, K_z=0, C_y=0, C_z=0) on the rotor response for unbalance force (F_u=1N); (a) shaft having a disk attached and (b) uniform shaft.



(a) (b)

Figure 8. Effect of bearing parameters (K_y=150, K_z=150, C_y=0, C_z=0) on the rotor response for unbalance force (F_u=1N); (a) shaft having a disk attached and (b) uniform shaft.



(a) (b)

Figure 9. Effect of bearing parameters ($K_y=150,\ K_z=150,\ C_y=150,\ C_z=150$) on the rotor response for unbalance force ($F_u=1N$); (a) shaft having a disk attached and (b) uniform shaft.

4. CONCLUSION

A computer code written in MATLAB® language allows the study of the design parameters of bearings in a rotating shaft submitted to an unbalance force. Preliminary results show a considerable sensitivity to the support parameters in a rotating shaft vibration problem. The proposed program also allows the inclusion of a stiff disk to the problem, not considering the rotating inertia effect. Such study is introductory to a more complex model which may include the actual hydrodynamic bearing forces, gyroscopic effects of the rotating shaft and rotating inertia of stiff and heavy disk coupled to the shaft.

5. REFERENCES

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