NONLINEAR DYNAMIC MODELING AND ACTIVE VIBRATION CONTROL OF A FLEXIBLE STRUCTURE MOUNTED MANIPULATOR SYSTEM

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Abstract. FSMS is the abbreviation of Flexible Structure mounted Manipulator System. It consists of two interacting robotic manipulators, with a small rigid Micro-Manipulator (Mi-M) mounted serially on the top of a large flexible link Macro-Manipulator (Ma-M). The micro-macro robotic configuration (Mi-Ma) is often used to increase the reach capability of a dexterous small Mi-M, whose positional accuracy can be greatly disturbed by the flexibility of the supporting Ma-M. The objective of this work is the development of a nonlinear model based control law capable to damp the vibrations of the flexible supporting structure, and thereby improve the position accuracy of the Mi-M. The inertial damping of the flexible supporting structure (Ma-M) is due the reaction torque developed by the Mi-M in its flexible supporting structure. The FSMS dynamic model is obtained by the generalized hybrid Lagrange equations and the assumed modes method. A simulation model was implemented in Matlab/Simulink to investigate the dynamics with and without active vibration control. The simulation results are compared with experimental results obtained from a real time implementation of the proposed control law.

Keywords: nonlinear, control, FSMS

1. INTRODUCTION

Robotic manipulators developed to work in outer space are made of light weight, long reach optimized structural arms with high inherent flexibility. The robotic structures have low inertia and can reach high velocities, with reduced energy consumption and control cost. On the other hand, the structural vibrations can interfere with the end-effector position accuracy, and hence need to be controlled by passive or active means.

The active vibration damping with the use of imbedded sensor/actuator is more difficult to realize due to the required power levels of the actuators. Among available alternatives, the inertial damping of a micro-macro manipulator (Mi-Ma) is an attractive solution because it involves the control of the active degrees of freedom of the manipulator. One such configuration is the long-reach arm (LRA) structure comprised of a small rigid micro-manipulator (Mi-M) mounted on the tip of a large flexible macro-manipulator (Ma-M). These systems are also known as macro/micro manipulator (Mi-Ma) and the active vibration of the flexible macro manipulator is attained by the inertial reaction wrench (force/torque) control due to the rigid motion of the micro-manipulator. Since the macro part is only used to drive the micro to the intend position in the work space, it is usually considered the case of a stationary macro. This system corresponds to a flexible structure mounted micro-manipulator, abbreviated as FSMS (Nenchev et al., 1999; Trudnowski; Lew, 1996).

In Lew and Trudnowski (1996) the inertial damping was realized considering a simple FSMS with one dof rotational joint for the Mi-M and one flexible mode at the stationary base, which is the same approach adopted in the present work.

In Nenchev et al. (1999) it is described the control of a FSMS system called TREP. The system consists of a two-degree of freedom Mi-M mounted on a flexible stationary base. The FSMS model considers only one flexible mode of the flexible base. The active control strategy uses the concept of reaction null space (RNS) for a system with dynamic redundancy. The study of active control of a FSMS is of great interest to aerospace applications (Yoshida and Hashizume, 2001) and also to applications on nuclear waste dumping.

The work describes the experimental assembly of a Mi-M supported on the top of a flexible beam attached to a stationary base. The base is a large Ma-M assembled at the Mechatronics Laboratory of the University of Taubaté (UNITAU).

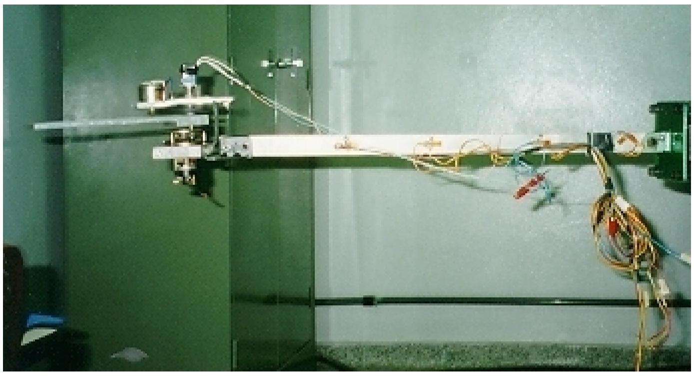

Figure 1. View of the FSMS System

The objective of this work is to compare the dynamic simulation of the FSMS with the *SIMULINK / MATLAB®*, without and with the nonlinear active control law implemented, with the available experimental results. The FSMS model corresponds to a simple cantilevered Euler-Bernoulli beam, actuated by the inertial reaction of a single rigid robotic joint mounted on the tip of the beam.

2. DYNAMIC MODELING

2.1. Assumed Modes Method

The motions of the flexible base the FSMS is described by a partial differential equation since it is a distributed parameter system. The continuous system is approximated by a discrete one using the assumed modes method, where the elastic deflection of the beam is described as a finite expansion of assumed space-dependent shape functions, $\phi_i(x)$, multiplied by unknown time-dependent modal coordinates, $q_{fi}(t)$. Te modal expansion is given by Eq.(1) where $\phi_i(x)$, $q_{fi}(t)$, are the i-th assumed mode shape, and modal coordinate, (x,t) are the independent space and time coordinates, respectively. In this work, the assumed mode shape is given by Eq.(2) as described in (Junkins et al., 1990),

$$w(x,t) = \sum_{i=1}^{n} \phi_i(x) q_{fi}(t) \tag{1}$$

$$\phi_i(x) = 1 - \cos\left(\frac{i\pi x}{L_f}\right) + \frac{1}{2}(-1)^{i+1}\left(\frac{i\pi x}{L_f}\right)^2 \tag{2}$$

where:

L_f: length of the flexible link [m];
n: the number of discrete modes or flexible d.o.f. of the link.

2.2. Equations of Motion of the FSMS

In the present study, The FSMS is mounted on a flexible stationary base (Costa et al., 2001), as illustrated in the Fig. (2). The Mi-M is a one link robotic joint with length, L_r, and one rotational dof, $q_r(t)$. The robotic joint is mounted on the tip of a cantilevered flexible beam, with length, L_f, and known physical properties (E, I, ρ) . The elastic deformation of the supporting beam is given by, w(x,t), with x in the range [0, L_f].

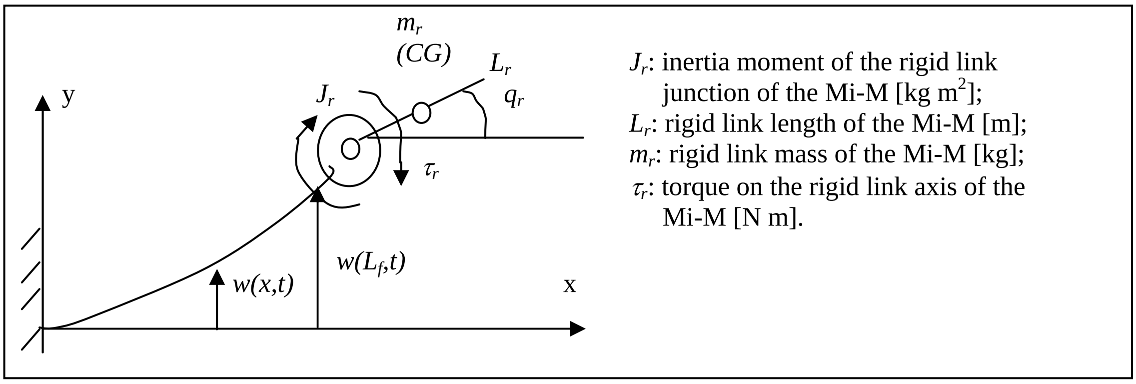

Figure 2 – Schematic view of the stationary FSMS.

Admitting small deformations, the position of the center of gravity (CG) of the rigid link with respect to an inertial coordinate system, described by the unit base vectors (i, j) is given by:

$$\vec{r}(t) = [L_f + \frac{1}{2} L_r \cos(q_r)]\ \hat{i} + [w(L_f, t) + \frac{1}{2} L_r \sin(q_r)]\ \hat{j} \tag{3}$$

The equations of motion are derived from the generalized Lagrange equations applied to a hybrid (rigid-flexible) system. One assumes that the motion takes place on the (x,y) plane, perpendicular to the gravitational field, and that the beam elastic deformation is only due to flexion in the y-direction, with no torsion or compression effects, i.e., the elastic motion $w(x,t)$ is on the direction of the y axis as shown in Fig. (2). The total kinetic and potential energies of the FSMS are given by:

$$T_r = \frac{1}{2} m_r \left|\dot{\vec{r}}(t)\right|^2 + \frac{1}{2} J_r \dot{q}_r^2 \quad \text{(rigid body K.E.)} \tag{4}$$

$$V_r = 0 \quad \text{(no gravity effects)} \tag{5}$$

$$T_f = \frac{1}{2} m_m\, \dot{w}^2(L_f, t) + \int_0^{L_f} \frac{1}{2} \rho(x)\, \dot{w}^2(x,t) dx \quad \text{(elastic K.E. of the beam)} \tag{6}$$

$$V_f = \int_0^{L_f} \frac{1}{2} EI(x) \left(w''(x,t)\right)^2 dx \quad \text{(elastic P.E. of the beam)} \tag{7}$$

where:

E: elasticity module of elasticity of the beam [N/m²];

I(x): inertial area moment of the flexible beam [m^4];

m_r: total mass of the rigid link including the dc-motor, tachometer, potentiometer and gear transmission supported by the flexible beam [kg];

J_r: rotary inertia of the drive and rigid link;

$\rho(x)$: mass per unity length of the flexible beam [kg/m];

$( )'$: represents space derivatives, while dotted quantities represent time derivatives

Applying the assumed modes expansion, and substituting Eq.(1) in the expressions of the total kinetic e potential energies and integrating over the elastic domain of the flexible link, one obtains the discrete Lagrangian function in terms of the generalized coordinate of the rigid body (q_r) and flexible modal coordinates (q_{fi}) of the link,

$$T(q_r, \dot{q}_r, \dot{q}_{fi}) = T_r + T_f \tag{8}$$

$$V(q_{fi}) = V_f \tag{9}$$

$$L(q_r, \dot{q}_r, q_{fi}, \dot{q}_{fi}) = T - V \tag{10}$$

The Lagrangian equation of motion is written as,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} = Q_j \tag{12}$$

where, the generalized torque or force, Q_j, of the FSMS system are deduced from the virtual work of the non-conservatives forces of the hybrid system (SCHILLING, 1990), defined as:

$$\delta W_{nc} = \left(\tau_r - B_r \dot{q}_r\right) \ \delta q_r - (\underline{B}_f^T \underline{\dot{q}}_f)\delta q_f \tag{13}$$

here, B_r is the viscous torque constant of the rigid link, and B_f represents the damping matrix of the flexible link with multiple flexible dof.

Applying Eq.(12) one obtains the FSMS equation of motion in the usual matrix form:

$$\underbrace{\begin{bmatrix} M_{rr} & \underline{M}_{rf} \\ \underline{M}_{fr} & \underline{\underline{M}}_{ff} \end{bmatrix}}_{\underline{\underline{M}}} \begin{Bmatrix} \ddot{q}_r \\ \underline{\ddot{q}}_f \end{Bmatrix} + \underbrace{\begin{bmatrix} 0 & \underline{0} \\ \underline{N}_{fr} & \underline{\underline{0}} \end{bmatrix}}_{\underline{\underline{N}}} \begin{Bmatrix} \dot{q}_r \\ \underline{\dot{q}}_f \end{Bmatrix} + \underbrace{\begin{bmatrix} B_r & \underline{0} \\ \underline{0} & \underline{\underline{B}}_f \end{bmatrix}}_{\underline{\underline{B}}} \begin{Bmatrix} \dot{q}_r \\ \underline{\dot{q}}_f \end{Bmatrix} + \underbrace{\begin{bmatrix} 0 & \underline{0} \\ \underline{0} & \underline{\underline{K}}_{ff} \end{bmatrix}}_{\underline{\underline{K}}} \begin{Bmatrix} q_r \\ \underline{q}_f \end{Bmatrix} = \begin{Bmatrix} \tau_r \\ \underline{0} \end{Bmatrix} \tag{14}$$

where,

$$M_{rr} = \frac{1}{4} m_r L_r^2 + J_r \tag{15}$$

$$\underline{M}_{rf\,j} = \underline{M}_{fr\,j}^T = \frac{1}{2} m_r L_r \phi_j(L_f)\cos(q_r) \tag{16}$$

$$\underline{\underline{M}}_{ff\,ji} = (m_r + m_m)\phi_j(L_f)\phi_i(L_f) + \int_0^{L_f} \rho\phi_j(L_f)\phi_i(L_f)dx \tag{17}$$

$$\underline{N}_{fr\,j} = -\frac{1}{2} m_r L_r \phi_j(L_f) sin(q_r)\dot{q}_r \tag{18}$$

$$\underline{\underline{B}}_{f\,j} = diag(B_{fj}) \tag{19}$$

$$\underline{\underline{K}}_{ff\,ji} = \int_0^{L_f} EI\phi_j''(x)\phi_i''(x)dx \tag{20}$$

$$\tau_r = f(q_r, q_{fi}) \tag{21}$$

3. NONLINEAR DYNAMIC CONTROL

The FSMS with a stationary flexible base belongs to the class of under-actuated robotic system, since the number of actively controlled dof (rigid joints of the Mi-M) is normally smaller that the total number of dof of the system, which include the passive elastic dof of the flexible beam as described by the flexible modes retained in Eq. (1). In the following equations, it is considered a SIMO FSMS with one control input and multiple output signals characterized by the available sensors: potentiometer, tachometer, strain-gage and accelerometer. The sensor outputs signals are all written in terms of the FSMS generalized coordinates, described by the rigid dof (q_r) and the multiple flexible modal coordinates, $\underline{q}_f = [q_{f1}, q_{f2}, \ldots, q_{fn}]^T$.

3.1. Vibration Control of the Flexible Link

The FSMS coupled equations of motion are used to derive a model based nonlinear control strategy to actively damp the vibrational energy of the flexible beam (Yoshida et al., 2001). This strategy consists to identify the dynamic reaction forces between the flexible link and the rigid body motion of the Mi-M.

The motion of the flexible supporting structure excited by the dynamic interaction with the Mi-M is obtained from the second matrix component of Eq. (14),

$$\underline{\underline{M}}_{ff}\ddot{\underline{q}}_f + \underline{M}_{fr}\ddot{q}_r + \underline{\underline{B}}_f\dot{\underline{q}}_f + \underline{N}_{fr}\dot{q}_r + \underline{\underline{K}}_{ff}\underline{q}_f = 0 \tag{22}$$

The absence of external actuators in the dynamic of the flexible link, makes it a "passive" system, except for the dynamic interaction term represented by the generalized force, Γ(t), written as

$$\underline{\Gamma}(t) = \underline{M}_{fr}\ddot{q}_r + \underline{N}_{fr}\dot{q}_r \tag{23}$$

If the dynamic reaction, $\underline{\Gamma}(t)$, is externally controlled by the Mi-M motion, the flexible dynamics of the supporting beam can now be actively controlled, according to:

$$\underline{\underline{M}}_{ff}\ddot{\underline{q}}_f + \underline{\underline{B}}_f\dot{\underline{q}}_f + \underline{\underline{K}}_{ff}\underline{q}_f = -\underline{\Gamma}(q_r) \tag{24}$$

The above equation shows that the Mi-M motion, $q_r(t)$, excites a generalized reaction force, Γ(t), dependent on the "active" dof and that it can be written as a time variation of a "generalized momentum" of the FSMS, Π(t):

$$\underline{\Gamma}(t) = \underline{M}_{fr}\ddot{q}_r + \underline{N}_{fr}\dot{q}_r = \frac{d}{dt}(\underline{M}_{fr}\dot{q}_r) = \frac{d\,\underline{\Pi}}{dt} \tag{25}$$

The active vibration control strategy of the flexible link consists of a control law for the movement for the Mi-M rigid link, such that the coupling momentum, $\underline{\Pi}(t)$, gives a controlled force able of actively damp the vibrational modes of the flexible link. This control law can be obtained from the dynamic model of the FSMS, as a proportional plus derivative (PD) flexible control law:

$$\underline{\Gamma}(t) = \underline{\underline{G}}_f^D\,\dot{\underline{q}}_f(t) + \underline{\underline{G}}_f^P\,\underline{q}_f(t) \tag{26}$$

Substituting the control law in the equation of the flexible motion, Eq. (24), one obtains the closed loop equation for the dynamics of the flexible link:

$$\underline{\underline{M}}_{ff}\,\ddot{\underline{q}}_f + (\underline{\underline{B}}_f + \underline{\underline{G}}_f^D)\,\dot{\underline{q}}_f + (\underline{\underline{K}}_{ff} + \underline{\underline{G}}_f^P)\,\underline{q}_f = 0 \quad (27)$$

The control law derivative gain, G_f^D, modify the damping factor of the flexible link, while the proportional gain, G_f^P, modify the stiffness of the structure and consequently its natural frequencies. Under special conditions, the resulting closed loop equations can be rewritten in terms of uncoupled modal equations leading to the independent modal space control (IMSC) of the FSMS,

$$\ddot{q}_f + 2\,\xi\,\omega_{nf}\,\dot{q}_f + \omega_{nf}^2\,q_f = 0 \quad (29)$$

where, ξ is the active damping coefficient and ω_{nf} the natural frequency for each controlled flexible mode.

3.2. Implementation of the Active Control of the Flexible Link

The implementation of the active vibration control corresponds to control the acceleration of the Mi-M (Costa et al., 2001). The desired angular acceleration is obtained equating Eq.(25) and Eq.(26), resulting

$$\ddot{q}_{rd}(t) = \underline{M}_{rf}^{+}(\underline{\underline{G}}_f^D\dot{\underline{q}}_f + \underline{\underline{G}}_f^P\underline{q}_f - \underline{N}_{fr}\dot{q}_r) \quad (28)$$

In the case of active control with multiple flexible dof, the implementation of the acceleration control needs the implementation of an observer for the flexible modal coordinate vector, $\underline{q}_f(t)$, and the implementation of a pseudo-inverse operation for the nonlinear coupling term, $\underline{M}_{rf}$.

The controlled motion of the Mi-M, as prescribed by Eq. (28), is implemented by controlling the motor torque, $\tau(t)$. The joint actuator is a dc-motor described by the electrical impedance of the armature (R_a, L_a), the motor torque constant (K_t) and the back-emf constant (K_e). The nonlinear coupling term between the Mi-Ma is considered as a perturbation torque. The closed loop control can be implemented in the form of a position servomechanism with a PID control law, or a direct torque control using the armature current feedback and a power current amplifier.

In this work, it was considered the active control of the first flexible mode. In this case $M_{rf}$is a scalar and the control law described in Eq.(28) can be readily applied considering the strain gage signal as proportional to the flexible coordinate system (Costa, 2001b).

4. EXPERIMENTAL ASSEMBLY

The FSMS experimental setup is shown in Fig. (3). The system is digitally controlled with a PC microcomputer provided with AD/DA interfaces and a real time digital control, programmed in C-language. The microcomputer system possesses a data acquisition board, with 8 analog channels, and 12-bit AD converter with a maximum sample rate of 200 kHz. The dc servomotor is driven by a linear power amplifier. The output signals include a potentiometer to monitor the rigid link position, a tachometer to measure the joint angular velocity, a strain-gage to measure the flexible link elastic deformation and a piezoelectric accelerometer to monitor the vibrations of the beam. The experimental set up was assembled at the Mechatronics and Robotics Laboratory of UNITAU, and was previously described by Grandinetti (1998).

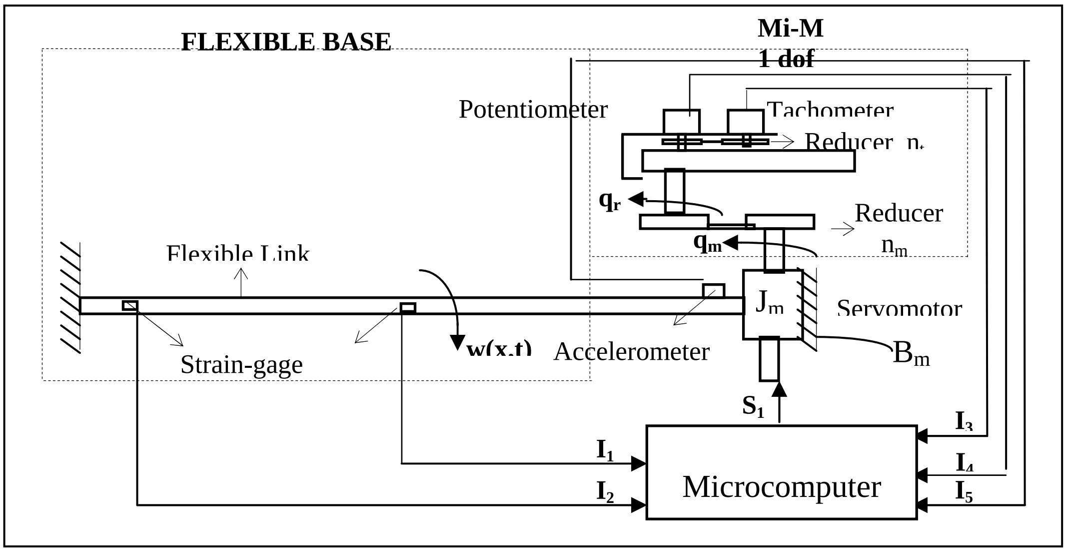

Figure 3. Experimental Assembly of the Stationary Base FSMS

As shown in the Fig. (3), the experimental setup includes:

a) flexible link : a cantilevered aluminum beam;
b) a one joint Mi-M;
c) strain gage sensors disposed on half bridge;
d) one turn angular potentiometer;
e) dc tachometer;
f) piezoelectric accelerometer conditioned by a charge amplifier and a vibration pre-amplifier with double integrator;
g) dc servomotor with permanent magnet;
h) linear power amplifier;
i) AD/DA converters;
j) PC microcomputer.

The Mi-M motion is controlled a dc servomotor, driven by a linear power amplifier. A gear reduction (n_m), with a 20:1 transmission factor, is localized between the servomotor output axis (q_m) and the Mi-M joint (q_r). A pulley and belt transmission system, with a relation of 5:1 (n_r), is used to transmit the Mi-M joint motion to the dc-tachometer. Three strain-gage sensors are placed along the beam, at three different positions, $\frac{1}{4}L_f, \frac{1}{2}L_f$ and $\frac{3}{4}L_f$, respectively. A piezoelectric accelerometer is used to monitor the vibrations at the tip of the flexible link.

5. ACTIVE CONTROL SIMULATIONS AND EXPERIMENTAL RESULTS

The dynamic motion of the FSMS, with one flexible degree of freedom, was simulated with the Matlab/Simulink^R program. The FSMS equations of motion were simulated with and without the nonlinear active control. The physical parameters of the FSMS used in the simulation studies are summarized in the Table 1, below.

Table 1. FSMS physical parameters

Modulus of elasticity	E	7.1 x 10^{10}	N/m^2
Moment of inertia	I	2.97 x 10^{-9}	m^4
Mass of the rigid link	m_r	0.45	kg
Length of the rigid link	L_r	0.30	m
Length of the flexible link	L_f	0.81	m
Mass per unit length of the link	ρ	0.32	kg/m
Width of the flexible link	e_b	5.20	mm
Height of the flexible link	h_b	25.4	mm

The FSMS system was excited with a perturbation in the form of a doublet pulse, giving rise to a lightly-damped oscillatory response of the flexible base. Figures (4) and (5) show a comparison between the experimental response and the simulation result measured by the strain gage signal (elastic response of the beam), without the active control law. The open loop response are quite similar corresponding to an oscillation that lasts for approximately 40 s. The tip deflection of the beam is obtained multiplying the assumed shape function by the elastic deformation signal.

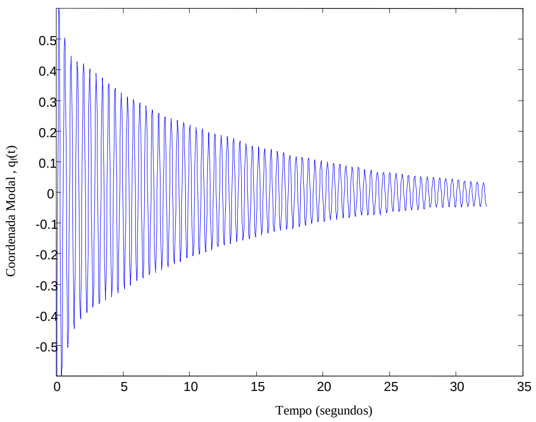

Figure 4. Experimental response of the flexible link without active vibration control, (Grandinetti, 1998).

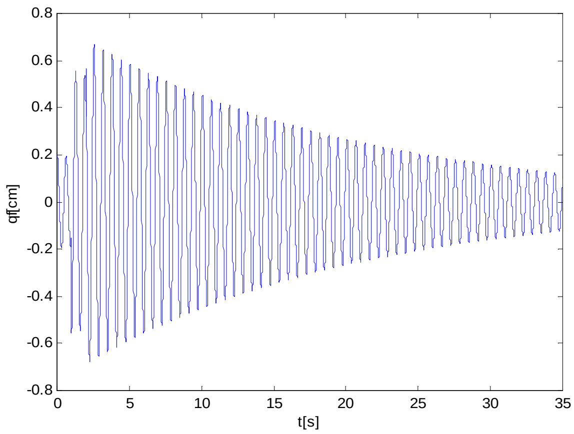

Figure 5. Simulation of the flexible dof, $q_f(t)$, without active vibration control.

The experimental result of the inertial damping is shown in Fig. (6). In this figure, 1000 sample points corresponds to approximately 32 seconds in real time. The vibrations are reduced in half the time corresponding to the case without control. Figure (7) shows the simulation result of the closed loop vibration control.

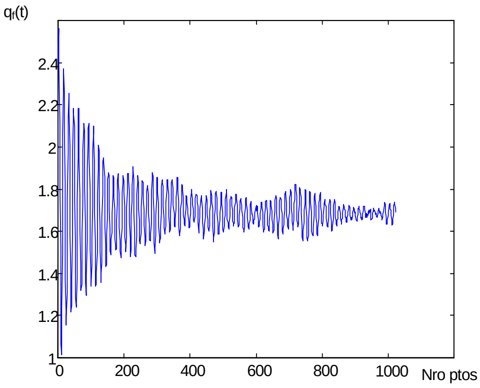

Figure 6. Implementation in real time with damping of 0.02, total time 32,29 s (Grandinetti, 1998)

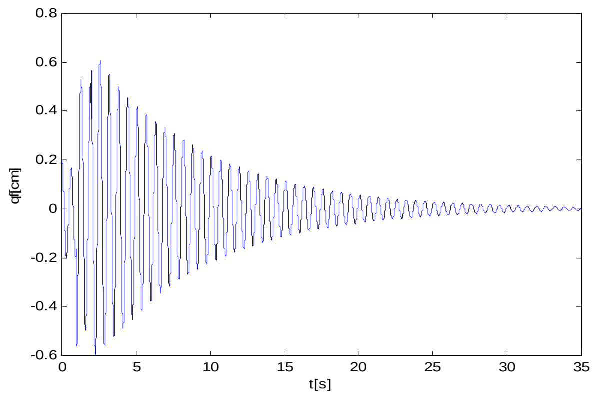

Figure 7. Simulation of the flexible dof, $q_f(t)$, with damping of 0,020

The comparison between simulation and experimental results indicate that the present FSMS model can reproduce the observed dynamics and corresponds to the real system. The nonlinear control law performed qualitatively well, attending the FSMS model predictions.

6. CONCLUSIONS

This work addresses the inertial damping of a FSMS by active control of a Mi-M mounted on a flexible beam attached to a stationary base. The work applied the assumed mode technique to represent the discrete modes of the system. The assumed shape function gave coherent result as compared to the real system behavior.

Effort was concentrated on the simulation and characterization of a control strategy to compensate the effects of the dynamic coupling between the flexible beam and the Mi-M rigid link. The main objective was to reproduce the results of inertial damping available in the literature. The effectiveness of the active control law was tested by monitoring the inertial damping of a doublet perturbation applied to the flexible base. With the implementation of the active control the flexible damping and stiffness could be modified. The damping is brought off by the inertial interaction force acting between the Mi-M and the flexible beam.

The inertial damping was introduced by closed loop control of the Mi-M rigid link, using the reference signal calculated by the nonlinear vibration control law and by feedback of the actual angular position of the Mi-M. The servo loop was controlled with a PID control law, to minimize the static acceleration error in the system. Further work is in progress and includes: implementation of a FSMS model with many flexible modes and a Mi-M with more dof; to improve the inertial damping by tuning the derivative and proportional gains of the nonlinear control law.

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