LINEAR MATRIX INEQUALITIES FOR VIBRATION ATTENUATION IN SMART STRUCTURES

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Abstract. *Light structures are, also, usually lightly damped, which can cause large amplitude vibration. Any disturbances in these systems can degrade the demanded performance of mechanical systems. An atractive methodology for attenuation structural vibrations is to use active control techniques with pieozoelectric sensors and actuators coupled in a base structure, wich has been convicciøned to call smart structure. Many control strategies and techniques have been used in active vibration control. This paper deals with the use of linear matrix inequalities (LMI) framework. It is discussed: the optimal placement of actuators; the model reduction; and the robust controller design for active damping through techniques involving LMI. To illustrate the methodology, it was considered a cantilever beam with bonded piezoelectric actuators. It was designed a robust controller considering parametric variation described by polytopic uncertainty. Usually, the order of the model is large enough to cause loss of efficiency in the controller, so, a reduced model with observation and control spillover effects was applied. The results demonstrated the vibration attenuation in the structure by controlling only some modes and the increased damping ratio in the bandwidth of interest.*

Keywords: *LMI, smart structures, active damping, robust control.*

1. INTRODUCTION

Vibration reduction is an important goal in a variety of engineering applications, mainly in modern structures of huge space vehicles and aircraft. In special, active vibration control (AVC) in mechanical structures has great practical interest because it needs excellent dynamic behavior to guarantee stability and light structures, in order to reduce cost. However, these two requirements are often contradictory, because light structures have low degree of internal damping, which hinder the accuracy requirements, (Yan and Yam, 2002).

It is possible to use modern techniques of active control associated with intelligent materials to execute these demands. These materials are composed by piezoelectric ceramic (PZT, Lead Zirconate Titanate), commonly used as distributed actuators, and piezoelectric plastic films (PVDF, PolyVinyliDeno Floride), highly indicated for distributed sensors, (Clark et al., 1998).

The design process of such structures encompasses three main phases: structural design; optimal placement of sensor and actuator; and controller design. Consequently, for optimal design purposes, the structure, the placement of sensor/actuator, and the controller have to be considered simultaneously (Lopes Jr. et al., 2003).

We have chosen a recent control technique involving linear matrix inequalities (LMI) due its advantages when compared to conventional techniques. Once formulated in terms of LMI a problem can be solved efficiently by convex optimization algorithms, for example, using interior-point methods, (Gahinet et al., 1995). Besides, few researchers explore the use of LMI in the structural control community. In this sense, Sana and Rao (2000) utilized a cantilever beam with pieozoelectric actuator and sensor distribuided to design an ouput feedback controller to increase damping of some modes using LMI. However, the resulting matrix inequalities involved bilinear matrix inequalities (BMI) in an unknown variables, and, hence it is not a convex optimization problem. So, the BMI could not be solved directly by software package of convex optimization. In this case, it is necessary to used iterative methods, as for instance, cone complementarity linearization algorithm (El Ghaoui et al., 1997) that is a high cost procedure.

In another way, Gonçalves et al. (2002) controlled a two degree-of-freedom (2DOF) mechanical system comparing the H_2 and H_∞ optimal control with state-feedback via LMI, in a procedure of solution proposed by Peres (1997). Gonçalves et al. (2003a) and (2003b) simulated a state-feedback synthesis using classical LMI, described in Boyd et al. (1994), considering norm-bound linear diferential inclusions (LDI) for a 2DOF mechanical system and for a fixed-fixed aluminum beam, respectively.

The proposal of the present work is to ilustrate a state-feedback synthesis via LMI for uncertainties parametric rejection described by politopic LDI (PLDI) in a procedure that was first proposed by Geromel et al. (1991). This technique was numerically applied in a truss structure in Silva et al. (2004).

The actuators optimal locations will be find by using H_∞ norm. In the following, a brief review of modal state-space model is shown and a LMI structural controller is proposed for vibration attenuation, considering only a small number of natural modes. Since, not every states usually are available as measurement, it was implemented a dynamic observer, also, using LMI. Analytical models determined by finite element method (FEM) with eletromechanical coupling, usually, have large number of degrees of freedom (dof). So, for the controller design it was considered a reduced model with observation and control spillover effects to overcome this numerical approximation. The paper concludes with an application using a beam structure for illustration purposes.

2. STATE-FEEDBACK DESIGN VIA LMI

A linear differential inclusion (LDI) system, in modal state-space form, considering the matrices with appropriate dimensions and assumed to be known is given by:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}(t)\,\mathbf{x}(t) + \mathbf{B}_1\,\mathbf{w}(t) + \mathbf{B}_2\,\mathbf{u}(t), \quad \mathbf{A}(t) \in \Omega \\ \mathbf{y}(t) &= \mathbf{C}\,\mathbf{x}(t) \end{aligned} \tag{1}$$

where $\mathbf{A}(t)$ is the dynamic matrix, $\mathbf{B}_1$ is the matrix of disturbance input, $\mathbf{B}_2$ is the matrix of control input, $\mathbf{C}$ is the output matrix, $\mathbf{w}(t)$ is the vector of exogenous input, $\mathbf{u}(t)$ is the vector of control input, $\mathbf{y}(t)$ is the output vector, and Ω is a polytope that is described by a list of vertexes in a convex space **Co**, (Boyd et. al., 1994). More information about this topic can be found in Gawronski (1998).

A reduced-order model can be obtained by truncating the states considering the canonical modal decomposition. From the Jordan canonical form, it can be obtained:

$$\begin{Bmatrix} \dot{\mathbf{x}}_c(t) \\ \dot{\mathbf{x}}_r(t) \end{Bmatrix} = \begin{bmatrix} \mathbf{A}_c(t) & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_r \end{bmatrix} \begin{Bmatrix} \mathbf{x}_c(t) \\ \mathbf{x}_r(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{B}_{1c} \\ \mathbf{B}_{1r} \end{bmatrix} \mathbf{w}(t) + \begin{bmatrix} \mathbf{B}_{2c} \\ \mathbf{B}_{2r} \end{bmatrix} \mathbf{u}(t)$$

$$\mathbf{y}(t) = [\mathbf{C}_c \quad \mathbf{C}_r] \begin{Bmatrix} \mathbf{x}_c(t) \\ \mathbf{x}_r(t) \end{Bmatrix} \tag{2}$$

where the subscripts $(.)_c$ and $(.)_r$ are related to controlled (low frequency modes) and residual modes (high frequency modes), respectively. Considering parametric variations only in low frequency modes, the dynamic matrix related with the controlled modes is described by PLDI as

$$\mathbf{A}_c(t) \in \Omega, \qquad \Omega = \mathbf{Co}\{\mathbf{A}_{c,1}, \cdots, \mathbf{A}_{c,v}\} \tag{3}$$

where v is the number of vertexes of the polytopic system. The problem to be investigated is the state-feedback control with the linear control law $\mathbf{u}(t) = \mathbf{G}\mathbf{x}_c(t)$, where $\mathbf{G}$, the state-feedback gain, must be found. The state-space equation related with the controlled modes, described by eq. (2), can be rewritten in the closed-loop:

$$\dot{\mathbf{x}}_c(t) = (\mathbf{A}_c(t) + \mathbf{B}_{2c}\mathbf{G})\,\mathbf{x}_c(t) + \mathbf{B}_{1c}\,\mathbf{w}(t) \quad , \quad \mathbf{A}_c(t) \in \Omega \tag{4}$$

Equation (4) is quadratically stabilizable (via linear state-feedback) if and only if the following LMI are feasible:

$$\mathbf{Q} > \mathbf{0}, \quad \mathbf{A}_{c,i}\mathbf{Q} + \mathbf{Q}\mathbf{A}_i^T + \mathbf{B}_{2c}\mathbf{Y} + \mathbf{Y}^T\mathbf{B}_{2c}^T < \mathbf{0}, \quad i = 1,2,\cdots,v \tag{5}$$

where: the symbols > **0** and < **0** means positive and negative defined, respectively; $\mathbf{A}_{c,i}$ is the *ith* vertex of the polytopic system, with i=1,..., v; **Q** is a symmetric, positive and defined matrix; and **Y=GQ**, (Boyd et. al., 1994).

For a given feedback gain **G**, the output energy of the system from eq. (2) is upper bounded by $\mathbf{x}_c(0)^T\mathbf{Q}^{-1}\mathbf{x}c(0)$ if **Q** satisfies the LMI:

$$\mathbf{Q} > \mathbf{0}, \quad \begin{bmatrix} \mathbf{A}_{c,i}\mathbf{Q} + \mathbf{Q}\mathbf{A}_{c,i}^T + \mathbf{B}_{2c}\mathbf{Y} + \mathbf{Y}^T\mathbf{B}_{2c}^T & \mathbf{Q}\mathbf{C}_c^T \\ \mathbf{C}_c\mathbf{Q} & -\mathbf{I} \end{bmatrix} \le \mathbf{0}, \quad i = 1,2,\cdots,v \tag{6}$$

where **I** is the identity matrix and $\mathbf{x}_c(0)$ is the initial condition. Regarding **Y** as a variable, one can find a state-feedback gain that guarantees output energy less than β by solving the LMI problem:

$$\begin{bmatrix} \mathbf{Q} & \mathbf{x}_c(0) \\ \mathbf{x}_c(0)^T & \beta \end{bmatrix} > \mathbf{0} \tag{7}$$

where β is the bounded output energy.

When the initial condition, $\mathbf{x}_c(0)$, is known, it is also possible to define an upper bound for the norm of the control input. Therefore, the constraints $\|\mathbf{u}\| \le \mu$ is enforced at all times t>0 if the LMI below hold, (Folcher and Ghaoui, 1994):

$$\begin{bmatrix} 1 & \mathbf{x}_c(0)^T \\ \mathbf{x}_c(0) & \mathbf{Q} \end{bmatrix} \ge \mathbf{0}, \quad \begin{bmatrix} \mathbf{Q} & \mathbf{Y}^T \\ \mathbf{Y} & \mu^2\mathbf{I} \end{bmatrix} > \mathbf{0} \tag{8}$$

where μ is the maximum value of the amplitude of the control input. In the same way, it can be possible to impose a decay rate, α, on the closed-loop:

$$\mathbf{Q} > \mathbf{0}, \quad 2\alpha\mathbf{Q} + \mathbf{A}_{c,i}\mathbf{Q} + \mathbf{Q}\mathbf{A}_{c,i}^{T} + \mathbf{B}_{2c}\mathbf{Y} + \mathbf{Y}^{T}\mathbf{B}_{2c}^{T} < \mathbf{0}, \qquad i = 1,2,\cdots,v \tag{9}$$

Besides, the optimal decay rate can be found solving the generalized eigenvalue problem (GEVP), (Boyd et al., 1993). In summary, the controller design is the result of the following convex optimization problem, considering μ and β known and α*=-α:

$$\begin{gathered}
\underset{\alpha\ \mathbf{Q}\ \mathbf{Y}}{\text{minimize}}\ \alpha^{*} \\
\begin{bmatrix} \mathbf{A}_{c,i}\mathbf{Q} + \mathbf{Q}\mathbf{A}_{c,i}^{T} + \mathbf{B}_{2c}\mathbf{Y} + \mathbf{Y}^{T}\mathbf{B}_{2c}^{T} & \mathbf{Q}\mathbf{C}_{c}^{T} \\ \mathbf{C}_{c}\mathbf{Q} & -\mathbf{I} \end{bmatrix} \le \mathbf{0} \\
\begin{bmatrix} \mathbf{Q} & \mathbf{x}_{c}(0) \\ \mathbf{x}_{c}^{T}(0) & \beta \end{bmatrix} > \mathbf{0} \\
\begin{bmatrix} 1 & \mathbf{x}_{c}(0)^{T} \\ \mathbf{x}_{c}(0) & \mathbf{Q} \end{bmatrix} \ge \mathbf{0} \qquad\qquad i = 1,2,\cdots,v \\
\begin{bmatrix} \mathbf{Q} & \mathbf{Y}^{T} \\ \mathbf{Y} & \mu^{2}\mathbf{I} \end{bmatrix} > \mathbf{0} \\
\mathbf{Q} > \mathbf{0} \\
\mathbf{A}_{c,i}\mathbf{Q} + \mathbf{Q}\mathbf{A}_{c,i}^{T} + \mathbf{B}_{2c}\mathbf{Y} + \mathbf{Y}^{T}\mathbf{B}_{2c}^{T} < 2\alpha^{*}\mathbf{Q}
\end{gathered} \tag{10}$$

This problem can be solved using interior-point methods, (Gahinet et al., 1995). For each initial condition, the input **u**(t) and the output **y**(t) assure:

$$\forall t \ge 0, \begin{cases} \|\mathbf{u}\| < \mu e^{-\alpha t} \\ \|\mathbf{y}\| < \beta e^{-\alpha t} \end{cases} \tag{11}$$

The optimal feedback gain is given by:

$$\mathbf{G} = \mathbf{Y}\mathbf{Q}^{-1} \tag{12}$$

where **Y** and **Q** are optimal solution from LMI problem given by eq. (10).

3. DYNAMIC OBSERVER VIA LMI

Some states can be not available for feedback, since a limited number of sensor is available, or it can still have state variables with difficult access, or even not measurable directly. In this sense, it is essential the observers design. In this work is considered the design of a deterministic observer to estimate the modal states not available. So, the input control is:

$$\mathbf{u}(t) = \mathbf{G}\bar{\mathbf{x}}_{c}(t) \tag{13}$$

where $\bar{\mathbf{x}}_{c}(t)$ is the controlled modal state vector estimated. One can write the linear equation of modal observer in the form, (Meirovitch, 1990):

$$\dot{\bar{\mathbf{x}}}_c(t) = \mathbf{A}_c(t)\bar{\mathbf{x}}_c + \mathbf{B}_{2c}\mathbf{G}\bar{\mathbf{x}}_c(t) + \mathbf{B}_{1c}\mathbf{w}(t) + \mathbf{L}(\mathbf{C}_c\bar{\mathbf{x}}_c(t) - \mathbf{y}(t)) \tag{14}$$

where **L** is the observer gain matrix, which can be obtained by different techniques. In the present work we, also, use LMI. It is possible to find an observer gain through the solution of the following LMI, (Boyd et al., 1994):

$$\mathbf{P} > \mathbf{0}, \qquad 2\gamma\mathbf{P} + \mathbf{A}_{c,i}^T\mathbf{P} + \mathbf{P}\mathbf{A}_{c,i} + \mathbf{W}\mathbf{C}_c + \mathbf{C}_c^T\mathbf{W}^T < \mathbf{0}, \quad i = 1,2,\cdots,v \tag{15}$$

where γ is the decay rate of the observer, with $\gamma >> \alpha$. To every **P** and **W** satisfying these LMI, there corresponds a stabilizing observer. The observer gain is given by:

$$\mathbf{L} = \mathbf{P}^{-1}\mathbf{W} \tag{16}$$

where **P** and **W** are solution from LMI problem given by eq. (15).

Therefore, the sensor signals include contributions from controlled and residual modes, so the output vector is, (Meirovitch, 1990):

$$\mathbf{y}(t) = \mathbf{C}_c\mathbf{x}_c(t) + \mathbf{C}_r\mathbf{x}_r(t) \tag{17}$$

After some mathematical manipulations, described in full details in Meirovitch (1990), these equations can be written in matrix form:

$$\begin{gathered}
\begin{Bmatrix} \dot{\mathbf{x}}_c(t) \\ \dot{\mathbf{x}}_r(t) \\ \dot{\mathbf{e}}_c(t) \end{Bmatrix} = \begin{bmatrix} \mathbf{A}_c(t) + \mathbf{B}_{2c}\mathbf{G} & \mathbf{0} & \mathbf{B}_{2c}\mathbf{G} \\ \mathbf{B}_{2r}\mathbf{G} & \mathbf{A}_r & \mathbf{B}_{2r}\mathbf{G} \\ \mathbf{0} & -\mathbf{L}\mathbf{C}_r & \mathbf{A}_c(t) + \mathbf{L}\mathbf{C}_c \end{bmatrix} \begin{Bmatrix} \mathbf{x}_c(t) \\ \mathbf{x}_r(t) \\ \mathbf{e}_c(t) \end{Bmatrix} + \begin{bmatrix} \mathbf{B}_{1c}(t) \\ \mathbf{B}_{1r}(t) \\ \mathbf{0} \end{bmatrix} \mathbf{w}(t) \\
\mathbf{y}(t) = \begin{bmatrix} \mathbf{C}_c & \mathbf{C}_r & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{x}_c(t) \\ \mathbf{x}_r(t) \\ \mathbf{e}_c(t) \end{Bmatrix}
\end{gathered} \tag{18}$$

where:

$$\dot{\mathbf{e}}_c(t) = (\mathbf{A}_c(t) + \mathbf{L}\mathbf{C}_c)\mathbf{e}_c(t) - \mathbf{L}\mathbf{C}_r\mathbf{x}_r(t) \quad \text{and} \quad \mathbf{e}_c(t) = \bar{\mathbf{x}}_c(t) - \mathbf{x}_c(t) \tag{19}$$

The term $\mathbf{B}_{2r}\mathbf{G}$ is responsible for the excitation of residual modes by the control forces and it is known as *control spillover*. This term has no effect on the eigenvalues of closed-loop system and it cannot destabilize the system, although it can cause some degradation in the performance. On the other hand, the term $-\mathbf{L}\mathbf{C}_r$ can produce instability in the residual modes. This effect is known as *observation spillover*. However, a small modal damping, inherent in the structure, is often sufficient to overcome the observation spillover effect. Another way to reduce this effect is to use a large number of sensors or to filter the sensor signals, in order to screen out the contribution of residual modes, (Meirovitch, 1990).

4. APPLICATION EXAMPLE

To verify the proposed methodology, an aluminum cantilever beam, as shown on fig. (1), was considered. The system is discretized by FEM with 24 elements (2 dof by node). The properties of the beam are given in tab. (1). The number of electrical dof changes as a function of the number of

PZT considered (2 dof by PZT), (Lopes Jr. et. al., 2003). The PZT size is equal to the discrete finite element size. The properties of PZT, based on material designation PSI-5A-S4 (Piezo Systems, Inc.), are given in tab. (2). The FEM model used considers the eletromechanical coupling between PZTs and host structure, but it was not shown in the formulation for clearness proposal, for details in this point see Rocha et al. (2004). In order to test the proposed optimization method a pair of PZT actuador were bonded on the beam surface in the optimal location. The objective in this test was to control the first two vibration modes. It was considered to be possible to bond PZT actuator in all elements, while the optimal location for the displacement sensor to state-feedback considered all nodes on vertical direction.

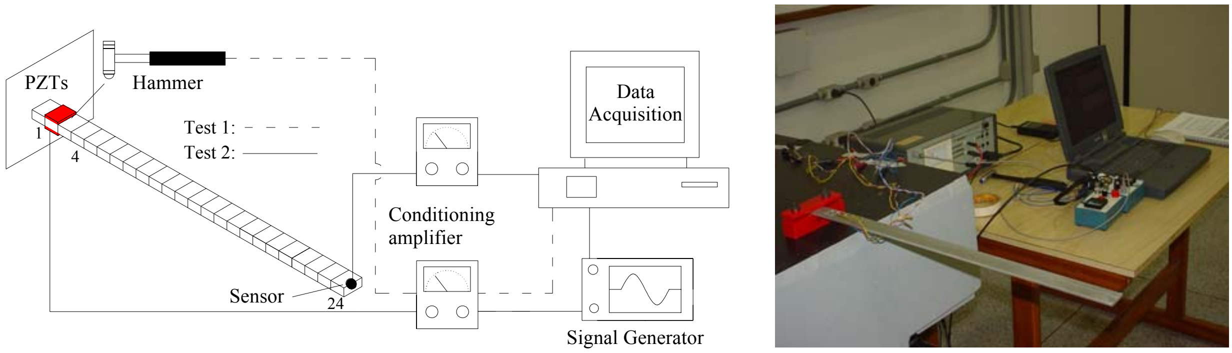


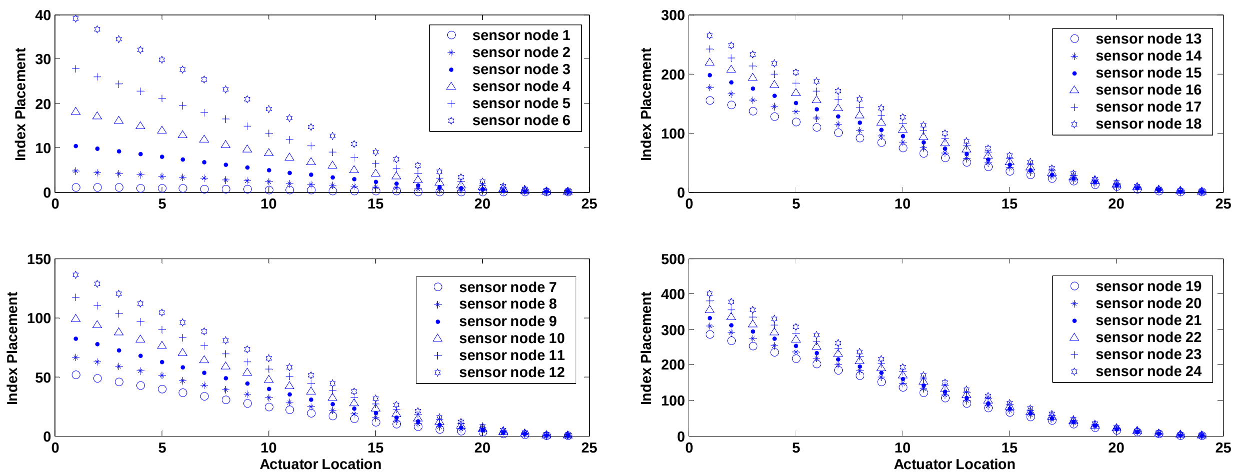
Figura 1. Schematic diagram of the measurement setup.

Table 1. Beam properties and dimensions.

Dimensions (m)	Length	Width	Thickness
	0.48	0.025	0.003
Mass Density (kg.m^{-3})	2710		
Young's Modulus (GPa)	70		

Table 2. PZT properties and dimensions.

Dimensions (m)	Length	Width	Thickness
	0.02	0.025	0.00267
Young's Modulus (GPa)	63		
Mass Density (kg.m^{-3})	7650		
Dielectric Constant, d_{31}, (m.V^{-1})	-190e-12		



(a) sensor in the 12 first nodes (b) sensor in the 12 last nodes

Figure 2. Placement indices for control the two first modes versus PZT location.

The placement indices computed from H_∞ norm for each candidate position of sensor/actuator for the first two modes are shown in fig. (2). The largest value index was found for PZT actuator bonded on element 1 and sensor on node of the free end of the beam. In our case the placement of the actuator in the first element is not a practical location, then it was collacted on element 2.

The frequency response functions (FRF) comparing the experimental results and the modeling with and without eletromechanical coupling considerations are shown in fig. (3), for excitation with impact hammer, H_1, and PZT actuators, H_2, respectively. So, the transfer functions H_1 and H_2 are relative with the state-space realization ($\mathbf{A}$,$\mathbf{B}_1$, $\mathbf{C}$) and ($\mathbf{A}$,$\mathbf{B}_2$, $\mathbf{C}$), respectively.

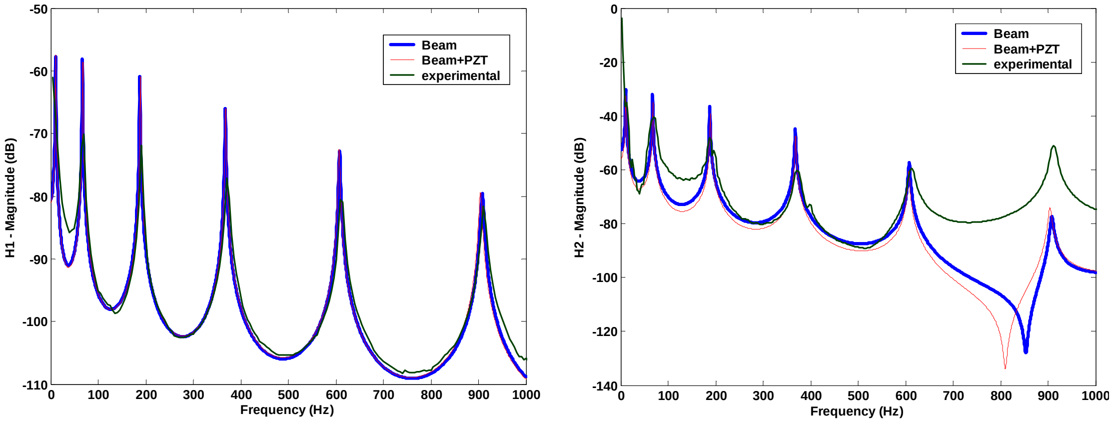
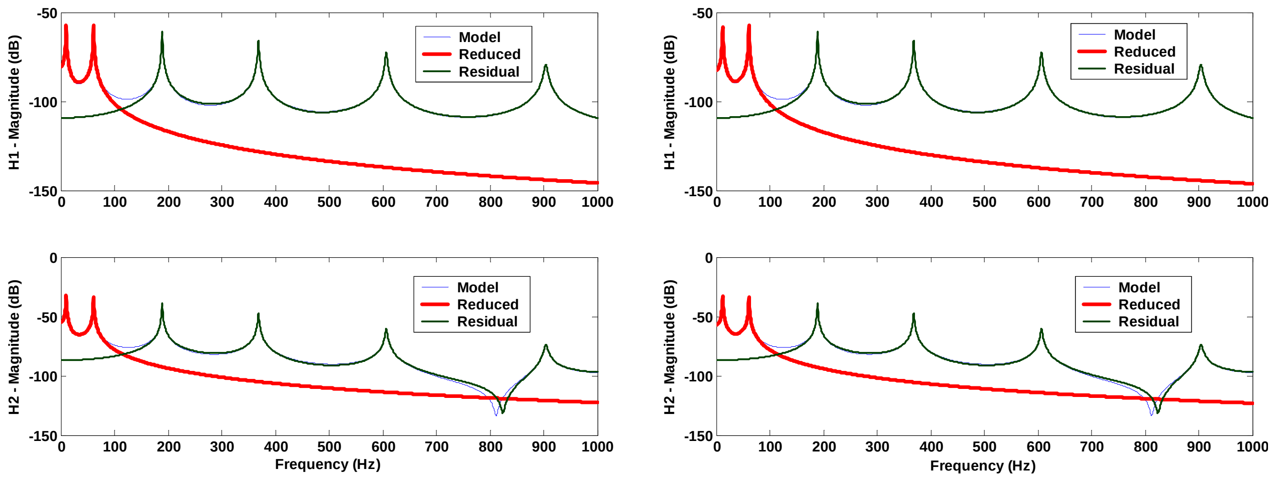


Figure 3. Comparing experimental FRF and simulated with and without pieozoelectric coupling.

For active control design proposal, the state-space matrices must be of low order. Otherwise, it should compromise the control performance. In this sense, it was considered a fourth order model by truncating the modes. One of the biggest advantage of this approach is the possibility of to deal with constraints and uncertainties simultaneously. It was considered, in this example, that the system can have a possible variation of ±10% in the first and second natural frequencies. So, we have 2 uncertainty parameters:

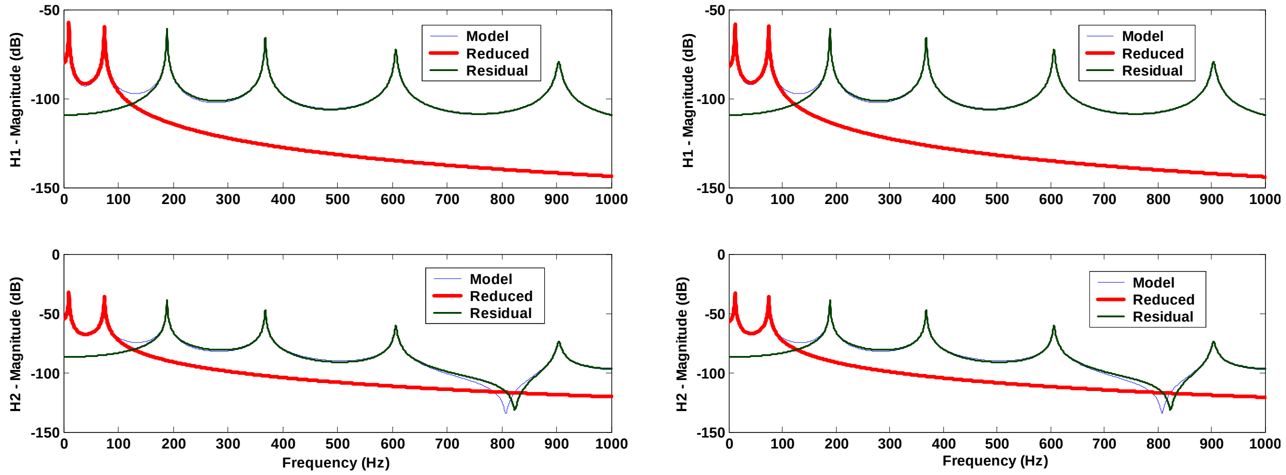
$$\omega_1 \in \left[\omega_1^{min} = 0.9\omega_1 \quad \omega_1^{max} = 1.1\omega_1\right], \quad \omega_2 \in \left[\omega_2^{min} = 0.9\omega_2 \quad \omega_2^{max} = 1.1\omega_2\right],$$

and 4 vertexes of a polytopic system (V1, V2, V3, and V4). The FRF magnitude plot H_1 and H_2 for reduced and residual model are shown in fig. (4) and (5) for all four open-loop vertexes.



(a) V1 system. (b) V2 system.

Figura 4. FRFs of the complete model, reduced models and residual modes for polytopic vertexes V1 and V2.



(a) V3 system. (b) V4 system.

Figura 5. FRFs of the complete model, reduced models and residual modes for polytopic vertexes V3 and V4.

The LMI regulator (controller + observer) was designed considering all vertex system simultaneously. In this case, it was obtained a robust controller that mathematically guarantees the performance specifications in all convex space. The regulator is obtained from the solution of LMI problem of equations (10) and (15), with $\mathbf{x}_C(0)=[-0.009\ 0\ 0\ -0.009]^T$, $\mu= 0.5$, $\beta=1$ and $\gamma=6\alpha$. Figure (6) compares the FRF magnitude plots for open-loop and closed-loop system for all four vertex. The ressonance peaks of the controlled modes are reduced, as a result of the active damping. Furthermore, the amplitude of other modes, which are not explicitly included in the controller, are reduced occasionally. Nevertheless, some peaks increase the magnitude, because the controller leads to control spillover.

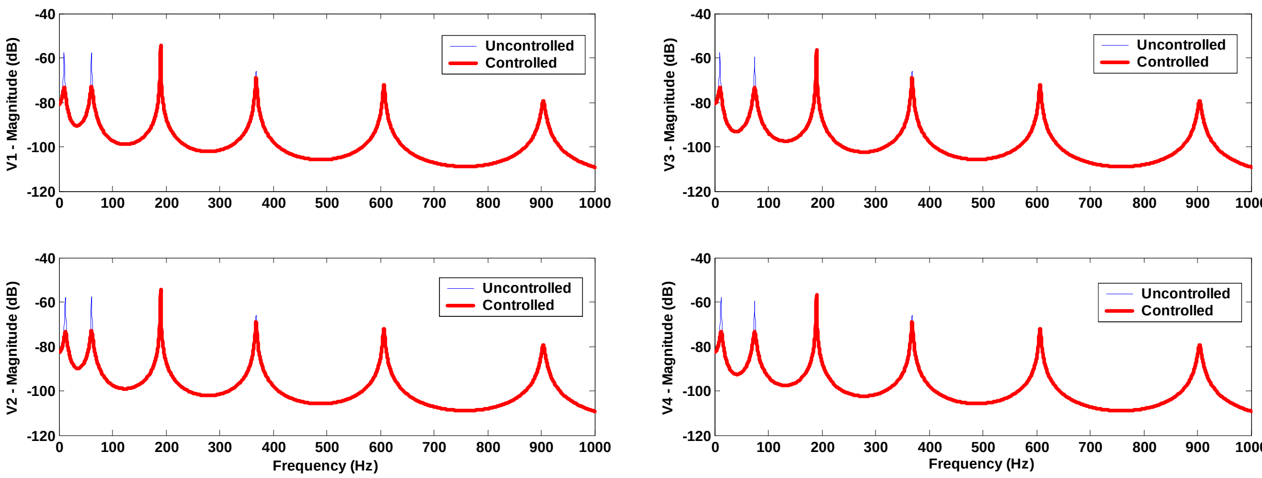


Figure 6. FRF for uncontrolled and controlled system considering polytopic vertexes.

Figure (7) shows the modal state in time domain for the controlled system (in the nominal condition) considering an impulse disturbance. Clearly, one observes the low influence of the high frequency dynamics in the structural control performance. Comparing the modal magnitude of residual modes, in fig. (7), we observe that spillover effect exist, but it is small when compared with modal magnitude in controlled modes.

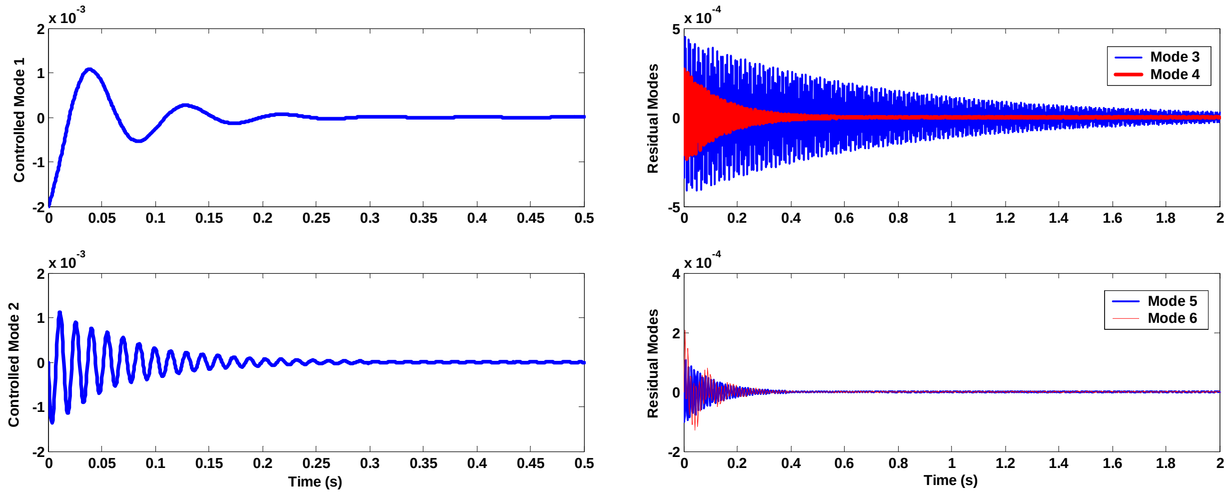


Figure 7. Closed-loop for controlled and residual modes in time domain in nominal conditions.

5. CONCLUSIONS

An LMI regulator (controller + observer) was proposed for vibration attenuation using only a small number of modes. To illustrate the proposed procedure was considered the control of a smart structure modeled by FEM. In a first phase the model was verified by comparing the experimental and numerical models. Its important to note that eletromechanical coupling can be considered as an uncertainty, and it should facilitate the modeling phase. The robust controller using reduced order model was designed to reject parametric variations, however other uncertainties should be computed. The results demonstrated the improvement of the system performance. The methodology developed can be extended to others practical systems, considering different constraints and different uncertainties. This combination seems to be the great differential of the LMI approach in active vibration control applications.

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