A Numerical Simulation of 3D Dynamic Green's Functions for Viscoelastic Anisotropic Solids

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Abstract
This article presents a numerical study of Green's and Influence functions for three-dimensional viscoelastic anisotropic solids in frequency domain. The equations of motion in a cylindrical set of coordinates are transformed using both Fourier and Hankel integral transforms. The time domain is transformed to the frequency domain by means of a conventional Fourier transform and the space coordinates are transformed to the wave number domain by means of a coupled Hankel transform and Fourier series in angular coordinate. A numerical study concerning the importance of the anisotropy in the behavior of such functions is also considered. The ability of the scheme to reproduce several boundary conditions is also considered.

Keywords: Green's functions, boundary element method, dynamic soil-structure interaction, anisotropy

1. INTRODUCTION

The dynamic behavior of the unlimited continuous media is fundamental to modeling several types of problems in structural mechanics where contact between the structure and the underlying supporting soil exists. The dynamic response of the underlying soil is needed to quantify the so-called geometric damping *(Wolf, 1985)*. Such a response is frequently obtained considering the soil as an unlimited domain. Refined models for studying the dynamic soil-structure interaction (DSSI) often require the inclusion of the energy which is taken away from the source as stress waves originated mainly in soil-foundation interface travel throughout the medium *(Gazetas, 1979).*

DSSI problems arise in a variety of practical situations involving foundation dynamics of machines and structures such as the study of vibrations of heavy machinery, rail and subway lines, roads, etc. The vibrations coming from the source are transmitted throughout the soil towards the immediate vicinity *(Wolf, 1985)* and dissipated through the several types of damping (hysteretic, geometric, etc). In present days innumerous industries have been obligated to transfer the location of their machines for environmental causes since vibrations emanating from such equipments affect directly the security and well been of the installations nearby. An adequate modeling of the phenomena arising from DSSI is, therefore, a starting point for looking to isolate or, at best, reduce to a minimum, vibration problems in such industrial plants.

The dynamic behavior of such unlimited media is modeled through the so-called Sommerfeld´s radiation condition also known as geometric damping *(Sommerfeld, 1949; Richart et alii, 1970; Mesquita, 1989)*. The numerical methods based on domain discreteness such as Finite Differences (FDM) and the Finite Element Method (FEM) have a fundamental problem when modeling such unlimited domains since their meshes cannot extend to the infinite *(Beskos, 1987; Barros, 2001)*. The resulting meshes in these methods must be truncated, originating an artificial boundary which causes wave reflection and spurious eigenvalues in dynamic analysis because such reflected waves violate the Sommerfeld´s condition. An efficient alternative for modeling such unlimited domains is the Boundary Element Method (BEM) in its direct and indirect versions *(Dominguez, 1993)*.

The most adopted version of the BEM uses the so-called full-space solutions witch are obtained via solution of the differential operator governing the problem together with certain boundary conditions. In fact, the name full-space solutions applies if only radiation condition is required for the solution and no other boundary exists. The usage of the full-space solutions in BEM allows the fulfillment of the radiation condition in an automatic way but requires that any other boundary should be discretized. This need tend to cause numerical problems in situations where a frontier extends toward infinite, which is a common situation in DSSI problems *(Gazetas, 1979)*.

Another efficient alternative is the usage of Green´s and Influence functions directly in BEM integral formulation. Such Green´s functions are obtained in a similar way as the full-space solutions but are more specific since these functions also fulfill specific boundary conditions in certain critical boundaries such as traction-free surfaces ou interfaces where the load distribution is known or estimated. The main advantage in the usage of Green´s functions is that these specific boundary conditions are also fulfilled automatically in addition to the radiation condition. This implies that only the surfaces having boundary conditions witch differ from those originally imposed in the Green´s functions need to be discretized. This characteristic of the formulation of the BEM using Green´s functions greatly reduces the size of the [G] and [H] matrices in BEM.

The dynamic response of the anisotropic media in frequency domain received a considerate attention between 1940 and 1950. Stoneley (1949) and Synge (1956) studied the problem of elastic wave propagation in transversally anisotropic media. An interesting result in Singe´s analysis is that surface (or Rayleigh) waves exists in the medium only if one of the material principal axes is normal or is contained in a plane parallel to the surface. Latter, Achenbach (1973) obtained an integral representation for the displacement field in an elastic half-space via double Fourier transform in a rectangular set of coordinates. Wang and Rajapakse (1992) presented some results concerning Green´s and Influence functions in static and dynamic problems in transversely isotropic materials. Latter, these authors applied this set of results in the analysis of certain boundary value problems of structures resting on the surface of an anisotropic half-space.

2. GREEN´S FUNCTIONS IN CYLINDRICAL COORDINATES

The equations of motion for a viscoelastic, transversely isotropic medium in terms of displacement potentials $\phi(r,\theta,z)$, $\psi(r,\theta,z)$ e $\chi(r,\theta,z)$ are (Achenbach, 1973):

$$
\begin{gathered}
(c_{13}+c_{44})\nabla^2\frac{\partial^2\chi}{\partial z^2}+c_{11}\nabla^2\nabla^2\phi+c_{44}\nabla^2\frac{\partial^2\phi}{\partial z^2}+\rho\omega^2\nabla^2\phi=0 \\
\tfrac{c_{11}-c_{12}}{2}\nabla^2\nabla^2\psi+c_{44}\nabla^2\frac{\partial^2\psi}{\partial z^2}+\rho\omega^2\nabla^2\psi=0 \\
(c_{13}+c_{44})\nabla^2\frac{\partial^2\phi}{\partial z^2}+c_{44}\nabla^2\nabla^2\chi+c_{33}\nabla^2\frac{\partial^2\chi}{\partial z^2}+\rho\omega^2\nabla^2\chi=0
\end{gathered}
\tag{1}
$$

where coefficients c_{ij} represent the elements of the elastic properties. These potentials are related to the displacement fields in cylindrical coordinates as:

$$u_r(r,\theta,z) = \frac{\partial\phi}{\partial r} + \frac{1}{r}\frac{\partial\psi}{\partial\theta} \quad , \quad u_\theta = \frac{1}{r}\frac{\partial\phi}{\partial\theta} - \frac{\partial\psi}{\partial r} \quad \text{and} \quad u_z = \frac{\partial\chi}{\partial z} \tag{2}$$

The constitutive law for a transversely isotropic medium tensor in a cylindrical set of coordinates can be found in Lekhnitskii (1963):

$$\begin{Bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \\ \sigma_{\theta z} \\ \sigma_{rz} \\ \sigma_{r\theta} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(c_{11} - c_{12}) \end{bmatrix} \begin{Bmatrix} \varepsilon_{rr} \\ \varepsilon_{\theta\theta} \\ \varepsilon_{zz} \\ 2\varepsilon_{\theta z} \\ 2\varepsilon_{rz} \\ 2\varepsilon_{r\theta} \end{Bmatrix} \tag{3}$$

where σ_{ij} and ε_{ij} are the stress and the strain tensors admitting both harmonic excitation and response with angular frequency ω in such a way that total displacements can be represented as $U_i(r,t) = u_i(r)\exp(i\omega t + \varphi)$ for $i=r,z,\theta$.

The solution for the set of equations (1) in terms of the displacement potentials can be obtained following the steps:

1. The displacement potentials ϕ, ψ e χ are given in terms of a discrete Fourier series in angular coordinate θ as (Wolf, 1985):

$$\begin{aligned} \phi(r,\theta,z) &= \sum_{m=0}^{\infty} \phi_m(r,z)\cos(m\theta) + \phi_m^* \operatorname{sen}(m\theta) \\ \psi(r,\theta,z) &= \sum_{m=0}^{\infty} \psi_m(r,z)\operatorname{sen}(m\theta) - \psi_m^* \cos(m\theta) \\ \chi(r,\theta,z) &= \sum_{m=0}^{\infty} \chi_m(r,z)\cos(m\theta) + \chi_m^* \operatorname{sen}(m\theta) \end{aligned} \tag{4}$$

2. Usage of Hankel transform (H-transform) in radial coordinate as defined in Sneddon (1972) as:

$$\bar{f}(\lambda) = \int_0^{\infty} f(r)\,\mathbf{J}_m(\lambda r) r\,dr \quad \Leftrightarrow \quad f(r) = \int_0^{\infty} \bar{f}(\lambda)\,\mathbf{J}_m(\lambda r)\lambda\,d\lambda \tag{5}$$

where $J_m(\lambda r)$ represents a Bessel function of the first kind and m^{th}-order. Thus, for the potentials:

$$\left\{\phi_m(r,z),\psi_m(r,z),\chi_m(r,z)\right\} = \int_0^\infty \left\{\phi_m^{(m)},\psi_m^{(m)},\chi_m^{(m)}\right\} J_m(\lambda r)\lambda d\lambda \tag{6}$$

After substituting equations (4) and (6) in (1) one gives the equations of motion in the transformed domain λ:

$$\begin{aligned}
\kappa\frac{d^2\chi_m^{(m)}}{dz^2}+\frac{d^2\phi_m^{(m)}}{dz^2}-\left(\beta\lambda^2-\delta^2\right)\phi_m^{(m)} &= 0\\
\frac{d^2\psi_m^{(m)}}{dz^2}-\left(\varsigma\lambda^2-\delta^2\right)\psi_m^{(m)} &= 0\\
-\kappa\lambda^2\frac{d^2\phi_m^{(m)}}{dz^2}+\alpha\frac{d^2\chi_m^{(m)}}{dz^2}-\left(\lambda^2-\delta^2\right)\chi_m^{(m)} &= 0
\end{aligned} \tag{7}$$

where parameters α, β, κ , ς e δ are defined by:

$$\alpha=\frac{c_{33}}{c_{44}} \qquad \beta=\frac{c_{11}}{c_{44}} \qquad \kappa=\frac{\left(c_{13}+c_{44}\right)}{c_{44}} \qquad \varsigma=\frac{c_{11}-c_{12}}{2c_{44}} \qquad \delta^2=\frac{\rho\omega^2}{c_{44}} \tag{8}$$

It is also useful to define an admentionalized frequency in terms of material property c_{44} and external frequency ω as:

$$a_0 = a\omega\sqrt{\frac{\rho}{c_{44}}} \tag{9}$$

where ***a*** is a characteristic dimensional constant. It is also important to note that the choice for such potential functions does not uncouple the original equations of motions (7) as would expect, due to the anisotropic behavior of the solution. If, on the other hand, a isotropic behavior was considered, the formulations due to Wolf (1985), Ron Pak (1987) and Romanini (1996) would be sufficient to find such potentials directly.

The solutions of the equations set (7) is, therefore, obtained using the same separation process used by Almeida Barros (1995). It should be noted, however, that the usage of the H-transform allows the second equation to be solved promptly and in such a way that this solution is independent of the remaining ones. Therefore, following the steps (1) and (2) outlined above, one can obtain the solution for the potential harmonics in λ-domain as:

$$\begin{aligned}
\phi_m^{(m)}(\lambda,z) &= \omega_1 A_1 e^{-\delta\xi_1 z}+\omega_1 B_1 e^{\delta\xi_1 z}+\omega_2 A_2 e^{-\delta\xi_2 z}+\omega_2 B_2 e^{\delta\xi_2 z}\\
\psi_m^{(m)}(\lambda,z) &= A_3 e^{-\delta\xi_3 z}+B_3 e^{\delta\xi_3 z}\\
\chi_m^{(m)}(\lambda,z) &= A_1 e^{-\delta\xi_1 z}+B_1 e^{\delta\xi_1 z}+A_2 e^{-\delta\xi_2 z}+B_2 e^{\delta\xi_2 z}
\end{aligned} \tag{10}$$

where $\xi_{1,2}$ and ξ_3 are given by:

$$\xi_{1,2} = \pm\sqrt{\frac{\gamma\zeta^2-\alpha-1\pm\sqrt{\Xi}}{2\alpha}} \tag{11}$$

$$\xi_3 = \pm\sqrt{\varsigma\zeta^2-1}\,. \tag{12}$$

and $\Xi = \left(1+\alpha-\gamma\zeta^2\right)^2 - 4\alpha\left(1+\beta\zeta^4-\beta\zeta^2-\zeta^2\right)$. In addition, factors ω_1 and ω_2 are given by:

$$\omega_{1,2} = \frac{\alpha\xi_{1,2}^2 - \zeta^2 + 1}{\kappa\zeta^2} \tag{13}$$

In the equations (10), terms A1...B1 represent arbitrary functions to be solved accordingly to a prescribed boundary condition set. Taking terms ξ_i , i=1,2,3 in such a way that $Re(\xi_i) \geq 0$, functions A_i stands for wave amplitudes moving towards +z (away from the source) and B_i stands for wave amplitudes moving towards -z (closing the source

2.1 Displacement fields in Hankel domain:

Displacement fields u_r, u_θ e u_z are given in λ-domain as a combination of integration constants A_i and B_i, i=1,2,3 and the harmonics of the transformed potentials. Applying the Fourier series expansion of such potentials for azimuth angle θ it is possible to obtain formulae for displacement fields as:

$$\begin{aligned}
u_r &= \phi_{,r} + \frac{1}{r}\psi_{,\theta} = \sum_{m=0}^{\infty}\left(\phi_{m,r} + \frac{m}{r}\psi_m\right)\cos(m\theta) \\
u_\theta &= \frac{1}{r}\phi_{,\theta} - \psi_{,r} = -\sum_{m=0}^{\infty}\left(\frac{m}{r}\phi_m + \psi_{m,r}\right)\mathrm{sen}(m\theta) \\
u_z &= \chi_{,z} = \sum_{m=0}^{\infty}\chi_{m,z}\cos(m\theta)
\end{aligned} \tag{14}$$

where:

$$\begin{aligned}
\phi_m(r,z) &= \int_0^\infty \phi_m^{(m)} J_m(\lambda r)\lambda d\lambda = \int_0^\infty \left(\sum_{i=1}^{2}\omega_i A_i e^{-\delta\xi_i z} + \omega_i B_i e^{\delta\xi_i z}\right) J_m(\lambda r)\lambda d\lambda \\
\psi_m(r,z) &= \int_0^\infty \psi_m^{(m)} J_m(\lambda r)\lambda d\lambda = \int_0^\infty \left(A_3 e^{-\delta\xi_3 z} + B_3 e^{\delta\xi_3 z}\right) J_m(\lambda r)\lambda d\lambda \\
\chi_m(r,z) &= \int_0^\infty \chi_m^{(m)} J_m(\lambda r)\lambda d\lambda = \int_0^\infty \left(\sum_{i=1}^{2} A_i e^{-\delta\xi_i z} + B_i e^{\delta\xi_i z}\right) J_m(\lambda r)\lambda d\lambda .
\end{aligned} \tag{15}$$

Substituting equations (15) in (14) it is possible to obtain displacement fields u_r, u_θ e u_z as:

$$\begin{aligned}
u_{rm} &= \int_0^\infty \bar{u}_{rm}\lambda d\lambda = \int_0^\infty \left(a_1A_1e^{-\delta\xi_1 z} + a_1B_1e^{\delta\xi_1 z} + a_2A_2e^{-\delta\xi_2 z} + a_2B_2e^{\delta\xi_2 z} + a_3B_3e^{\delta\xi_3 z} + a_3B_3e^{\delta\xi_3 z}\right)\lambda d\lambda \\
u_{\theta m} &= -\int_0^\infty \bar{u}_{\theta m}\lambda d\lambda = -\int_0^\infty \left(a_4A_1e^{-\delta\xi_1 z} + a_4B_1e^{\delta\xi_1 z} + a_5A_2e^{-\delta\xi_2 z} + a_5B_2e^{\delta\xi_2 z} + a_6B_3e^{\delta\xi_3 z} + a_6B_3e^{\delta\xi_3 z}\right)\lambda d\lambda \\
u_{zm} &= \int_0^\infty \bar{u}_{zm}\lambda d\lambda = \int_0^\infty \left(-a_7A_1e^{-\delta\xi_1 z} + a_7B_1e^{\delta\xi_1 z} - a_8A_2e^{-\delta\xi_2 z} + a_8B_2e^{\delta\xi_2 z}\right)\lambda d\lambda
\end{aligned} \tag{16}$$

where $a_1,...,a_7$ are functions obtained as combination of Bessel functions and admentionalized elastic characteristics given in (8).

2.2 Stress fields in Hankel domain:

In a similar way as treated above, one can obtain expressions for stresses fields in λ-domain using (14) and considering the expansions of the potentials using the Fourier harmonic expansion. In this paper only expressions of σ_{rz} , $\sigma_{\theta z}$ e σ_{zz} will be present, since this expressions will be sufficient for the solving process of the integration constants when one of the material principal direction is parallel to the z-axis.

a) constants for $\sigma_{zz}(r,\theta,z)$:

Assuming material linear behavior and expressing strain tensor ε_{ij} using the displacement fields as given above, it is possible to obtain the displacement fields as combination of the potential functions given in (15). Therefore:

$$\sigma_{zz} = \sum_{m=0}^{\infty} \sigma_{zzm} \cos(m\theta) \qquad \text{where} \qquad \sigma_{zzm} = \int_0^{\infty} \sigma_{zzm}^{(m)} \lambda d\lambda \tag{17}$$

The harmonics $\sigma_{zzm}^{(m)}$ in λ-domain are given by:

$$\frac{1}{c_{44}} \sigma_{zzm}^{(m)} = \sum_{i=1}^{2} b_{2i} \left(A_i e^{-\delta\xi_i z} + B_i e^{-\delta\xi_i z} \right) \quad \text{e} \quad b_{2i} = \left[\alpha\delta^2\xi_i^2 - (\kappa - 1)\omega_i \lambda^2 \right] J_m(\lambda r) \quad \text{for } i=1,2 \tag{18}$$

b) constants for $\sigma_{\theta z}(r,\theta,z)$:

Using again the displacement fields in terms of Fourier harmonics series one can write:

$$\sigma_{\theta z} = \sum_{m=0}^{\infty} \sigma_{\theta zm} \operatorname{sen}(m\theta) \quad \text{where} \quad \sigma_{\theta zm} = \int_0^{\infty} \sigma_{\theta zm}^{(m)} \lambda d\lambda \tag{19}$$

and, in a similar way, the harmonics $\sigma_{\theta zm}^{(m)}$ are given by:

$$\frac{1}{c_{44}} \sigma_{\theta zm}^{(m)} = \sum_{i=1}^{3} b_{4i} \left(A_i e^{-\delta\xi_i z} - B_i e^{-\delta\xi_i z} \right) \tag{20}$$

where:

$$b_{4i} = (1 + \omega_i)\delta\xi_i \frac{m}{r} J_m(\lambda r) = (1 + \omega_i)\delta\xi_i \left(J_{m-1} + J_{m+1} \right) \quad \text{for } i=1,2 \tag{21}$$

c) constants for $\sigma_{rz}(r,\theta,z)$:

In the same way as before, one can write:

$$\sigma_{rz} = \sum_{m=0}^{\infty} \sigma_{rzm} \cos(m\theta) \qquad \text{where} \qquad \sigma_{rzm} = \int_0^{\infty} \sigma_{rzm}^{(m)} \lambda d\lambda \tag{22}$$

and the harmonics $\sigma_{\theta zm}^{(m)}$ are given by:

$$\frac{1}{c_{44}} \sigma_{rzm}^{(m)} = \sum_{i=1}^{3} b_{5i} \left(-A_i e^{-\delta\xi_i z} + B_i e^{-\delta\xi_i z} \right)$$

where:

$$b_{5i} = (1+\omega_i)\delta\xi_i \frac{\lambda}{2}(J_{m-1} - J_{m+1}) \qquad \text{for } i=1,2 \tag{23}$$

$$b_{53} = \delta\xi_3 \frac{\lambda}{2}(J_{m-1} + J_{m+1})$$

The general formulation described above can be used for determination of any integration constant set (A_i, B_i} accordingly to the prescribed boundary condition functions if, and only if, the H-transform of these prescribed conditions exists. As an example of such precess, one will obtain the components of the Green´s tensor G_{iZ} for a vertical axi-symmetric loading acting on the surface of an anisotropic half-space. The loading and displacement/stress representation through harmonic analysis requires only the fundamental (m=0) harmonic, as described in figure 1.

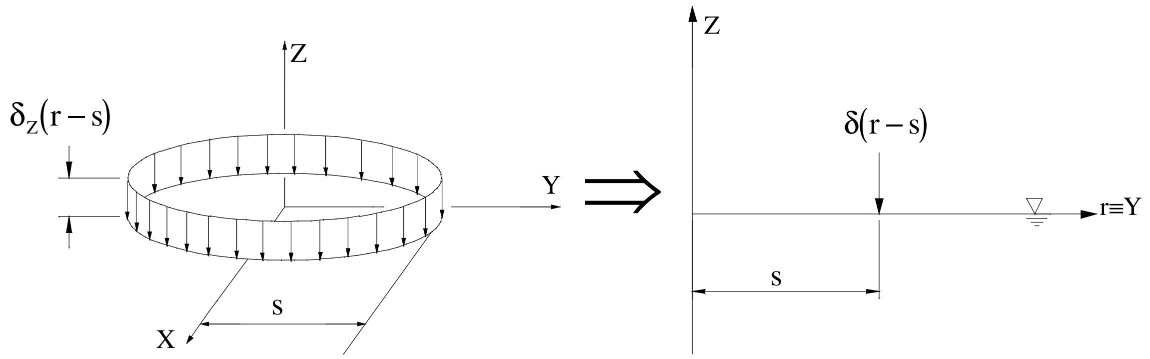

Figure 1: Geometry of the axi-symmetric case with vertical loading

The boundary value problem in stress (PVCT) can be described mathematically as:

$$PVCT \begin{cases} \sigma_{rz}, \sigma_{zz} = 0 & \text{para} \quad z=0; r \geq 0 \ \text{ e } \ r \neq s \\ \sigma_{zz} = \delta(r-s) & \text{para} \quad z=0; r=s \end{cases} \tag{24}$$

Applying radiation condition directly in the displacement fields in λ-domain one can have immediately $B_i = 0$ for i = 1,...,n since no wave will travel toward the source. The solution of the PVCT for the remaining integration constants A_i is:

$$A_1(\lambda) = \frac{b_{32}}{b_{21}b_{32} - b_{31}b_{22}} \frac{sJ_0(\lambda s)}{c_{44}} \tag{25a}$$

$$A_2(\lambda) = \frac{-b_{31}}{b_{21}b_{32} - b_{31}b_{22}} \frac{sJ_0(\lambda s)}{c_{44}} \tag{25b}$$

and, for displacement fields:

$$\overline{u}_{r0} = \overline{G}_{rz} = \frac{\omega_1 b_{32} e^{-\delta\xi_1 z} - \omega_2 b_{31} e^{-\delta\xi_2 z}}{b_{21}b_{32} - b_{31}b_{22}} \frac{sJ_0(\lambda s)}{c_{44}} \tag{26a}$$

$$\overline{u}_{z0} = \overline{G}_{zz} = \frac{b_{32} e^{-\delta\xi_1 z} + b_{31} e^{-\delta\xi_2 z}}{b_{21}b_{32} - b_{31}b_{22}} \frac{sJ_0(\lambda s)}{c_{44}} \tag{26b}$$

$$\overline{u}_{\theta 0} = \overline{G}_{\theta z} = 0 \tag{26c}$$

3. NUMERICAL EVALUATION OF THE GREEN´S TENSOR

The Green´s tensor for vertical, axi-symmetric loading case described above was evaluated numerically using both the Clenshaw-Curtiss method for finite range $[0,\lambda_{FIN}]$ and Wynn extrapolation method for the infinite range $[\lambda_{FIN}\ ,\ \infty]$ where λ_{FIN} is defined as 1.2 times the maximum pole λ_{MAX} in the integration kernels in equations (25) and (26). The results were also evaluated in three different frequencies a_0=0.50, a_0=1.00 e a_0=3.00. All presented results correspond to a maximum absolute error of 1.0e-6. The number of integration points for a_0=1.0 was about 500 points for the finite interval and 200 points for the infinite part although this number have suffered some variation due to the distance taken to the loading ring radius s_0. Materials with different degrees of anisotropy were analysed and their elastic constants were summarised in table 1, normalised with c_{44}. Table 2 presents anisotropy indices as defined in Lekhnitskii (1963).

The analysis of the results indicates that the degree of anisotropy plays a significant role in both real and imaginary part of the Green´s tensor. The part $Re(G_{ij})$ suffer an abrupt variation in the point of loading ring s_0=1 due mainly the discontinuous stress field which is expected to exist in this region. This change in solution profile is more gradual for composite materials results since these materials are much more rigid in transversal (3) direction than the others as indicated by the c_{33} constant in table 1. All three sets of results correspond to a hysteretic damping coefficient η=0.01

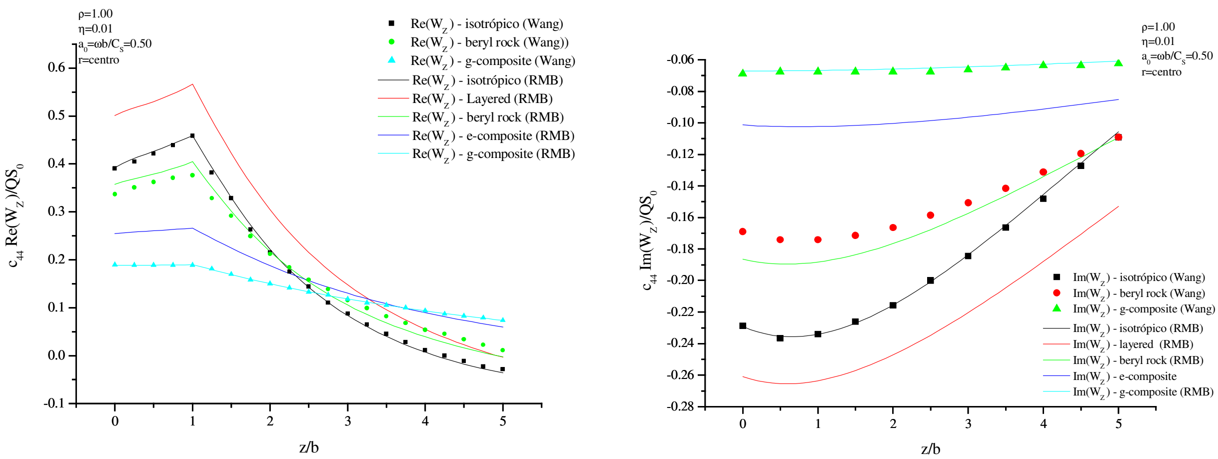

Figure 2: Vertical normalized displacement Gzz due to a vertical concentrated ring with radius s0=1. Results for a0=0.50 and η=0.01.

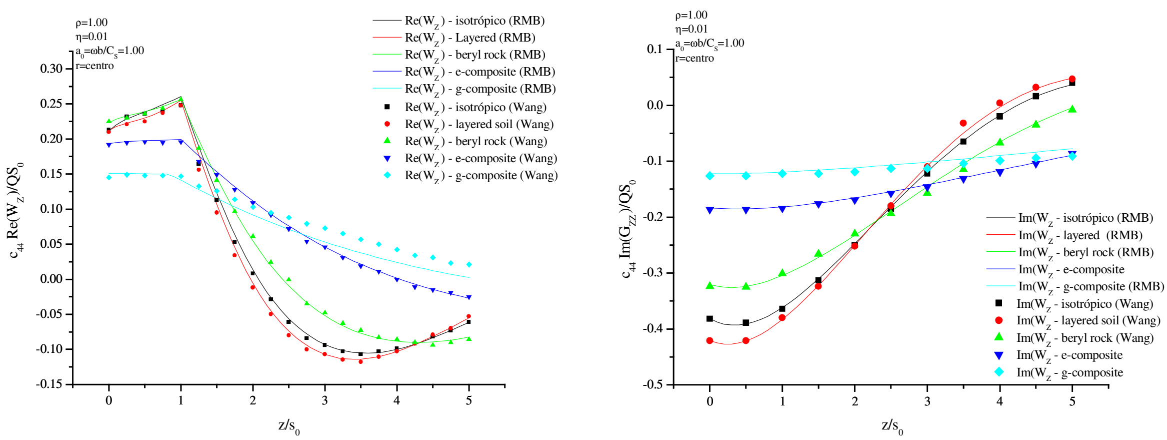

Figure 3: Vertical normalized displacement Gzz due to a vertical concentrated ring with radius s0=1. Results for a0=1.00 and η=0.01.

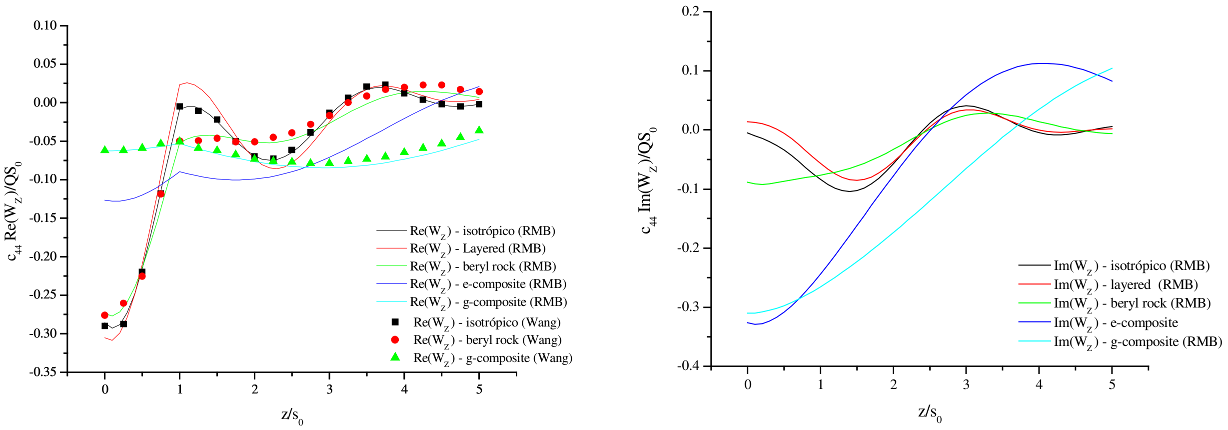

Figure 4: Vertical normalized displacement Gzz due to a vertical concentrated ring with radius s0=1. Results for a0=3.00 and η=0.01.

Isotropic case in table 1 corresponds to Lamè constants $\lambda=\mu=0.50x10^4$ N/mm^2 ($\nu=0.25$). The first layered soil corresponds to alternate and compacted layers of chalk ($CaCO_3$) and silicates. Second type stands for the same alternated chalk/silicates but is non-compacted. Composite materials represent epoxy/fibreglass (e-composite) and graphite/epoxy (g-composite) groups.

Table 1: mechanical characteristics of the materials used in Figures 2,3,and 4

Material	$\bar{c}_{11}$	$\bar{c}_{12}$	$\bar{c}_{13}$	$\bar{c}_{33}$	c_{44} (10^4 N/mm^2)
isotropic	3.00	1.00	1.00	3.00	1.00
layered[1]	4.46	1.56	1.24	3.26	1.40
layered[2]	2.11	0.43	2.58	0.47	1.40
rock (beryllium)	4.13	1.47	1.01	3.62	1.00
e-composite	3.17	1.40	1.11	10.04	0.47
g-composite	2.02	0.68	0.07	21.17	0.41

Table 2: anisotropy indices for materials in table 1

Material	n_1	n_2	n_3
isotropic	1.00	1.00	1.00
layered[1]	0.7309	1.45	1.987
layered[2]	0.818	0.60	1.755
rock (beryllium)	0.876	1.33	1.12
e-composite	3.167	0.885	1.054
g-composite	10.473	0.671	0.307

4. CONCLUSIONS

Green´s functions usage in the boundary element method is a well-known alternative for treatment of several PVCT´s in which boundary conditions are too complex to be treated with the conventional full-space fundamental solutions. Such situations arise, for instance, when dealing with several common problems of the dynamic soil-structure interaction. However, a cumbersome theoretical formulation and very special numerical procedures not always trivial in engineering practice must be used, particularly when anisotropic materials are considered. However many cases of practical interest can be analyzed with simple axi-symmetric functions which can be represented with only the fundamental harmonic in Fourier series. The results presented in figures 2,3 and 4 show a good agreement with available literature.

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