SOME ASPECTS OF REPRESENTATION OF SOLIDS THROUGH LATTICE MODELS

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***Abstract**. This article describes the modeling of beams and plates through spatial lattice models by using a proper arrangement of trusses which take into account large displacements, rotations and strains. The use of truss elements to represent a non-linear solid behavior have the benefit of allowing complex matters, like the kinematics, to be approached in a simpler way, since these elements admit only extension, contraction and rigid body motion. The analysis shown here uses an in house finite element program capable to take into account the major features of a solid behavior. The studied cases comprise the representation of a cantilever beam, a simply supported beam and a plate with the developed non-linear truss finite element. The results are compared with numeric solutions given by a commercial finite element program. The validity of the discretization of beams and plates via the non-linear truss elements is here demonstrated.*

***Keywords:** Lattice Models, Truss Structures, Large displacements.*

1. INTRODUCTION

Structures composed by interlinked groups of trusses have been much explored in both old and recent works. Due to the low cost and light weight, lattice structures may be employed in dayly applications, e.g., truss panels used in civil engineering and in future aerospace structures (Burgardt and Cartraud, 1999).

Of course that it is necessary to measure the material mechanical properties of these trusses made by a repetition of a given pattern cell. Accordingly, Chiras et al (2002) verified that the core panel of a tri-dimensional tetragonal beryllium-cooper alloy pattern cell is capable of supporting bending and compression loads at lower weight than competing structures. In this octet topology, in regions where the core experiences shear, the trusses experience only compression or tension. The authors highlighted the substantial differences that arise in the responses of trusses undergoing tension or compression. The trusses in tension continue to strain harden beyond yield, while trusses in compression exhibit buckling after a small plastic strain, limiting their load capacity.

Wallach and Gibson (2001) implemented an experimental protocol with pentagonal aluminum cells. The authors measured the mechanical properties of a “solid-like” truss panel. By compressing a panel made from beam and truss elements, they concluded that the error in the maximum displacement associated with neglecting bending moments was 1.4%, approximately.

Wicks and Hutchinson (2001) designed truss core panels by optimizing/minimizing its weight. The optimized plates were compared with similarly optimized honeycomb core plates fashioned from the same material, with the truss lattice presenting a rather greater global stiffness.

Another aspect of the use of lattice is in the modeling of real solids structures. It is possible to represent a solid by a certain pattern comprising truss members. This is only feasible nowadays if the trusses are analyzed using a computer program and even more attractive if the finite element technique is chosen. The advantages are manifold, like the possibility of representing complex non-linear solid behaviour and the analysis of brittle fracture, like in concrete.

Indeed, the lattice model has been used to represent straightforwardly the material heterogeneity in concrete at the meso-level (Lilliu and van Mier, 2003). Bolander and Le (1999) used a spring network model to study crack propagation in reinforced concrete structures, concluding that their lattice model was an effective framework for a crack propagation criterion. Van Mier and van Vliet (1999), utilizing a triangular pattern cell of beams elements, explored the influence of the mesh refinement and concluded that the results could be improved by decreasing element length.

Many others researchers (Raghuprasad et. al., 1998; Psakhie et. al., 2000; Mohamed and Hansen, 1999; Ibrahimbegovic and Delaplace, 2003 and Iturrioz et. al., 2000) worked in the modeling of concrete by lattice structures utilizing different pattern cells.

Chen et. al. (1998) develop a generalized continuum model for cellular materials based on the equivalence of macroscopic strain energy to the microscale tensile and bending energy in cell walls. By postulating a maximum-tensile stress failure criterion for cell walls, they predicted analytically the fracture toughness of cellular materials with hexagonal, triangular and square lattice. The results indicated that square lattice provided a rather small fracture toughness.

Damage in composite materials caused by solid-particle erosion was simulated using a computational model by Chen and Li (2003). The material system was discretized and mapped onto a discrete lattice with each cell representing a solid particle. During erosion, a lattice cell might move under the influence of the solid particles moving through the target material. It was demonstrated that the model was effective for investigating the mechanism responsible for material erosion, as well as helpful for establishing the relationship between the microstructure of a material and its wear behavior.

Ignatovich et al. (2000) used a lattice model to represent, in a crashworthiness analysis, vehicle and passenger. An optimization was proposed to determine the geometrical parameters of the pattern cell by minimizing the acceleration of the passenger. The analysis suggested that lattice models can be tuned to match at least some aspects of the behavior of more complex structural system.

Riera and Iturrioz (1995) determined the dynamic response of elastoplastic plates and shells subjected to impulsive loading, employing a discrete representation of the continuum associated with an explicit integration scheme in time domain.

Lattice structure can be replaced by a continuum model like a beam or plate, mainly for analysis of large repetitive lattices (Burgardt and Cartraud, 1999 and Moreau and Caillerie, 1998). However, the approach followed here is based on the substitution of a solid by truss like lattice. It is necessary to establish an analytical method to evaluate the parameters of the chosen pattern cell which will form the structure (Fraternali et. al., 2002; Ostoja-Starzewski, 2002 and Hrennikoff, 1941).

In this present work, we seek to use a cell pattern to reproduce the same volume and stiffness as in the original solid structure, under conditions of static load and large displacements. The cell pattern is based on Hrennikoff (1941).

The next section presents the chosen pattern cell and the evaluation of its geometrical parameters. It follows the simulation of three structures, two beam-like trusses and one plate-like truss, which are submitted to static load. The displacements obtained via the lattice model and a finite element analysis using beam and plates finite elements are compared.

2. MODELS

The motivation which impelled Hernnikoff (1941) to develop a method named by himself the Framework Method, was the mathematical difficulties which made the solution of differential equations of the theory of elasticity impossible in many cases. The method may be applied, for example, to problems of two-dimensional stress, bending of plates, bending of cylindrical shells and the general case of three-dimensional stress.

The framework, formed by bars arranged according to a definite pattern, have the same external outline and boundary restraints, and is subjected to the same loads as the solid body, the loads being all applied at the joints. He demonstrated that if the length of bars is made infinitesimal, the framework will represent a complete similar mechanical model of the solid prototype, with identical displacements, strains, and cell stress (which is different from stress in individual bars).

According to Hernnikoff (1941), the necessary and sufficient condition for the equivalence of the infinitesimal framework and the solid material is the deformability of the two to be equal.

Here, we replace a beam and a plate by a lattice built by Hernnikoff's method.

Two pattern cells were chosen to represent bi and tri-dimensional solids, i.e. the beam and the plate, respectively.

The first cell is present in Fig. (1) and it has two parameters, the areas of the side and of the diagonal truss bars, which can be found from the length of the cells, *a*, and the thickness of the beam, *t*.

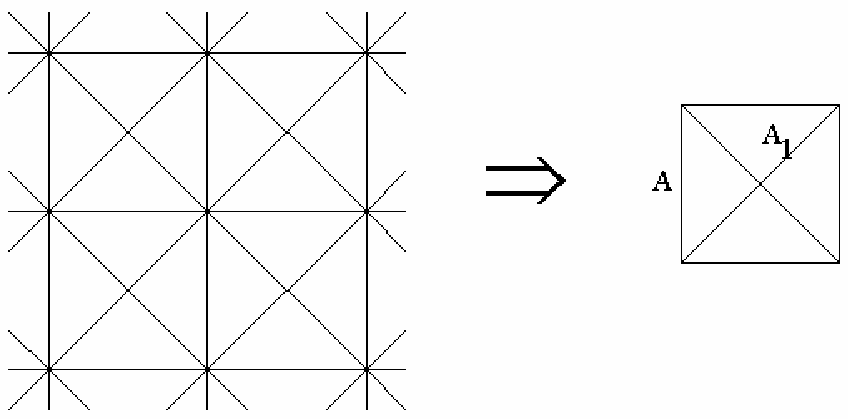

Figure 1. Bi-dimensional pattern cell for beam representation

The framework method point out that, for the square pattern cell presented above and for Poisson's ratio equal to $^1/_3$, the areas of the orthogonal and of the diagonal truss bars were, respectively

$$A = \frac{3}{4}\,at \tag{1}$$

$$A_1 = \frac{3}{4\sqrt{2}}\,at \tag{2}$$

It is important to note that the bars lying on the boundary of the framework, in order to satisfy conditions of deformability, must have one half of the greatest area.

As for the plate representation, it requires a tri-dimensional truss lattice composed by tri-dimensional cells, Fig. (2). This is due to the out of plane bending resistance.

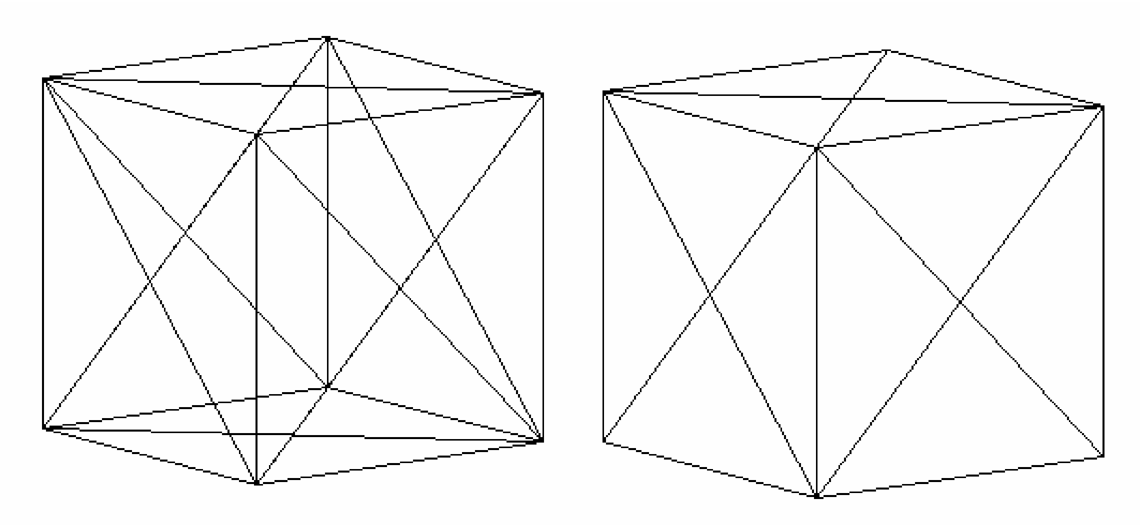

Figure 2. Tri-dimensional pattern cell for plate representation.

For the tri-dimensional cells, the two parameters presented still were the ones necessary to build the lattice. For a cubic cell of Poisson's ratio equal to ¼, edge *a* and plate thickness *t*, the areas are given by

$$A = \frac{2}{5}a^2 \tag{3}$$

$$A_1 = 0.5657a^2 \tag{4}$$

Again, we note that the bars belonging to faces of the lattice, i.e. the boundaries, have one half of the internal area of the members presented above and the bars in the lattice edges should have one quarter of the internal bars area.

3. SIMULATIONS

A beam with length of 1m, thickness of 5mm, height of 100mm, Young's modulus of 210GPa and Poisson's ratio of $^1/_3$, was simulated considering two different boundary conditions: a cantilever and simple supported beam. The truss lattice model representing the beam has 250 bi-dimensional pattern cells, Fig. (3).

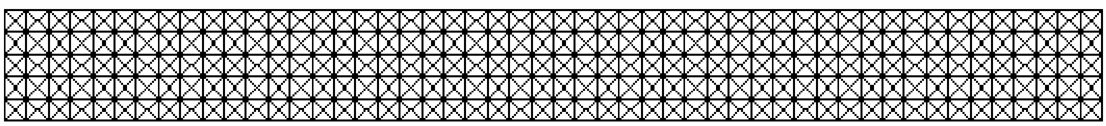

Figure 3. Bi-dimensional truss lattice model representation of the beam

The truss lattice model representing the plate was built by arranging 400 tri-dimensional pattern cells, Fig. (4). The plate is square of 100mm side and 5mm width, Young's modulus of 210GPa and Poisson's ratio of $^1/_4$ and had one of its side clamped.

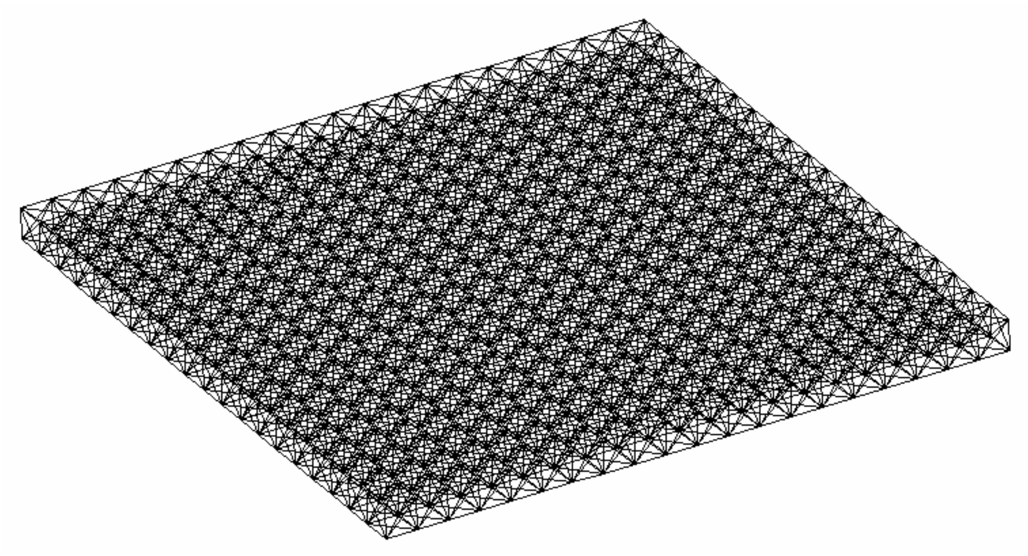

Figure 4. Tri-dimensional truss lattice model representation of the plate

The results of the simulations and comparisons between the solids and the lattice models are presented in the next sections.

4. RESULTS

The beams and the plate were also simulated with finite beam and shell elements available in ABAQUS. For the cantilever beam, a concentrated static load of 250KN was applied. For the simply supported case, the load was of 2500KN, applied in the middle of the beam. A concentrated load of 30KN was applied to one free vertex of the plate.

The beam lattice model was simulated using a in house non-linear truss finite element developed by Driemeier *et al*, 2004. The plate lattice model used the truss finite element available in the ABAQUS code.

The final configurations for all three tests are presented in Fig. (5), Fig. (6) and Fig. (7).

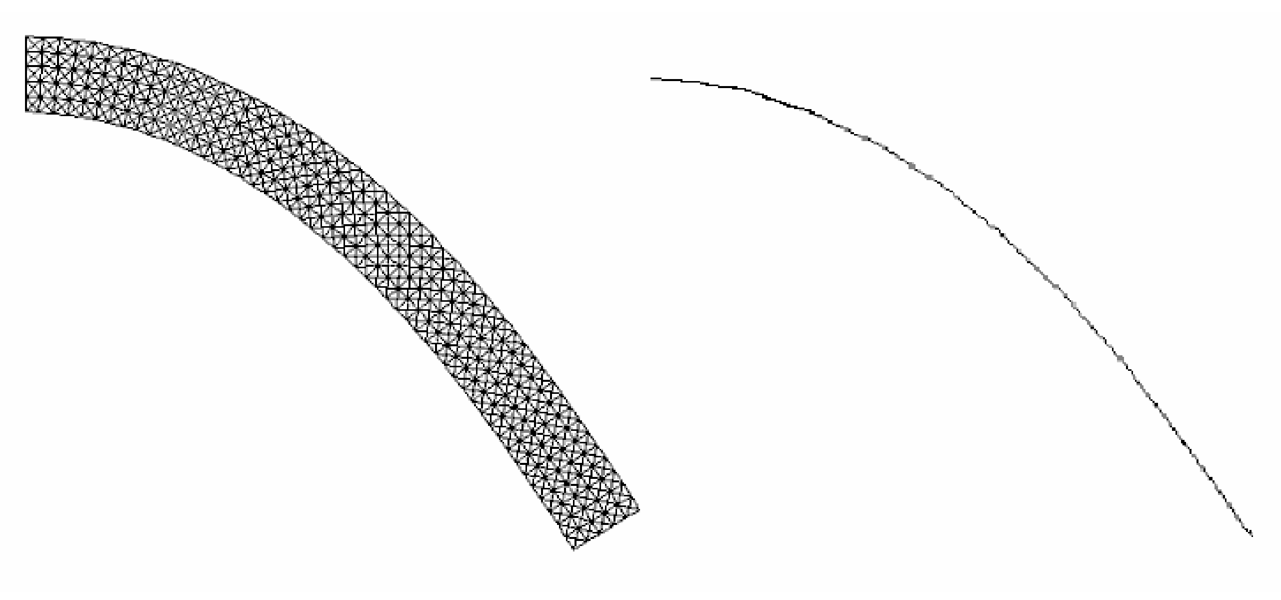

Figure 5. Final configuration for the cantilever modeled by truss lattice (left) and by beam elements (right)

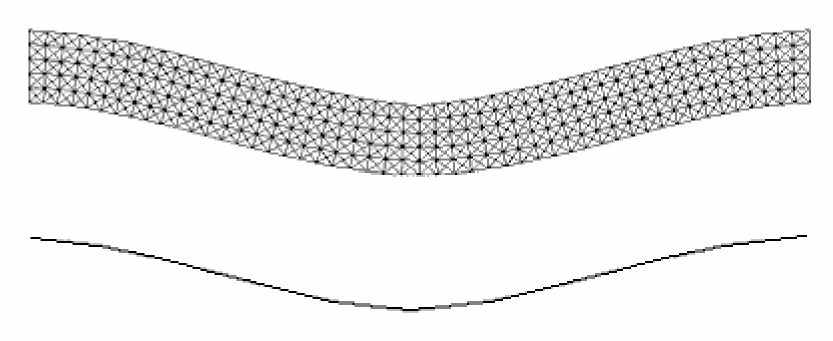

Figure 6. Final configuration of the beam modeled by truss lattice (upper) and beam elements (lower)

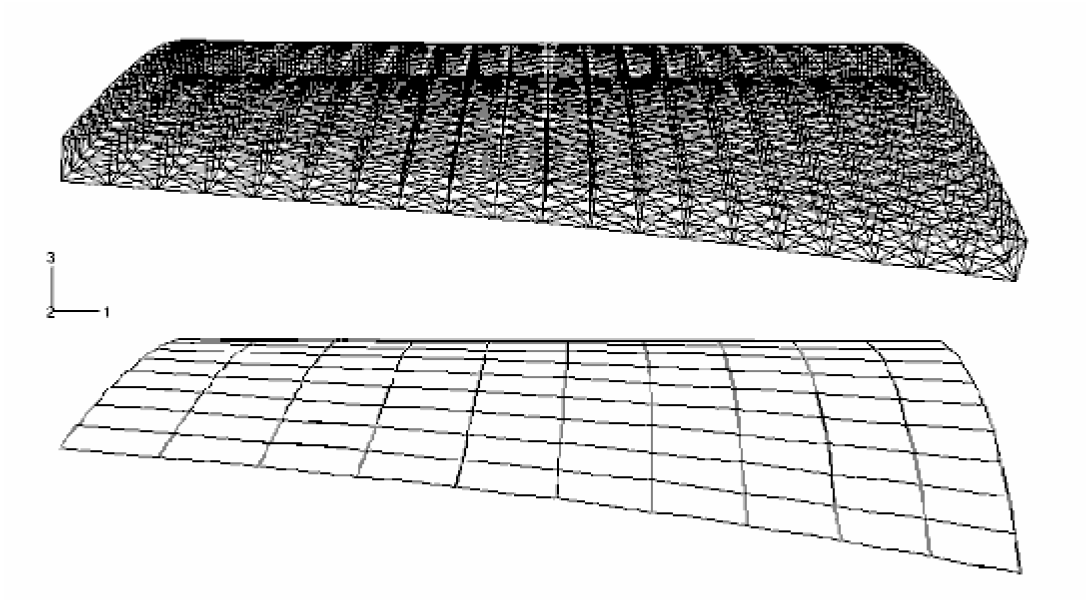

Figure 7. Final configuration for the plate modeled by truss lattice (upper) and shell elements (lower)

Table 1 presents the final displacement as obtained using the lattice model and normal structural finite element model.

Table 1. Displacements of the loaded points for lattice model and solid structure

		Structural finite elements (mm)	Truss Lattice Models (mm)	Differences (%)
Cantilever	Horizontal	0.2434	0.2531	3.9
	Vertical	0.5959	0.6044	1.4
Simple supported beam		0.1023	0.1033	1.0
Plate		0.0249	0.0257	3.2

4. CONCLUSIONS

This work offers an overview in the discretization of solids by lattice models using the Framework Method. The aim was investigate the validity of the model when applied to a non-linear structural response.

Comparisons were made between the results of lattice models and of structural finite elements meshes. The results found for both discretization of the beam and plate are shown to be accurate enough.

Hence, to use the framework method and to represent a solid by a lattice are accurate and attractive. This technique can be further explored in problems related to material failure.

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