# Post Severance Analysis of Impulsively Loaded Beams.

**André Weiss and Marcílio Alves**

Group of Solids Mechanics and Structural Impact
Department of Mechatronics and Mechanical Systems Engineering
University of São Paulo – Brazil.
Av Prof Mello Moraes 2231 - São Paulo - SP - 05508-900

*Abstract: This paper aims to analyse the plastic behavior of beams subjected to impulsive loads until complete severance, through the comparison between numerical and analytical models. For the numerical model, the failure criterion is based on the maximum plastic deformation, while for the analytical model, the failure criterion is based on the maximum beam transverse displacement, evolved from the Continuum Damage Mechanics. Some agreement between the analytical and numerical results were obtained and thoroughly discussed.*

Key words: impulsive load, rigid plastic method, plastic beams

## 1. Introduction

The inelastic behavior of impulsively loaded beams may be very usefull to the automobilistic, aerospacial, offshore, nuclear and civil industries, as well as in the military field. The analysis of impulsively loaded beams is a topic studied through the last three decades by many researchers(Shen and Jones, 1989; Jones and Alves). Among the first important works we may include the experimental works of Menkes and Opat (1973), and the analytical works by Jones (1971, 1976).

The impulsively loaded beam analysis usualy uses the rigid-plastic or rigid-perfectly plastic material models, due to the great simplicity resulting on the analytical solutions. These models admit a material without any elastic deformation, or that the strains in the material are solely plastic. Alongwith, the perfectly plastic model admits a constant flow stress, namely $\sigma_0$.

These models have shown to be very efficient when predicitng a beam plastic behavior (N. Jones, Structural Impact, 1989). Nevertheless, these models require a plastic hinge lenght definition in order to permit the strain calculation, a parameter which is often required in failure models.

An interesting feature of the dynamic lodaded beam analysis is that shear effects may be dominant even for long beams. The influence of shear effects in the behavior of impulsive loaded beams have been studied by many authors, (Yu and Jones, 1991; Li and Jones, 1995; Yu and Chen, 2000(a); Wierzbicki and Xeng, 2003), and the iteraction of membrane, shear and flexural effects in clamped beams still need attention.

A structure may be said to fail when presents any permanent deformation, when it does not support any load increase, or when presents material rupture. In the present work we assume failure occurs when the beams presents complete severance, or when it becomes completely free from its supports. The usual criteria applied in analytical or numerical methods for predicting the material failure are based on the equivalent plastic deformation, maximum shear tension, plastic deformation energy density, etc.

As one may note, all these criteria are based on the classic Continuum Mechanics, and its efficiency have been compared in Yu and Chen, 2000(b). A more simple criterion for a beam is just to say that material separation occurs when the beam moves a given fraction of its height. Recently, it has been shown that such a failure criterion may derived from Continuum Damage Mechanics (Jones and Alves (2004)).

The finite element numerical modeling of dynamic loaded strucutres with material failure may be very usefull even for analytical solutions development. Some features, such as the triaxiality, may be assessed through a numerical simulation and then employed in the analytical solution.

In this work we will compare an analytical solution developed in an early work for simply suported impulsively loaded beams, Jones and Alves (2004), and the numerical results obtained through a finite element simulation. The rupture condition is analised comparing the results of critical velocity, residual velocity, kinetic energy and time to failure.

## 2. Analytical Model

The analytical model is based on Jones and Alves (2004), who show that the failure criterion discussed below is derived from CDM. This failure criteria is simply a relation between beam transverse displacement and beam height, such that failure occurs when

$$W_s = KH \qquad \textbf{eq. (1)}$$

where Ws is the beam displacement in the support and K is a material property, but shown to be:

$$k = \frac{3}{4}\left(\sqrt{3}-\sqrt{2}\right)\left(\frac{E\bar{S}D_{cr}}{R_{\bar{\upsilon}}\sigma_0^2}+\frac{\varepsilon_D}{2}\right) \qquad \textbf{eq. (2)}$$

where $D_{cr}$ is the critical damage, $\bar{S}$ is a material parameter, $\varepsilon_D$ is the treshold damage strain and $R_{\bar{\upsilon}}$ is defined by

$$R_{\bar{\upsilon}} = \frac{2}{3}\left(1+\bar{\upsilon}\right)+3\left(1-\bar{\upsilon}\right)\left(\frac{\sigma_h}{\sigma_{eq}}\right)^2 \qquad \textbf{eq. (3)}$$

where the quadratic term is the stress triaxiality, taken null here when assuming a pure shear state, and  is the Poisson’s constant.
Assuming a plastic hinge lenght of

$$l_Q = \frac{3-\sqrt{6}}{2}H = 0.275H \qquad \textbf{eq. (4)}$$

it is possible to obtain a relation between the parameter K and the limit equivalent strain (Jones and Alves, 2004)

$$\varepsilon_{eq} = \frac{4}{3\left(\sqrt{3}-\sqrt{2}\right)}K \qquad \textbf{eq. (5)}$$

and, taken the material properties from the same reference, K=0.18, which results a limit strain of $\varepsilon_{eq} = 0.755$

It is shown that, for the simply supported beam, there exists three different mechanisms of deformation depending on the value of the parameter υ defined by:

$$\upsilon = \frac{LQ_0}{2M_0} \qquad \textbf{eq. (6)}$$

where $M_0$ and $Q_0$ are the bending moment and transverse shear force that, independently, make the beam section completely plastic.
Consider now a simply supported beam loaded along all its span by an initial velocity. For sufficiently high velocity, the beam will fail from its support. This occurs at time

- Time until severance: $t_{fa} = \frac{3mL^2}{8\upsilon^2 M_0}\left(V_0 - \sqrt{\frac{V_0^{\,2} - 16kH\upsilon^2 M_0}{3mL^2}}\right)$

which is associated with a critical velocity

- Critical velocity: $$V_{0C} = \sqrt{\frac{16kH\upsilon^2 M_0}{3mL^2}}$$

After failure, the beam will move freely and after a transient phase it will adquire a rigid body velocity such that the following ratio is valid

- Final to initial velocities ratio: $$\frac{V_{rb}}{V_i} = 1 - \frac{3}{4\upsilon}\left(1 - \sqrt{1 - \frac{16kH\upsilon^2 M_0}{3mL^2 V_i}}\right)$$

The equations above where employed to compare with the numerical results present below.

## 3. Numerical Model

The numerical model employed was developed with the comercial software Abaqus/Explicit, v6.2. The beam was modeled with plane stress four node quadrilateral elements, named CPS4R, with reduced integration. It is required a refined mesh for monitoring eficiently the failure evolution at the suport. The suport is modeled like the beam but much stiffer by asserting a high elastic modulus in order to avoid its deformation

The symmetry of the model is used to reduce the toal element number. It is imposed 0.5mm for the elements lenght for all analysis. The mesh for the beam with 100mm lenght is shown in figure 1. The only load aplied is an initial velocity through the beam span.

The failure criterion in the numerical simulation is based on the maximun equivalent plastic strain. This implies that the element fails when its equivalent plastic strain reaches a certain threshold. After this instant, all the element structural strenght is taken null.

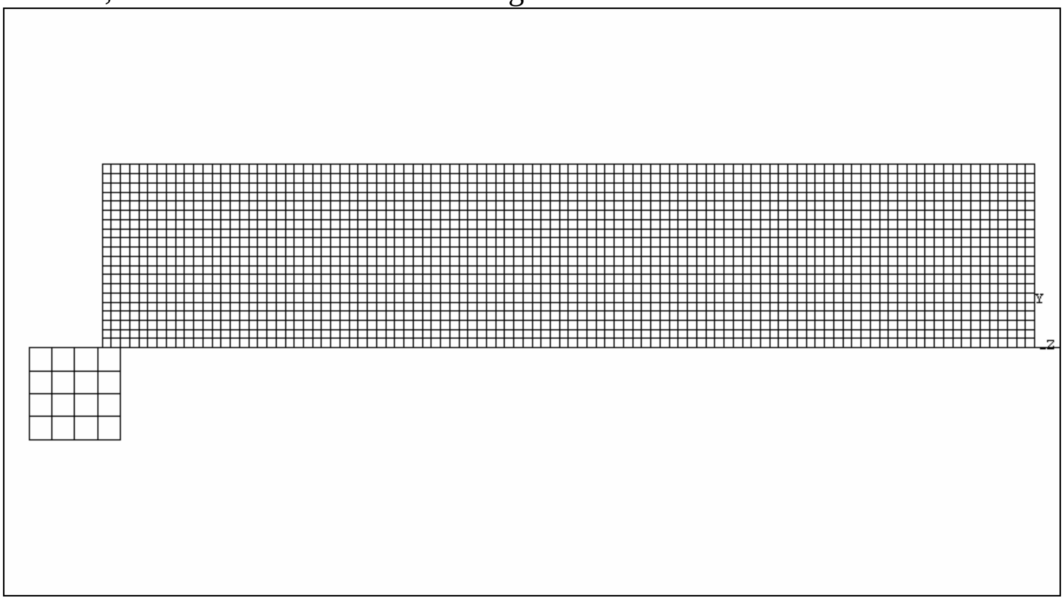


**Figure 1 Mesh detail of the υ=5 model.**

## 4. Results

The beams were taken to be failed just when the transversal displacement of the plastic hinge had reached the limit imposed through the parameter K. It must be noted that, at this moment, the plastic strain criteria had not been reached by any element of the model, so, though the analytical model predicts its failure, the beam hasn’t yet failed in the numerical model.

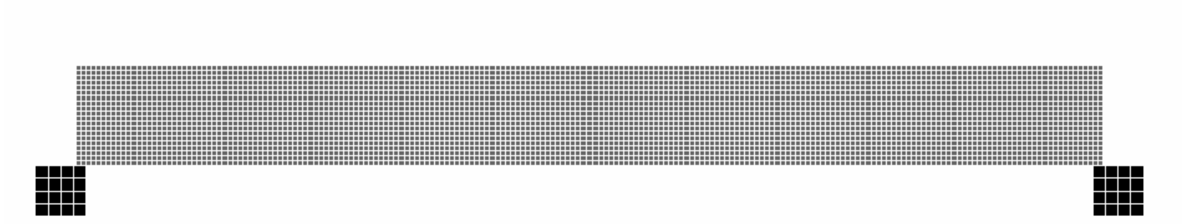
With this criteria, the following results were obtained: time of failure and rigid body (residual) velocity.

The beams analysed are listed in table 1.

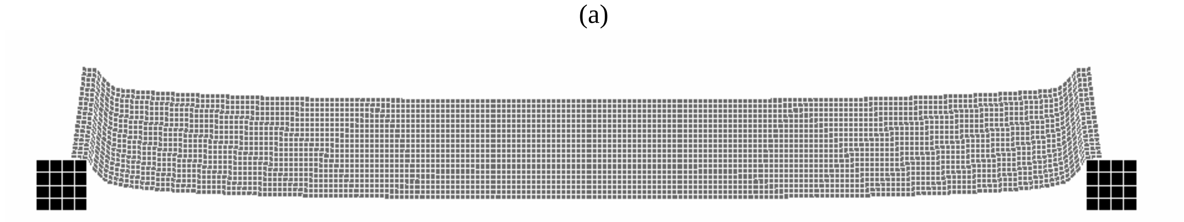
**Table 1 Beams numerically simulated.**

| Viga | 2L (mm) | H (mm) | B (mm) | $\nu$ | $\sigma_0$(Mpa) | $\varepsilon_{cr}$ |
|---|---|---|---|---|---|---|
| A | 30 | 10 | 10 | 1.5 | 300.0 | 0.755 |
| B | 50 | 10 | 10 | 2.5 | 300.0 | 0.755 |
| C | 100 | 10 | 10 | 5.0 | 300.0 | 0.755 |
| D | 200 | 10 | 10 | 10.0 | 300.0 | 0.755 |

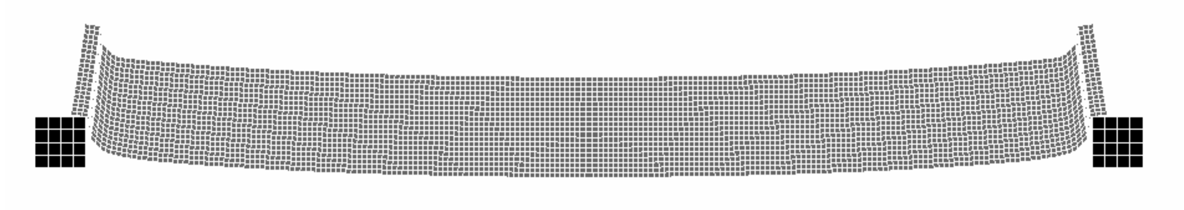
Figure 2 shows a sequence of deformation profiles at the failure time and after. It may be noted that, even after the beam severance, the beam bends. This is due to the transient phase proved to occur in Jones and Alves, 2004.



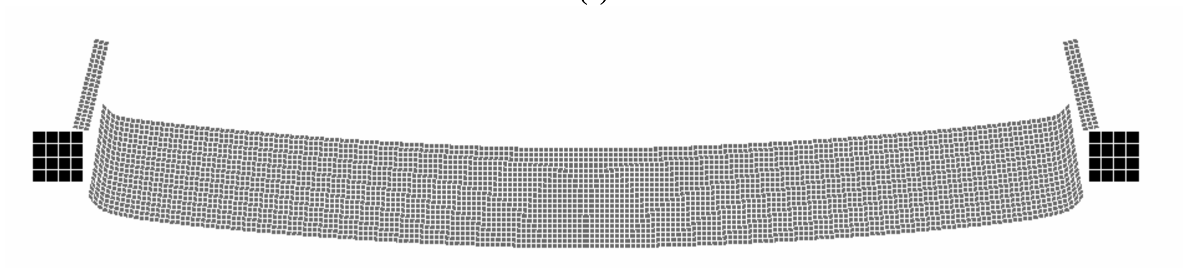
(a)



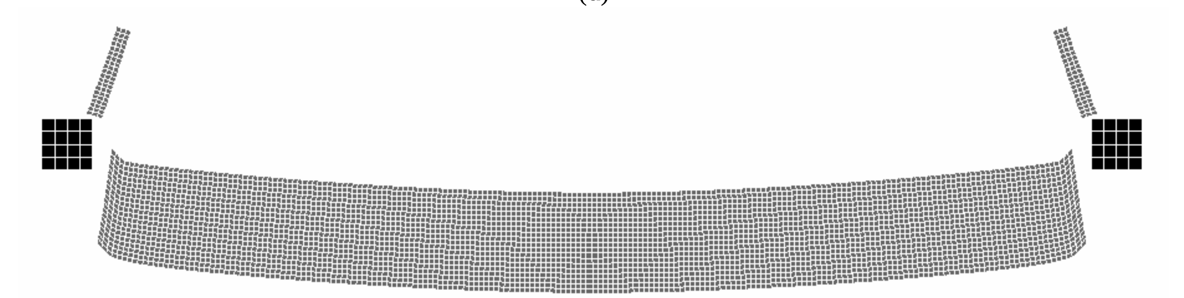
(b)



(c)



(d)



(e)

**Figure 2 Beam ($\upsilon$=5) deformation: (a) t=0; (b) $t=t_{fa}$; (c) $t=1.5t_{fa}$; (d) $t=3t_{fa}$; (e) $t=5t_{fa}$.**

The dashed lines in Figure 3 are related to the numerical results and the continuum lines indicate the analytical results. It presents the time to failure as a function of the initial velocity. It may be seen significant differences between the numerical and the analytical models. This is attributed to the use of the different failure criterion in the analytical model and in the FE code.

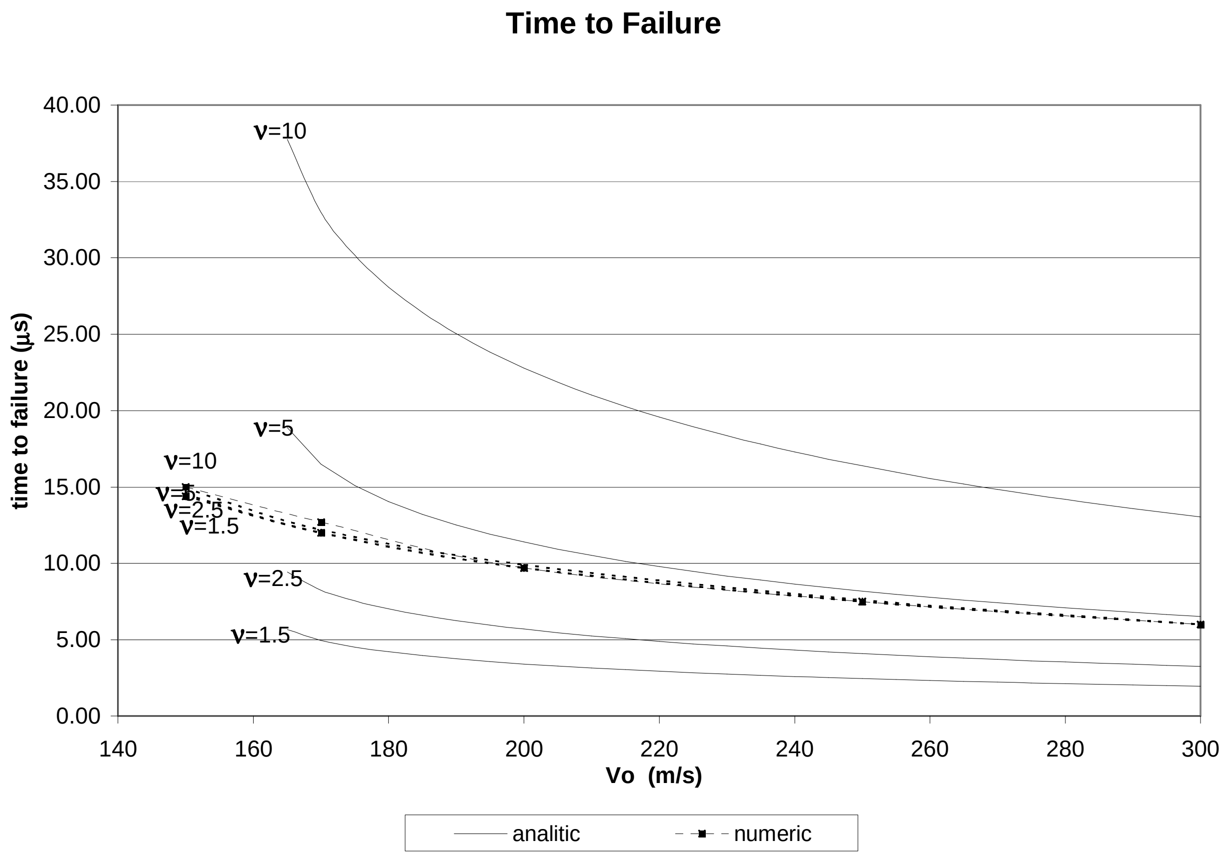



**Figure 3 Time to failure as a function of the initial velocity.**

Figuer 4 shows the initial to final velocities ratio as a function of the initial velocity. It may be seen that the diference between the models reduce as the initial velocity increases, as expected.

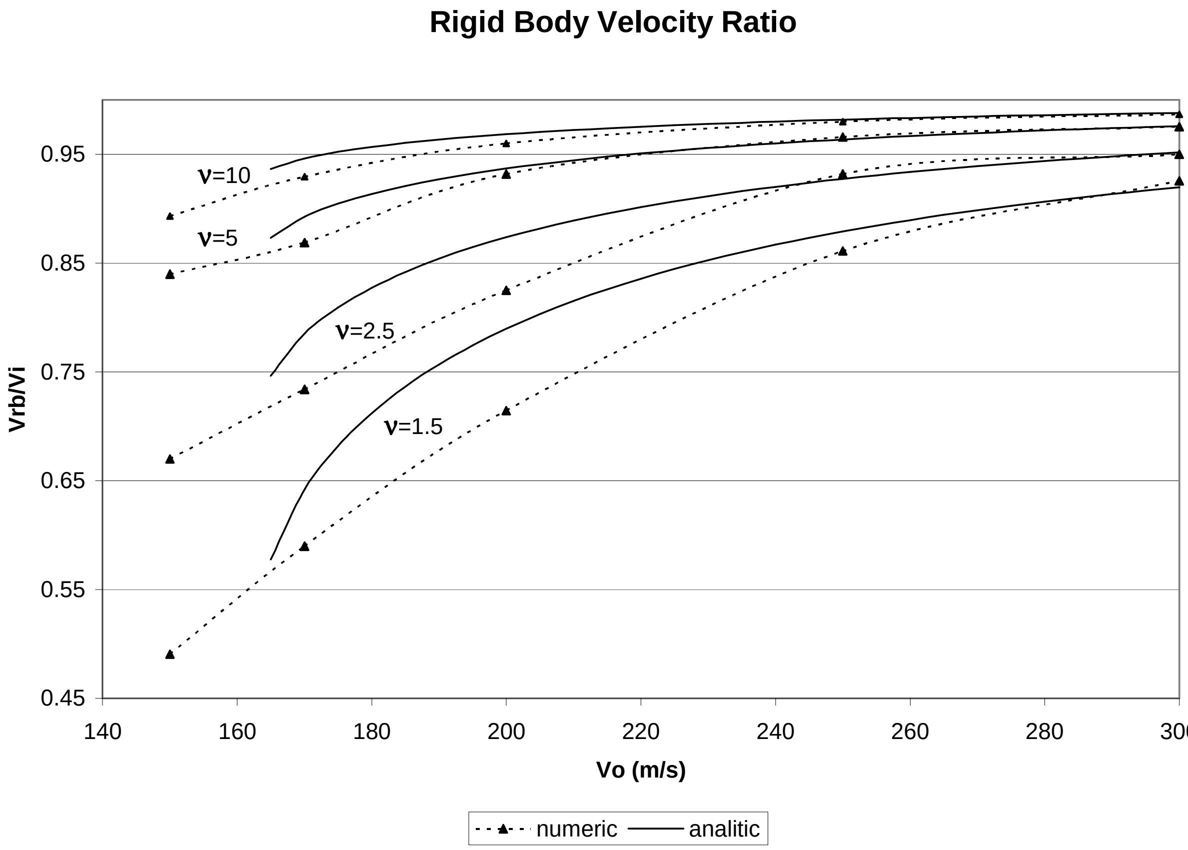



**Figure 4 Final to initial velocities ratio as a function of the initial velocity.**

Table 2 shows the equivalent plastic strain as obtained by the numerical results applied to the analytical solution. This means that, when the numerical model has severed, its transversal displacement was measured in the plastic hinge.

**Table 2 Equivalent plastic strain as obtained from the numerical results in eq.5**

| beam | velocity (m/s) | | | | |
|---|---|---|---|---|---|
| | 150 | 170 | 200 | 250 | 300 |
| A | 1.049 | 1.049 | 1.091 | 0.986 | 0.713 |
| B | 1.049 | 1.091 | 1.133 | 0.713 | 0.881 |
| C | 1.007 | 1.133 | 0.881 | 0.755 | 0.881 |
| D | 1.468 | 1.175 | 1.049 | 0.839 | 0.755 |

## 5. Discussion

It may be noted from figure 3 that the time failure for the numerical model almost does not depend on the aspect ratio υ, whereas in the analytical model it does. Nevertheless, figure 4 shows a good agreement for the rigid body velocity after severance and its dependence on the aspect ratio.
An important feature of the numerical results is the influence of the boundary condition. For all models, just a small portion of the beam was in contact with the support. This would cause just a small inertial effect, for its inertia corresponds just to some 1% to 5%, depending on the beam modelled. But it was found that the plastic deformation of the elements in contact with the support introduces an axial component to the strain of the elements near the contact region. This may be the cause of the severance for beams with initial velocity lower than the critical velocity calculated through the analytical model.

Another feature is the difficulty to establish the exact moment of failure. The transversal displacement is not uniform through the beam height and here it was taken at the upper surface. Also, it must be measured at some distance from the support. Here we have taken the distance equivalent to the plastic hinge lenght as calculated with the analytical model, though it does not correspond exactly to the plastic region, for the causes above cited.
All this features make difficult to fully compare the numerical and analytical models. Nevertheless, while the analytical model seems to be a reasonable way to predict the behavior of impulsively loaded beams, unfortunately, no experimental result is available in the literature.

**References**


N. Jones. Structural Impact. Cambridge Press, Cambridge,1989.

T. Wierzbicki, N. Jones (editors). Sructural Failure. John Wiley & Sons, 1989

J. Lemaitre. A Course on Damage Mechanics. Springer, 1992.

N. Jones and M. Alves, Post Severance Analysis of Impulsively Loaded Beams. *International Journal of Solids and Structures, 2004 (in press)*

N. Jones. A theoretical study of the dynamic behavior of beams and plates with finite deflections. *Int. Journal of Solids and Structures*, 7: 1007-1029, 1971.

S. B. Menkes, H. J. Opat. Broken beams. *Exp. Mechanics*, 13:480-486, 1973.

N. Jones. Plastic failure of ductil beams loaded dynamically. *Journal of Engineering for Industry*, 98(1):131-136, 1976.

J. Yu, N. Jones. Further experimental investigation on the failure of clamped beams under impact loads. Int. Journal of Solids and Structures, 27(9):1113-1137, 1991.

W. Q. Shen, N. Jones. A failure criterion for beams under impulsive loading. *Int. Journal of Impact Engineering*, 12(1):101-121, 1992.

Q. M. Li, N. Jones. Blast loading of fully clamped beams with transverse shear effects. *Mech. Struct. And Mach.*, 23(1):59-86, 1995.

T. X. Yu, F. L. Chen. A further study of plastic shear failure of impulsively loaded clamped beams. *Int. Journal of Impact Engineering*, 24:613-629, 2000

T. X. Yu, F. L. Chen. Failure of structures under intense dynamic loading: modes, criteria and tresholds. *Int. Journal of Mechanical Sciences*, 42: 1537-1554, 2000.

T. Wierzbicki, T. Xeng. How the airplane wing cut through the exterior columns of the world trade center. *Int. Journal of imact Engineering*, 28: 601-625, 2003.