MODEL REDUCTION METHODS AS APPLIED TO VISCOELASTICALLY DAMPED FINITE ELEMENT MODELS

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Abstract: *This paper is devoted to procedures for the finite element modeling of structures incorporating viscoelastic materials, with emphasis placed on reduction methods intended for the reduction of the order of the finite element matrices of the damped systems to perform frequency response and time domain analyses. This work focuses the use of two methods of reduction: the Modal Reduction Method and the Modified Internal Balancing Method. To account for the frequency-dependent behavior of viscoelastic materials, the Golla-Hughes-McTavish (GHM) model is used. The paper is organized as follows: introductory comments are first presented regarding the use of viscoelastic models and the procedures for their inclusion in the finite element structural matrices. Next, the studied of methods of reduction of viscoelastic finite models is developed. Numerical simulations dealing with two-dimensional structures are presented to illustrate the use of the reduction methods.*

Keywords: *Viscoelastic Materials, Finite Element Modeling, Reduction Methods*

1. INTRODUCTION.

In the context of passive control of mechanical vibrations, the use of viscoelastic materials has been regarded as a very convenient strategy in many types of industrial applications. To model the dynamic behavior of complex structures incorporating viscoelastic dampers, finite element procedures have been combined with models intended to describe the typical dependence of the mechanical properties of viscoelastic materials with respect to the vibration frequency. An important class of such models is based on the addition of internal or dissipative variables to account for the viscoelastic behavior, such as the Anelastic Displacement Fields Model – ADF, proposed by Lesieutre (1992) and the Golla-Hughes-MacTavish model (1985; 1993). In general, the inclusion of internal variables leads to global systems of equations of motion whose numbers of degrees-of-freedom largely exceeds the order of the associated undamped system. As a result, the numerical resolution of such equations can require prohibitive computational effort. Moreover, the increased dimension of the analytical model can preclude its use for active control. This drawback can be circumvented by using the so-called model-reduction techniques in an attempt to reduce the order of the finite element matrices while preserving its capability to represent the dynamic behavior of the damped system (Salmanoff, 1997). The simplest, yet very useful, model-reduction method is the well-known Guyan Static Condensation Method, according to which reduction is achieved by partitioning the equations of motion in terms of master and slave coordinates. By neglecting the inertia associated to the slave coordinates, only the master coordinates are kept in the model. This reduction method is commonly used to remove insignificant physical coordinates such as rotation coordinates, but is not applicable to systems with damping (Yae and Inman, 1993). However, this technique produces a reduced system whose coordinates are a subset of the original coordinates system (Lam, 1997). Another reduction model method is the internal balancing method, proposed by Moore (1981). Based on this method, the system is derived in a state space equation.

The disadvantage of this method is that the coordinates of the reduced system are not a subset of the original coordinates, producing a system with no physical coordinates, (Salmanoff, 1997). To overcome this problem, Yae and Inman (1993) proposed a modified internal balancing method, which combines the desirable features of a reduced system whose coordinates are a subset of those of the original system, and can be applied to damped systems.

Another strategy, which has been used, is the so-called Modal Reduction Method (Trindade et al, 2000), according to which the system response is expressed as a linear combination of a reduced number of complex eigenvectors which are assumed to be the most significant for the characterization of the dynamic behavior in a given frequency band. Upon this assumption, the dimension of the state matrices of the reduced model correspond to the number of eigenvectors kept in the truncated series.

In the remainder, the modified internal balancing method and the modal reduction method are used to perform modal analysis, frequency response and time domain analyses of two-dimensional structures such as beams and trusses treated with viscoelastic material.

2. GOLLA-HUGHES-McTAVISH MODEL (GHM).

The GHM model was introduced by Golla and Hughes (1985) and modified by Golla McTavish (1993). According to this model, the modulus function in the Laplace domain is expressed as:

$$G(s) = G_r\left(1+\sum_{i=1}^{N_G}\alpha_i\frac{s^2+2\zeta_i\omega_i s}{s^2+2\zeta_i\omega_i s+\omega_i^2}\right) \tag{1}$$

where G_r is the static modulus, $\alpha_i, \zeta_i, \omega_i$ are the parameters of the i mini-oscillator, and N_G is the number of mini-oscillators. The dynamic modulus for this model is represented by the form:

$$G_u = G_r\left(1+\sum_{i=1}^{N_G}\alpha_i\right) \tag{2}$$

Consider the finite element equation of motion of a structure containing N degrees-of-freedom:

$$[M]\{\ddot{q}\}+[D]\{\dot{q}\}+[K]\{q\}=\{F\} \tag{3}$$

where $[M]\in R^{NxN}$ is the mass matrix (symmetric and positive definite), $[D]\in R^{NxN}$ (symmetric and nonnegative definite) is the viscous damping matrix, and $[K]\in R^{NxN}$ is the stiffness matrix (symmetric and nonnegative definite), $\{q\}\in R^N$ is the vector displacements and $\{F\}\in R^N$ is the loading vector. It is assumed that the structure contains both elastic and viscoelastic elements, so that the stiffness matrix can be decomposed as follows:

$$[K]=[K_e]+[K_v(s)] \tag{4}$$

where $[K_e]$ is the stiffness matrix corresponding to the purely elastic substructure and $[K_v(s)]$ is the stiffness matrix associated with the viscoelastic substructure. The inclusion of the frequency-dependent behavior of the viscoelastic material can be made by generating $[K_v(s)]$ for specific types of elements (rods, beam, plates, etc.) considering initially the elastic moduli $E(s)$ and $G(s)$ as constant. Then, using the so-called *elastic-viscoelastic correspondence principle* (Christensen, 1982), those moduli are factored out of the stiffness matrix and made dependent on the frequency according to the particular viscoelastic model adopted. By assuming a constant, frequency-independent, Poisson ratio for the viscoelastic material, $E(s)$ becomes proportional to $G(s)$ through the relation $G(s)=E(s)/2(1+\nu)$. Then, by writing $\left[K_v(s)\right]=G(s)\left[\bar{K}_v\right]$, equations (4) and (3) are combined to give the following equation in the Laplace domain:

$$\left(s^2[M]+s[D]+[K_e]+G(s)\left[\bar{K}_v\right]\right)\{q(s)\}=\{F(s)\} \tag{5}$$

Introduction Eq. (1) into the Eq. (5), ones writes:

$$\left(s^2[M]+s[D]+[K_e]+\left[K_v^0\right]\right)\{q(s)\}+\left[K_v^0\right]\left(\sum_{i=1}^{N_G}\alpha_i\frac{s^2+2\zeta_i\omega_i s}{s^2+2\zeta_i\omega_i s+\omega_i^2}\right)\{q(s)\}=\{F(s)\} \tag{6}$$

where $\left[K_v^0\right]=G_r\left[\bar{K}_v\right]$ is the static stiffness matrix corresponding to the viscoelastic substructure.

The GHM model can be represented by series of dissipation coordinates $\left\{q_i^G(s)\right\}(i=1,\ldots,N_G)$, that are related to elastic variables $\{q\}$ according to:

$$\left\{q_i^G(s)\right\}=\frac{\omega_i^2}{s^2+2\zeta_i\omega_i s+\omega_i^2}\{q\} \tag{7}$$

Introducing Eq. (7) into (6), after some manipulations and back transformation to time domain, the following coupled system of equation is obtained:

$$\left\{s^2[M]+s[D]+[K_e]+\left[K_v^\infty\right]\right\}\{q\}-\alpha\left[K_v^0\right]\left\{q^G\right\}=\{F\} \tag{8a}$$

$$s^2\left\{q^G\right\}+s2\zeta\omega\left\{q^G\right\}+\omega\left\{q^G\right\}-\omega^2\{q\}=\{0\} \tag{8b}$$

where $\left[K_v^\infty\right]=\left[K_v^0\right]\left(1+\sum_{i=1}^{N_G}\alpha_i\right)$ is the dynamic stiffness matrix corresponding to viscoelastic substructure.

The system of Eq. (8) can be combined to form the following coupled system of equation of motion in time of the matrix form:

$$[M_G]\{\ddot{q}_G\}+[D_G]\{\dot{q}_G\}+[K_G]\{q_G\}=\{F_G\} \tag{9}$$

where $\{q_G\}=\left[\{q\}^T\ \ \left\{q_1^G\right\}^T\ \ldots\ \left\{q_{N_G}^G\right\}^T\right]^T$, $\{F_G\}=\left[\{F\}^T\ \ \{0\}^T\ \ldots\ \{0\}^T\right]^T$,

$$[M_G]=\begin{bmatrix}[M] & 0 & \cdots & 0\\ 0 & \frac{\alpha_1}{\omega_1^2}\left[K_v^0\right] & 0 & \vdots\\ \vdots & 0 & \ddots & 0\\ 0 & \cdots & 0 & \frac{\alpha_{N_G}}{\omega_{N_G}^2}\left[K_v^0\right]\end{bmatrix},\ [D_G]=\begin{bmatrix}[D] & 0 & \cdots & 0\\ 0 & \frac{2\alpha_1\zeta_1}{\omega_1}\left[K_v^0\right] & 0 & \vdots\\ \vdots & 0 & \ddots & 0\\ 0 & \cdots & 0 & \frac{2\alpha_{N_G}\zeta_{N_G}}{\omega_{N_G}}\left[K_v^0\right]\end{bmatrix}\ \text{and}$$

$$[K_G]=\begin{bmatrix}[K_e]+\left[K_v^\infty\right] & -\alpha_1\left[K_v^0\right] & \cdots & -\alpha_{N_G}\left[K_v^0\right]\\ -\alpha_1\left[K_v^0\right]^T & \alpha_1\left[K_v^0\right] & 0 & \vdots\\ \vdots & 0 & \ddots & 0\\ -\alpha_{N_G}\left[K_v^0\right]^T & \cdots & 0 & \alpha_{N_A}\left[K_v^0\right]\end{bmatrix}.$$

where $[M_G],[D_G],[K_G] \in R^{t_G \times t_G}$ with $t_G = N(1+N_G)$, $\left[K_v^0\right] = G_r[K_v]$ is the static or low frequency stiffness matrix and $\left[K_v^\infty\right] = \left[K_v^0\right]\left(1+\sum_{N_G}\alpha_i\right)$ is the dynamic or high frequency stiffness matrix.

The inclusion of dissipation coordinates to compute the frequency-dependence of the dynamic behavior of the viscoelastic damped systems increases the order of the differential equation of motion such that the structural degrees-of-freedom are at least doubled. This increases the computacional time to compute the time responses of the system, and motivates the use of model reduction methods.

3. MODAL REDUCTION METHOD.

To develop the formulation pertaining the Modal Reduction Method it is convenient to transform the second-order system (9) into an equivalent first-order form (space-state model) with an output equation added as follows:

$$\{\dot{x}\} = [A]\{x\} + [B]\{F\} \tag{10}$$

$$\{y\} = [C]\{x\} \tag{11}$$

where $\{x\} = \left[\{q\}^T \ \ \{q_1^{GG}\}^T \ \cdots \ \{q_{N_G}^{GG}\}^T \ \ \{\dot{q}\}^T \ \ \{\dot{q}_1^{GG}\}^T \ \cdots \ \{\dot{q}_{N_G}^{GG}\}^T\right]^T$, $[K_T] = [K_e] + \left[K_v^\infty\right]$ and

$$[A] = \begin{bmatrix} [0] & [0] & \cdots & [0] & [I] & [0] & \cdots & [0] \\ [0] & [0] & \cdots & [0] & [0] & [I] & \cdots & [0] \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ [0] & [0] & \cdots & [0] & [0] & [0] & \cdots & [I] \\ -[M]^{-1}[K_T] & \alpha_1[M]^{-1}[R] & \cdots & \alpha_{N_G}[M]^{-1}[R] & [0] & [0] & \cdots & [0] \\ \omega_1^2[\Lambda]^{-1}[R]^T & -\omega_1^2[I] & \cdots & [0] & [0] & -2\zeta_1\omega_1[I] & \cdots & [0] \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \omega_{N_G}^2[\Lambda]^{-1}[R]^T & [0] & \cdots & -\omega_{N_G}^2[I] & [0] & [0] & \cdots & -2\zeta_{N_G}\omega_{N_G}[I] \end{bmatrix}.$$

Since [A] is a non-symmetric matrix, the following eigenvalue problems are associated to system (10):

$$[A][R_r] = [\Lambda][R_r] \tag{12}$$

$$[A]^T[R_l] = [\Lambda][R_l] \tag{13}$$

where $[\Lambda] = \mathrm{diag}(\lambda)$ is the spectral matrix formed by the complex eigenvalues, and $[R_l]$ and $[R_r]$ are the modal matrices whose columns contain the left- and right-handside eigenvectors, respectively, which are assumed to be normalized so as to satisfy:

$$[R_l]^T[R_r] = [I] \tag{14}$$

The left- and right-handside eigenvectors can be separated into a structural eigenvectors, $[R_e]$, and dissipative eigenvectors $[R_d]$ of the damped system, according to:

$$[R_r] = [[R_{re}][R_{rd}]];\ [R_l] = [[R_{le}][R_{ld}]] \tag{15}$$

In general, the dissipative eigenvectors are related to the dissipative coordinates and are overdamped, such that their contributions to the dynamic behavior of the system is small. In this circumstance, they can be neglected, and the original state can be represented by the contribution of the elastic eigenvectors, $\{x\} \approx [T_{re}]\{x_e\}$, (Trindade, 2000). The Eqs. (10) and (11) can be expressed as:

$$\{\dot{x}_e(t)\} = [A_R]\{x_e(t)\} + [B_R]\{u(t)\} \tag{16a}$$

$$\{y(t)\} = [C_R]\{x_e(t)\} \tag{16b}$$

where $[A_R] = [R_{le}]^T[A][R_{re}]$, $[B_R] = [R_{le}]^T[B]$ and $[C_R] = [C][R_{re}]$ are input and output state reduced matrices of the viscoelastic damped system.

4. INTERNAL BALANCING METHOD.

The internal balancing method is another important method of reduction of the damped systems. However, it does not guarantee that the reduced coordinates are a subset of the original coordinates of the system. This method is based on the controllabilitly and observability of each balanced state.

According to Moore (1981) the balanced internal system is defined such that the grammians are diagonal and equal. For the system expressed by Eqs. (10) and (11), the controllability and observability grammians, denoted by $[W_c]$ and $[W_0]$, can be defined so as to satisfy the Lyapunov stability equation (Salmanoff, 1997):

$$[A][W_c] + [W_c][A]^T = -[B][B]^T \tag{17}$$

$$[A]^T[W_0] + [W_0][A] = -[C]^T[C] \tag{18}$$

The Cholesky decomposition of matrix $[W_c]$ is performed according to the following equation:

$$[W_c] = [L_c][L_c]^T \tag{19}$$

The transformation matrix $[P]$ is given by:

$$P = [L_c][U][\Lambda]^{-1/2} \tag{20}$$

where $[\Lambda]$ e $[U]$ are the eigenvalues and eigenvectors of the eigenvalue problem $\left([L_c][W_0][L_c]^T\right)$.

The internal balanced model is given as:

$$\{\dot{\hat{x}}\} = [\hat{A}]\{\hat{x}\} + [\hat{B}]\{F\} \tag{21a}$$

$$\{y\} = [\hat{C}]\{\hat{x}\} \tag{21b}$$

where $[\hat{A}] = [P]^{-1}[A][P]$, $[\hat{B}] = [P]^{-1}[B]$, $[\hat{C}] = [C][P]$ and $[\hat{x}] = [P]^{-1}[x]$.

By definition of an internally balanced system, the quantities $W_c(P) = W_o(P) = \text{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_N\}$, the terms $\sigma_i\ (1, 2, \ldots, N)$ are a measure of the controllability of state i where N is the total number of degrees-of-freedom in the finite element model. In this sense, the internally balanced system (21a) can be partitioned into retained states, $\{\hat{x}_r\}$, and reduced states, $\{\hat{x}_d\}$, according to the states with high σ_i and small σ_i:

$$\begin{Bmatrix} \dot{\hat{x}}_r \\ \dot{\hat{x}}_d \end{Bmatrix} = \begin{bmatrix} \hat{A}_{rr} & \hat{A}_{rd} \\ \hat{A}_{dr} & \hat{A}_{dd} \end{bmatrix} \begin{Bmatrix} \hat{x}_r \\ \hat{x}_d \end{Bmatrix} + \begin{bmatrix} \hat{B}_r \\ \hat{B}_d \end{bmatrix} \{F\} \tag{22}$$

Next, performing the static reduction of the above system, which removes the undesirable states $\{\hat{x}_d\}$ from the equation, but retain the contribution to the dynamics of the system according to the states $\{\hat{x}_r\}$, the reduced model is obtained in following form:

$$\{\dot{\hat{x}}_r\} = \left[\hat{A}_{rr}\right]\{\hat{x}_r\} + \left[\hat{B}_r\right]\{F\} \tag{23a}$$

$$\{y\} = \left[\hat{C}_r\right]\{\hat{x}_r\} \tag{23b}$$

5. A FINITE ELEMENT FOR BEAMS TREATED WITH PASSIVE CONSTRAINING DAMPING LAYERS.

In this section the formulation of a three-layer sandwich beam element is presented, based on the original development by Lesieutre and Lee (1996). It is incorporated herein in models of two-dimensional beam systems treated with passive constrained layers. Figure 1 illustrates a three-node sandwich beam element of length L and width b (not indicated), which is formed by three layers: the base beam, the viscoelastic core and the constraining layer. In the same figure are also indicated the nodal degrees of freedom: transverse displacements, w_i, longitudinal displacements, u_i, and shear angles in the viscoelastic layer, $\beta_i \, (i = 1, ..., 3)$. The following assumptions are adopted:

a) the base beam and the constraining layer are assumed to be elastic with negligible transverse shear strain. Both extensional and bending stiffness are considered. Transverse and rotatory cross-section inertia is included.

b) for the viscoelastic core, classical Euler-Bernoulli hypotheses are augmented with the inclusion of a shear angle associated to transverse shear stress.

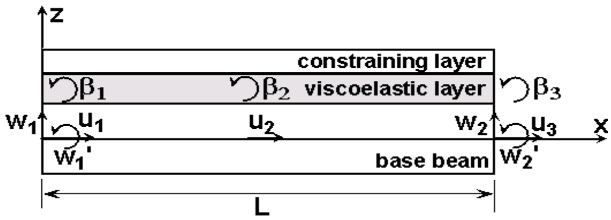


Figure 1. Three-layer sandwich beam element

The transverse displacement w_i is assumed to be the same for all the points lying on a same cross section and is interpolated by a cubic polynomial in x, according to:

$$w = w(x,t) = N_w(x)\bar{w}(t) \tag{25}$$

where $N_w(x) = \left[1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3} \quad x - \frac{2x^2}{L} + \frac{x^3}{L^2} \quad \frac{3x^2}{L^2} - \frac{2x^3}{L^3} \quad -\frac{x^2}{L} + \frac{x^3}{L^2} \right]$ and $\{w(t)\} = \left[w_1(t) \quad w_1'(t) \quad w_2(t) \quad w_2'(t) \right]^T$.

The longitudinal displacements u_i of the points lying on the middle plane of the base beam are interpolated by a quadratic polynomial in x:

$$u_0(x,t) = N_u(x)\bar{u}(t) \tag{26}$$

where $N_u(x)=\left[1-\frac{3x}{L}+\frac{2x^2}{L^2} \quad \frac{4x}{L}+\frac{4x^2}{L^2} \quad -\frac{x}{L}+\frac{2x^2}{L^2}\right]$ and $\{\bar{u}(t)\}=\left[u_1(t) \quad u_2(t) \quad u_3(t)\right]^T$.

The shear angle of the viscoelastic core β is interpolated consistently with u:

$$\beta(x,t) = N_u(x)\bar{\beta}(t) \tag{27}$$

where $\{\bar{\beta}(t)\}=\left[\beta_1(t) \quad \beta_2(t) \quad \beta_3(t)\right]^T$.

The strain energy accounting for extensional, bending and shear effects is given by:

$$U(t)=\frac{b}{2}\int_0^L\int_z\left[E(x,z)\varepsilon_{xx}^2+G(x,z)\varepsilon_{xz}^2\right]dzdx \tag{28}$$

By using the strain-displacement relations and taking into account approximations (25) to (27), the development of Eq. (28) leads to:

$$U(t)=\frac{1}{2}\left\{q^{(e)}\right\}^T\left[K^{(e)}\right]\left\{q^{(e)}\right\} \tag{29}$$

where $\left\{q^{(e)}\right\}=\left[\{\bar{u}\} \quad \{\bar{w}\} \quad \{\bar{\beta}\}\right]^T$ is the vector of element nodal degrees-of-freedom. The element stiffness matrix can expressed as follows:

$$\left[K^{(e)}\right]=\left[K_B^{(e)}\right]+\left[K_V^{(e)}\right]+\left[K_C^{(e)}\right] \tag{30}$$

where $\left[K_B^{(e)}\right],\left[K_V^{(e)}\right],\left[K_C^{(e)}\right]$ represent, respectively, the contributions of the base beam, viscoelastic core and constraining layer to the element stiffness matrix.

The kinetic energy including longitudinal, transverse and cross-section rotatory is:

$$T(t)=\frac{b}{2}\int_0^L\int_z\rho(x,z)\left(\dot{w}^2+\dot{u}^2\right)dzdx \tag{31}$$

By using approximations (25) to (27), the development of Eq. (31) leads to:

$$T=\frac{1}{2}\left\{\dot{q}^{(e)}\right\}^T\left[M^{(e)}\right]\left\{\dot{q}^{(e)}\right\} \tag{32}$$

where $\left[M^{(e)}\right]=\left[M_B^{(e)}\right]+\left[M_V^{(e)}\right]+\left[M_C^{(e)}\right]$ represent, respectively, the contribution of the base beam, viscoelastic core and constraining layer to the inertia matrices

By comparing Eqs. (4) and (30), one writes:

$$[K_e]=[K_B]+[K_C], \quad [K_v]=[K_V]=G(s)\left[\bar{K}_V\right] \tag{33}$$

where the matrix $[K_e]$ is formed by assembling the element stiffness matrices pertaining the base beam and the constraining layer, resulting in a constant, frequency independent matrix, and $[K_v]$ is the frequency dependent matrix related to the viscoelastic core. Expressions for theses matrices are fully developed by Lesieutre and Lee (1996).

6. NUMERICAL SIMULATIONS

In this section, numerical simulations are presented to illustrate the use of the model previously described to evaluate the damping performance of viscoelastic material and reduction methods as applied to two-dimensional structural systems. In the simulations that follow, the viscoelastic characteristics of commercially available 242F01, manufactured by 3M™ have been used (at 25ºC, and frequency band of 8 to 8000 Hz). Optimization was carried-out by using Genetic Algorithms, with populations of 800 individuals, allowing for 200 generations and using side-constraints. The values of the parameters obtained are given in Tab. 1, (Lima *et al*, 2003).

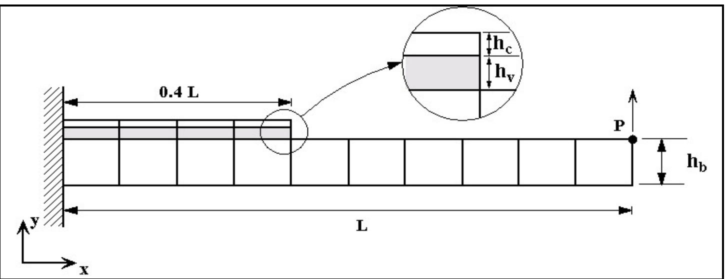
Table 1. Parameters of the GHM viscoelastic model identified for material 3M™ 242F01.

Mini-oscillator	1	2	3	4	5	6
αi	1.047	5.524	1.589	10.330	59.999	163.130
ζ_i	3911.898	323.891	48.414	30.544	14.627	4.763
ω_I [rad/s]	4943.062	6577.256	56363.554	45473.430	8601.413	57841.215
G_r [MPa]	0.079					

Table 2 presents the values of the physical and geometrical properties used to generate the FE model, which have been implemented based on the theory presented in Sections 5.

Table 2. Illustration of the FE model implemented for the beam with partial surface treatment.

Beam-Treated		
Base-beam	Viscoelastic core	Constraining layer
L = 0,5 m b =38×10^{-3} m	L = 0,5 m b =38×10^{-3} m	L = 0,5 m b =38×10^{-3} m
h_b = 45×10^{-4} m	h_v = 20x10^{-5} m	h_c = 5x10^{-4} m
E = 70,3×10^9 N/m	ρ = 1099,5 Kg/m^3 ν = 0,5	E = 70,3×10^9 N/m
ρ = 2750 Kg/m^3		ρ = 2750 Kg/m^3



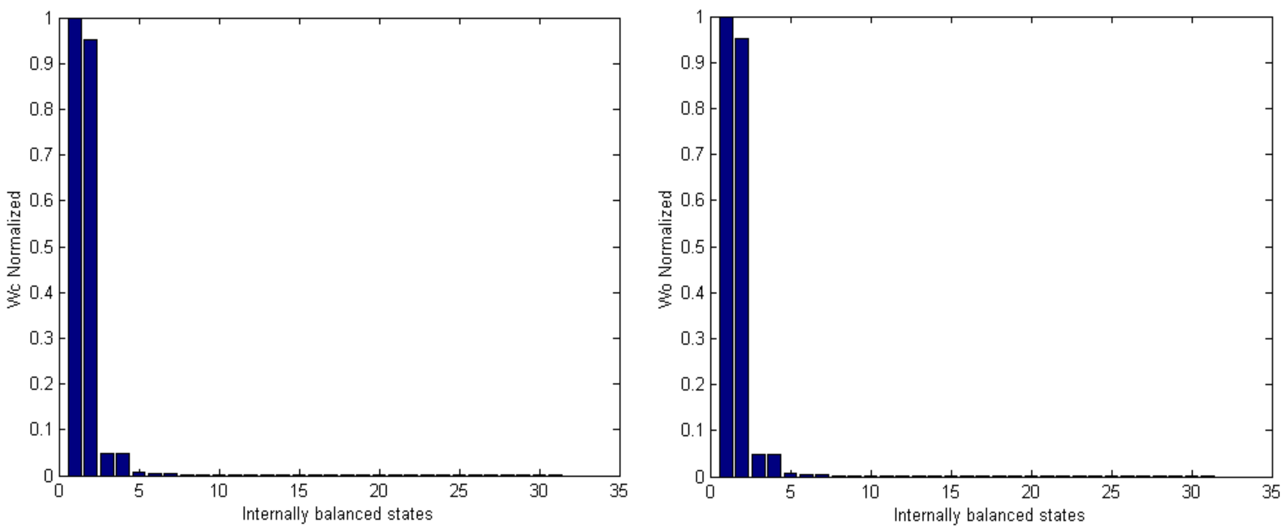


Figure 2. Wc and Wo versus internally balanced states for data set.

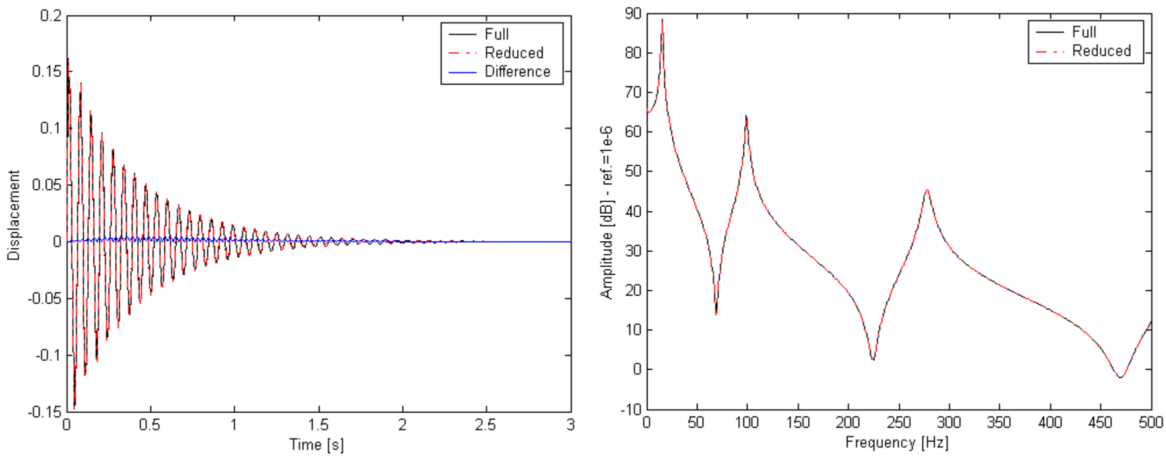


Figure 3. Time and FRFs for the full and reduced systems –internally balanced method

Figure 2 shows that the controllability and observability grammians in the balanced realization, Wc and Wo must be equal and diagonal, as predicted by the theory of internally balanced reduction method. The time response to an impulse excitation at the point P, and frequency response functions of the beam before and after reduction is shown in Fig. 3. Both impulse responses (full and reduced) appear as expected, where the oscillations are large initially, but decrease exponentially, leading to conclude that the internally balanced method is a viable method of modeling modal parameters of a viscoelastic material, as well as predicting responses, as can be see by the FRFs show in Fig. 3.

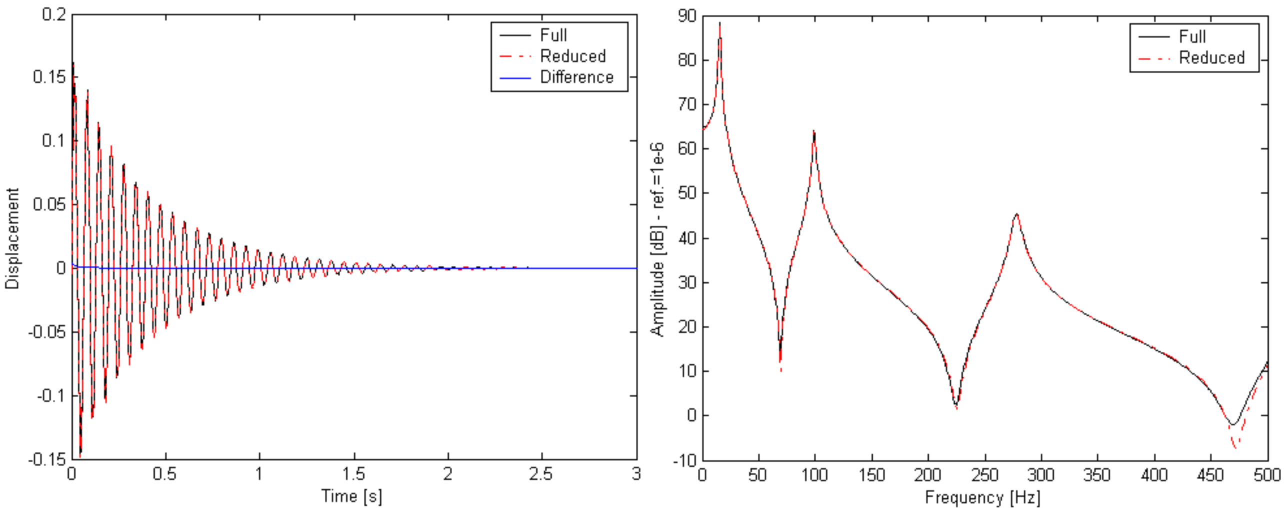


Figure 4. Time and FRFs for the full and reduced systems – modal reduction method

Figure 4 show the time response to an impulse excitation at the point P, and frequency response functions of the beam for the reduced model compared with those of full model. As can be seen, both impulse responses and FRFs appear as expected compared with the time and frequency responses obtained by the internally balanced method, leading to conclude that the modal reduction method is a viable method of modeling modal parameters of a viscoelastic material, as well as the internally balanced method in predicting the dynamic responses of the viscoelastically damped systems.

7. CONCLUSIONS

Finite element modeling procedures of structures incorporating viscoelastic materials that are able to reproduce the impulse response and FRFs before and after model reduction was implemented, with emphasis placed on the incorporation of models to account for the frequency-dependent behavior of viscoelastic materials and the implementation of two model order reduction methods: the internally balanced method and modal reduction method.

The numerical simulations presented enabled to illustrate the application of the reduction order procedures as a tool to evaluate the damping, time response and natural frequencies of the reduction viscoelastically damped systems as a dynamic representation of the larger order systems, reduction the time for processing the data and the analysis, and enable to the application of active control techniques to increase the damping effectiveness.

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