OPTIMIZATION OF THE HOLE-DRILLING STRAIN-GAGE ROSETTE POSITION IN NON-UNIFORM AND UNIFORM RESIDUAL STRESSES FIELDS

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Abstract: *Hole-drilling strain-gage rosette method is an experimental technique used to measure residual stresses near the surface of a mechanical component. Various types of rosettes are available with different geometric configurations and dimensions. The strain field high gradients observed near the hole, where the strain-gage rosette is bonded, makes the problem of choosing the optimal rosette position a critical one. In this work, a study of the effect of strain-gage rosette positioning is developed considering uniform and non-uniform residual stresses fields in a thin plate. The analytical analytic expressions developed for an elastic plate with a single hole by Kirsh and Tuzi are used to represent, respectively, a uniform and non-uniform (bending) residual stress fields.*

Keywords: *Residual Stresses, Hole-drilling, Strain-gage Rosette.*

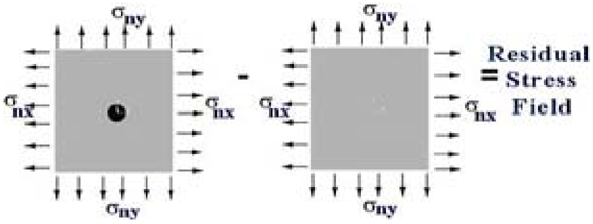
1. INTRODUCTION

It is well know that residual stresses from machining and fabrication components are those stresses that exist in the components before application of external loads. Almost all fabrication process including casting, forging, forming, machining, welding, or metallurgical changes arising from heat-treatment operations, introduce residual in the components. Nevertheless, the presence of residual stress is not usually considered in traditional design of mechanical components. The traditional design methodologies assume that a null stress state is present in the mechanical component before it is subjected to the operational loading and in the use of precise analytic and/or computational methods is not sufficient for a reliable structural integrity life prediction of a component.

There are several experimental methods to evaluate residual stresses in mechanical components, as hole-drilling, X-ray and ultrasonics (Withers & Bhadeshia, 2001). The hole-drilling method is used to evaluate residual stresses in a mechanical component by installing a strain-gage rosette on the surface of the piece and drill a small hole in the surface. Strains, which are relieved by this operation in the immediate vicinity of the hole, are measured by the strain-gage rosette. Finally, equations are used to transform the rosette strain-gages measurements in stresses. ASTM E-837 standard (ASTM, 1992) established this method. However several authors have developed studies in this field (Bathgate, 1968; Kabiri, 1984; Gomes, 1989; Beghini *et al.*, 1994; Fernandes, 2002; Fernandes & Castro, 2003).

2. HOLE-DRILLING METHOD - UNIFORM RESIDUAL STRESS FIELD

The present method for measuring residual stress, established by ASTM E-837 standard (ASTM, 1992), is restricted to uniform stress fields and geometry dimensions for which Kirsch (Kabiri, 1984) formula can be utilized. Kirsch proposed an analytic solution in 1898 for the stress distribution in the vicinity of a hole in an infinite thin plate considering a linear-elastic, isotropic, and homogeneous material subjected to a uniform bi-axial stress field. Residual stresses state can be evaluated subtracting the Kirsch analytic solution from the original stress state: $\sigma_{residual} = \sigma_{Kirsch} - \sigma_{original}$, from the knowledge of the magnitudes of the relieved strains, the diameter of the hole, modulus of elasticity (E) and Poisson ratio (ν) of the material. Figure 1 presents the methodology to evaluate residual stresses considering the Kirsch analytic solution, where σ_r, σ_t *e* σ_{rt} are the radial, tangential and shear stress and σ_{nx} σ_{ny} are the nominal stress in the *x* and *y* direction, respectively. *R* is the hole radius, *r* is the radial coordinate and θ is the angle measured in the counter-clockwise direction from *x* axis.



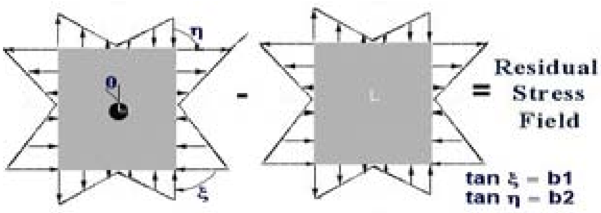
$$\begin{Bmatrix}\sigma_r\\ \sigma_\theta\\ \tau_{r\theta}\end{Bmatrix} = \begin{Bmatrix}\frac{\sigma_{nx}+\sigma_{ny}}{2}\left(1-\frac{R^2}{r^2}\right)+\frac{\sigma_{nx}-\sigma_{ny}}{2}\left(1-4\frac{R^2}{r^2}+3\frac{R^4}{r^4}\right)\cos 2\theta\\ \frac{\sigma_{nx}+\sigma_{ny}}{2}\left(1+\frac{R^2}{r^2}\right)-\frac{\sigma_{nx}-\sigma_{ny}}{2}\left(1+3\frac{R^4}{r^4}\right)\cos 2\theta\\ -\frac{\sigma_{nx}-\sigma_{ny}}{2}\left(1+2\frac{R^2}{r^2}-3\frac{R^4}{r^4}\right)\sin 2\theta\end{Bmatrix} - \begin{Bmatrix}\frac{\sigma_{nx}+\sigma_{ny}}{2}+\frac{\sigma_{nx}-\sigma_{ny}}{2}\cos 2\theta\\ \frac{\sigma_{nx}+\sigma_{ny}}{2}-\frac{\sigma_{nx}-\sigma_{ny}}{2}\cos 2\theta\\ -\frac{\sigma_{nx}-\sigma_{ny}}{2}\sin 2\theta\end{Bmatrix}$$

Figure 1. Relieved stresses for a uniform bi-axial uniform stress field.

3. HOLE-DRILLING METHOD – LINEAR BENDING RESIDUAL STRESS FIELD

In many practical situations, the assumption of a uniformly residual stress field is unrealistic and the dimensions are outside the range of the Kirsch formula. For example, the residual stresses generated by welding process in thick plates are highly non-uniform and do not satisfy the assumptions of the present method with regard to uniformity and dimension (Fernandes *et al.*, 2003).

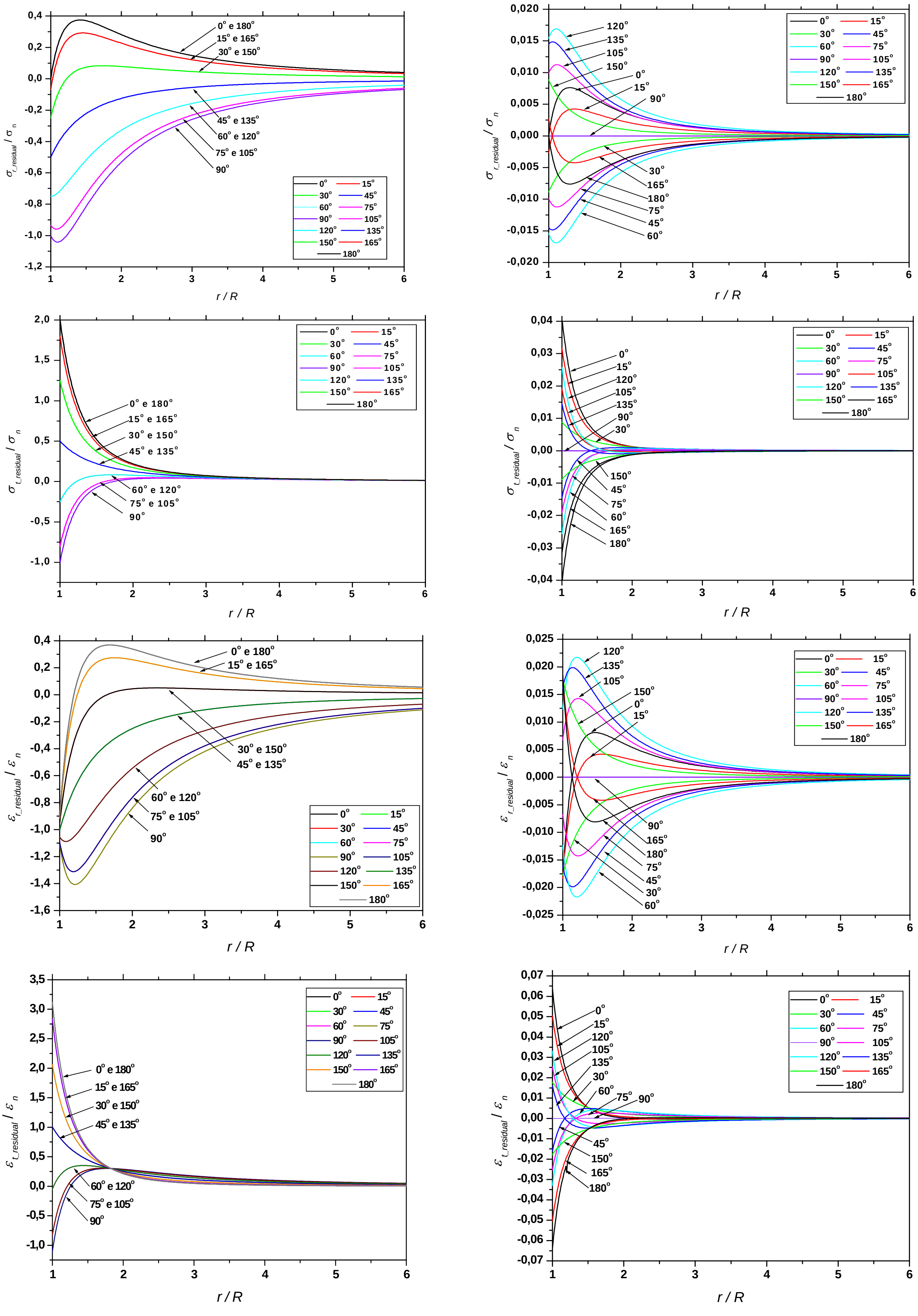
In 1984 Kabiri (Kabiri, 1984) propose a methodology using Tuzi analytic solution (Tuzi, 1930) for an infinite thin plate with a hole considering a linear-elastic, isotropic, and homogeneous material subjected to a non-uniform in-plane bi-axial bending loading. The state of residual stresses at the vicinity of the hole can be determined using a similar approach, shown in Figure 2, by subtracting Tuzi analytic solution from the original stress state: $\sigma_{residual} = \sigma_{Tuzi} - \sigma_{original}$.



$$\begin{Bmatrix}\sigma_r\\ \sigma_\theta\\ \tau_{r\theta}\end{Bmatrix} = \begin{Bmatrix}\left(r-4\frac{R^4}{r^3}+3\frac{R^6}{r^5}\right)(b_2\cos\theta+b_1\sin\theta)-\left(r-5\frac{R^4}{r^3}+4\frac{R^6}{r^5}\right)(b_2\cos^3\theta+b_1\sin^3\theta)\\ \left(\frac{R^4}{r^3}-3\frac{R^6}{r^5}\right)(b_2\cos\theta+b_1\sin\theta)-\left(r-\frac{R^4}{r^3}+4\frac{R^6}{r^5}\right)(b_2\cos^3\theta+b_1\sin^3\theta)\\ \left(r+2\frac{R^4}{r^3}-3\frac{R^6}{r^5}\right)(b_2\cos\theta-b_1\sin\theta)-\left(r+3\frac{R^4}{r^3}-4\frac{R^6}{r^5}\right)(b_2\cos^3\theta+b_1\sin^3\theta)\end{Bmatrix} + - \begin{Bmatrix}r(b_2\cos\theta+b_1\sin\theta)-r(b_2\cos^3\theta+b_1\sin^3\theta)\\ r(b_2\cos^3\theta+b_1\sin^3\theta)\\ r(b_2\sin\theta-b_1\cos\theta)-r(b_2\sin^3\theta-b_1\cos^3\theta)\end{Bmatrix}$$

Figure 2. Relieved stresses for a non-uniform linear (bending) bi-axial stress field.

It is worth to note that for this stress field, a strain-gage rosette with at least 3 strain-gages it is necessary (Kabiri, 1984; Fernandes, 2002). Figure 3 shows the normalized stress and strain distributions (radial: *r* and tangential: *t*) as a function of the ratio between radial distance and hole radius (*r*/*R*) for Kirsch and Tuzi solutions, considering a uniaxial stress state applied in the *y* direction.



(*a*) (*b*)

Figure 3. Normalized radial and tangential residual strains and stresses for a plate submitted to a one-dimensional nominal loading in the y direction. (*a*) Kirsh´s and (*b*) Tuzzi solutions.

It can be observed that strain distribution rapidly decreases as *r* rises. Also high strain gradients are observed in the vicinity of the hole, where high values are present.

4. INTEGRATION OF STRAIN-GAGE STRAINS

One important practical aspect associated with the hole-drilling method is the position of the strain-gage rosette. To improve the signal-noise ratio in the strain-gage measurements, the best position for strain-gage positioning is at the maximum strain radial deformation. However, at this position a high strain gradient is usually observed and, as the strain-gage measures the average of the strain developed through his area, a measurement error is present. To reduce this error and maximize the strain values measured small strain-gages must be used. Finally, is worth to note that in spite that tangential strains are larger than radial strains in the vicinity of the hole, strain-gages positioned at tangential direction should be avoided, because the drilling process induce plastic strains in this direction and, as a consequence, residual stresses. Therefore, radial strain-gages are the better option.

Kabiri (Kabiri, 1984) develop an analytic calculation to determine the strain value measured by each of the three strain-gages of a conventional rosette. To obtain the strain values, he performed an analytic integration of the radial deformations below the area of the strain-gages considering Kirsch and Tuzi analytic solutions. Figure 4 shows the model used with all the necessary parameters considering a general strain-gage rosette. Note in Figure 4*b* the stress transformation necessary to compute the integration.

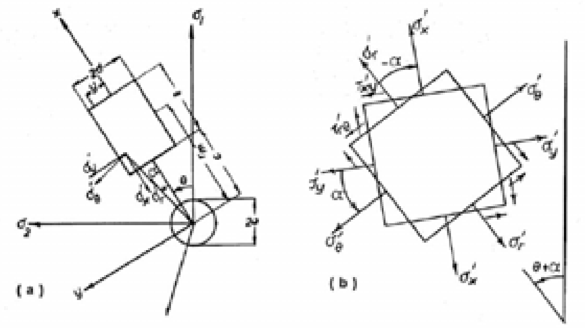


Figure 4. (*a*) Residual stress rosette grid. (*b*) Stress transformation at a point of the grid. (Kabiri, 1986).

From the analytic integration, the strain-gage deformation (ε_{SG}) for Kirsch analytic solution can be obtained:

$$\varepsilon_{SG} = \frac{\Delta_0^0}{E}\left(\sigma_{nx} + \sigma_{ny}\right) + \frac{\Delta_2^0}{E}\left(\sigma_{nx} - \sigma_{ny}\right)\cos 2\theta \tag{1a}$$

$$\Delta_0^0 = -\frac{(1+\nu)S_1R^2}{4ld} \tag{1b}$$

$$\Delta_2^0 = \frac{1}{2ld}\left[-(1+\nu)\left(R^2S_2 - \frac{3}{2}R^4S_3\right) + -(1-\nu)R^2S_1\right] \tag{1c}$$

$$S_1 = 2\tan^{-1}\left(\frac{c+l}{d}\right) - 2\tan^{-1}\left(\frac{c}{d}\right) \tag{1d}$$

$$S_2 = -2ld\left(\frac{d^2 - c^2 - cl}{\left(c^2 + d^2\right)\left(c^2 + 2cl + l^2 + d^2\right)}\right) \tag{1e}$$

$$S_3 = -\frac{2}{3}d\left(\frac{(c+l)\left(c^2 + d^2\right)^2 - c\left((c+l)^2 + d^2\right)^2}{\left(c^2 + d^2\right)^2\left(c^2 + 2cl + l^2 + d^2\right)^2}\right) \tag{1f}$$

From the analytic integration, the strain-gage deformation (ε_{SG}) for Tuzi analytic solution can be obtained:

$$\varepsilon_{SG} = \frac{\Delta_2^2}{E}\left(b_1 \sin\theta + b_2 \cos\theta\right) + + \frac{\Delta_3^1}{E}\left(b_1 \sin 3\theta - b_2 \cos 3\theta\right) \quad (2a)$$

$$\Delta_2^2 = -\frac{(1+\nu)R^4 T_3}{8ld} \quad (2b)$$

$$\Delta_3^1 = \left(\frac{(1+\nu)}{2ld}\right)\left(-\frac{3}{4}R^4 T_4 + R^6 T_5 - \frac{1-\nu}{1+\nu}\frac{R^4}{2}T_3\right) \quad (2c)$$

$$T_3 = \frac{dl\,(2c+l)}{\left((c+l)^2 + d^2\right)\left(c^2 + d^2\right)} \quad (2d)$$

$$T_4 = \frac{d}{3}\left[\frac{\left(d^2 - 3(c+l)^2\right)\left(c^2+d^2\right)^2 - \left((c+l)^2 + d^2\right)^2\left(d^2 - 3c^2\right)}{\left((c+l)^2 + d^2\right)^2\left(c^2+d^2\right)^2}\right] \quad (2e)$$

$$T_5 = \frac{d}{6}\left[\left(\frac{4d^2\left((c+l)^2+d^2\right)^2 - 3\left((c+l)^2+d^2\right)^3}{\left((c+l)^2+d^2\right)^5}\right) - \left(\frac{4d^2\left(c^2+d^2\right)^2 - 3\left(c^2+d^2\right)^3}{\left(c^2+d^2\right)^5}\right)\right] \quad (2f)$$

Figures 5 and 6 shows the residual stress and strain distribution in a strain-gage rosette type EA-XX-125RE-120 (MM, 2004) that has three strain-gages positioning at 0°, 90° and 225° for Kirsch and Tuzi analytic solutions, respectively. In these figures the plate is submitted to an uni-dimensional loading in the *y* direction. Constructive characteristics are presented in Table 1 where this rosette is listed as *SG2*.

5. OPTIMIZATION OF STRAIN-GAGE POSITION

Figure 7 shows the integrated strain-gage strain (ε_{SG}) below the area of strain gages, calculated from Eq. (1) and Eq. (2), as a function of (r/R) for 0°, 90° and 225°considering Kirsch and Tuzi analytic solutions. In these figures the plate is submitted to an uni-dimensional loading in the *y* direction . Three strain-gages configurations are considered in the analysis and their main characteristics are presented in Table 1. *SG2* and *SG3* is the commercial available Special Purpose Strain Gage **125RE** from Micro-Measurements (MM, 2004). The only difference between *SG2* and *SG3* is the hole diameter. *SG1* is a non-standard strain-gage presented only for comparison proposes.

Table 1. Strain-gages main characteristics.

Strain-gage	**Grid Length** (mm)	**Hole Diameter** (mm)	**Grid centerline radius** (mm)
SG1	1.0	2.05	-
SG2	3.18	1.5	5.13
SG3	3.18	2.05	5.13

In Fig. 7, vertical lines points to the optimal grid centerline radius positioning ($R_{optimal}$) associated with the maximum magnitude of measured strain ($\varepsilon_{optimal}$), representing the maximum strain-gage gain. Table 2 resumes this data.

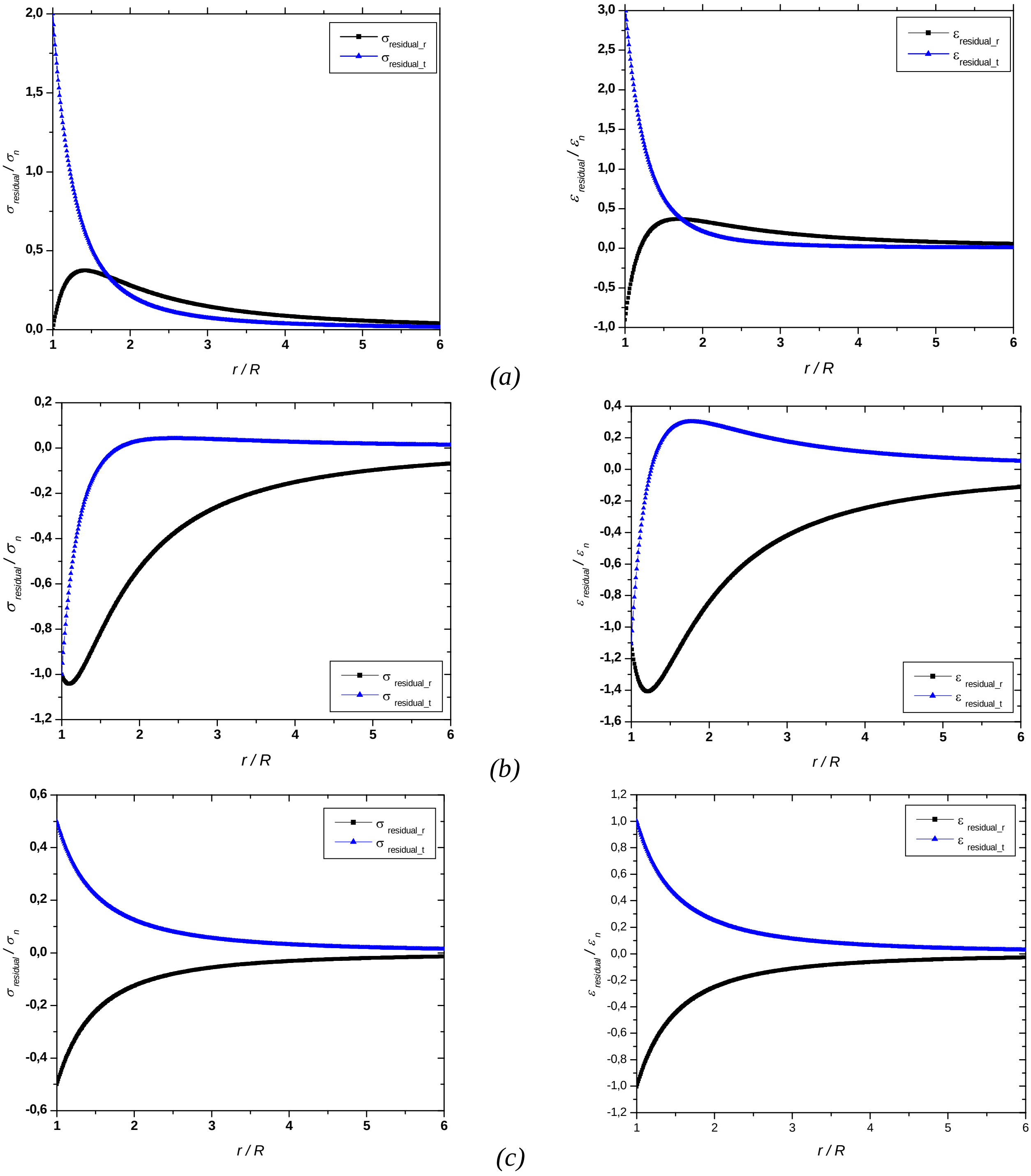


Figure 5. Residual stresses (left) and strains (right) distributions for Kirsch analytic solution considering an uni-dimensional loading in *(a)* 0^{o}, *(b)* 90^{o} and *(c)* 225^{o}.

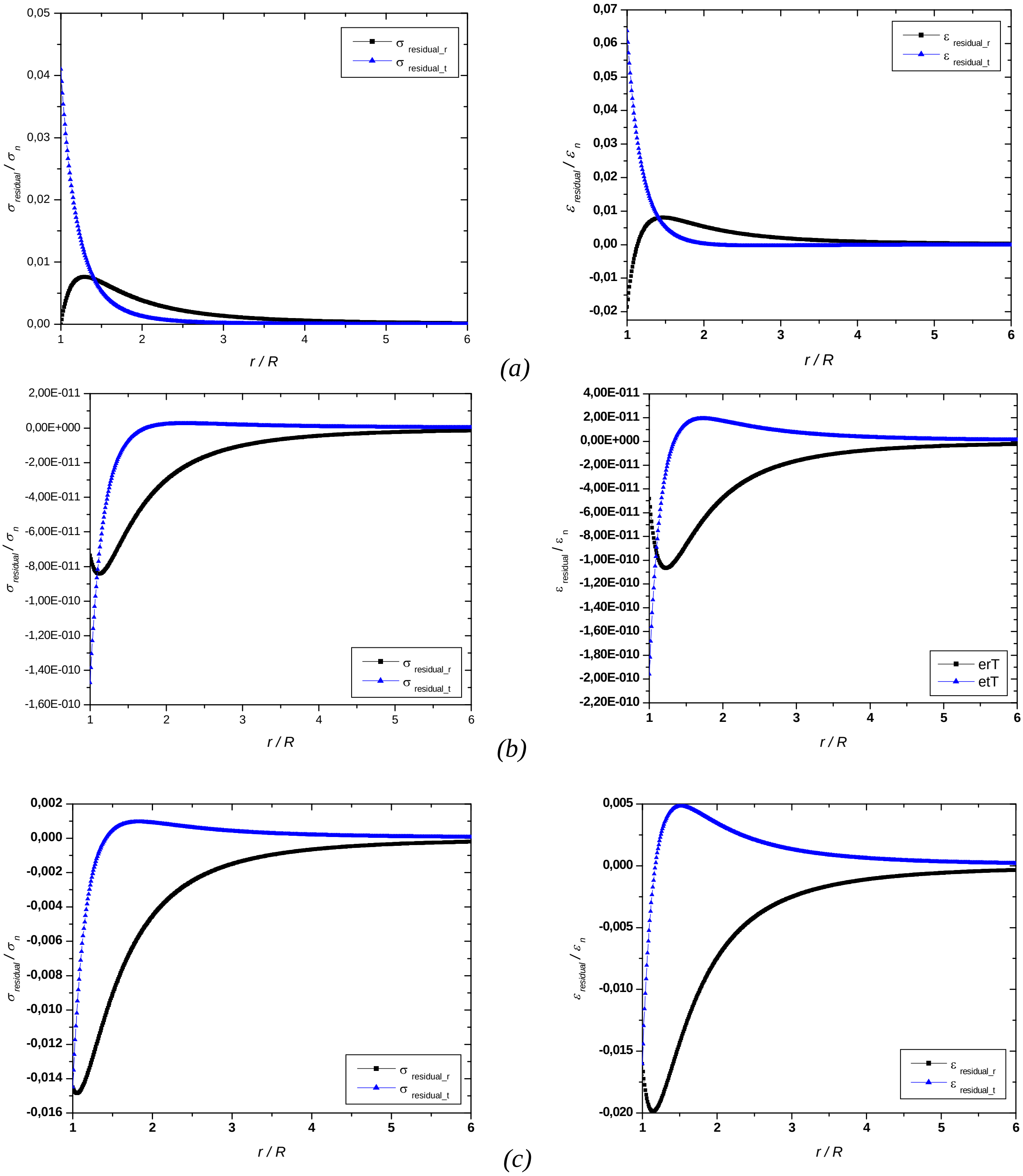


Figure 6. Residual stresses (left) and strains (right) distributions for Tuzi analytic solution considering an uni-dimensional loading in *(a)* 0^o, *(b)* 90^o and *(c)* 225^o.

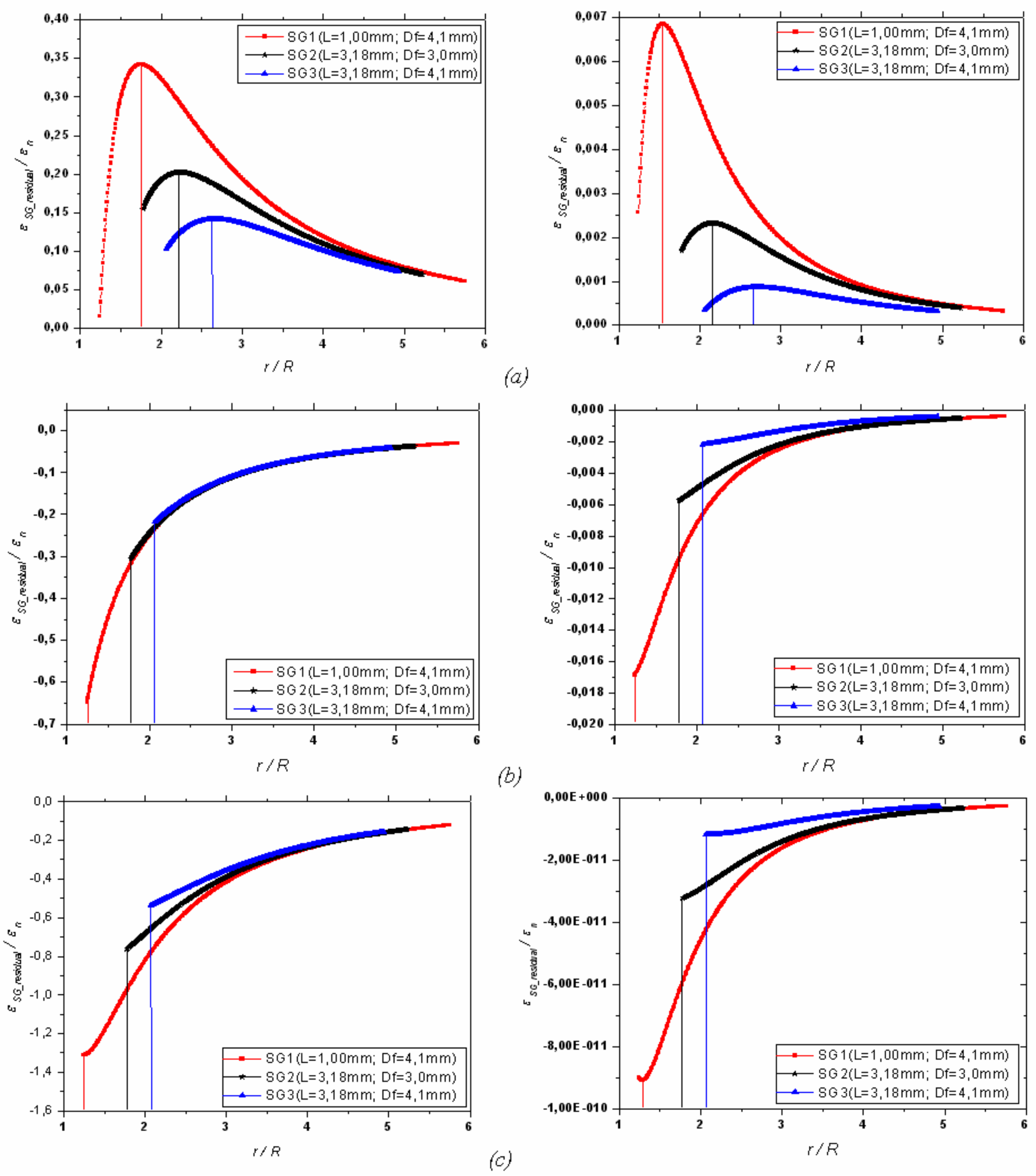


Figure 7. Integrated strain-gage strain as a function of (r/R) for 0°, 90° and 225° considering (*a*) Kirsch and (*b*) Tuzi analytic solutions.

Table 2. Optimal strain-gage positioning for *SG1*, *SG2*, *SG3* at 0°, 90° and 225°.

		0°		45°		90°	
		Kirsh	**Tuzi**	**Kirsh**	**Tuzi**	**Kirsh**	**Tuzi**
SG1	$R_{optimal}$ / R	1.77	1.54	1.24	1.24	1.24	1.30
	$\varepsilon_{optimal}$ / ε_n	0.3421	0.0069	-0.6474	-0.0169	-1.3103	$-9.1e^{-11}$
SG2	$R_{optimal}$ / R	2.23	2.16	1.78	1.78	1.78	1.78
	$\varepsilon_{optimal}$ / ε_n	0.2025	0.0023	-0.30429	-0.0057	-0.7630	$-3.2e^{-11}$
SG3	$R_{optimal}$ / R	2.62	2.71	2.06	2.06	2.06	2.06
	$\varepsilon_{optimal}$ / ε_n	0.1416	$8.68e^{-4}$	-0.2175	-0.0021	-0.5383	$-1.2e^{-11}$

Two comparision are developed considering the Kirsch analytic solution. The first one compares the responses between strain-gages *SG2* and *SG3* to study the influence of the strain-gage positioning on the gain. Two positions are considered: the optimal grid centerline radius positioning ($R_{optimal}$) and the original position (R_{MM}). Table 3 shows that for *SG3*, $R_{optimal}$ is very close to R_{MM}. Therefore the ratio between the strain measured at $R_{optimal}$ and at R_{MM} ($\varepsilon_{optimal.} / \varepsilon_{\mathrm{MM}}$) is close to unit. For *SG2* $\varepsilon_{optimal.} / \varepsilon_{\mathrm{MM}}$ presents higher values indicating that a better strain-gage gain can be obtained if the strain-gage is located at these positions. The second comparison considers the effect of strain-gage length. Strain-gages *SG1* and *SG3* presents the same characteristics except for its length. In this case, the ratio between the integrated strain values of *SG1* and *SG3* considering the optimal positions ($R_{optimal}$) for the 3 directions (0°, 90° and 225°) are, respectively, 2.42, 2.98 and 2.43. Therefore, a larger gain can be obtained using a smaller strain-gage. This is expected and is associated with the integration below the strain gage where a high radial strain gradient is present. For the results considering Tuzi analytic solution, a similar behavior is observed.

Table 3. Ratio between the strain measured at $R_{optimal}$ and at R_{MM} ($\varepsilon_{optimal.} / \varepsilon_{\mathrm{MM}}$) for *SG1*, *SG2*, *SG3* at 0°, 90° and 225° for the Kirsch analytic solution.

		$R_{optimal} / R$	R_{MM} / R	$\varepsilon_{optimal.} / \varepsilon_{\mathrm{MM}}$
SG2	**0°**	2.23	3.42	1.45
	45°	1.78		3.56
	90°	1.78		2.45
SG3	**0°**	2.62	2.50	1.01
	45°	2.06		1.41
	90°	2.06		1.20

The E-837 standard (ASTM, 1992) furnish the following equation to obtain the residual stresses values from the measured strains:

$$\sigma_{1,2} = \frac{\varepsilon_3 + \varepsilon_1}{4\overline{A}} \pm \frac{\sqrt{(\varepsilon_3 - \varepsilon_1)^2 + (\varepsilon_3 + \varepsilon_1 - 2\varepsilon_2)^2}}{4\overline{B}} \tag{3}$$

where σ_1 and σ_2 are the principal residual stresses, and $\overline{A} = -((1+\nu)/2E)\overline{a}$ and $\overline{A} = -(/2E)\overline{b}$. For a through the thickness hole, E-837 standard furnish a closed formula for parameters $\overline{a}$ and $\overline{b}$ (ASTM, 1992). For an uniform uni-axial or bi-axial residual stresses distribution, this approach furnishes stresses values very close to the applied ones, as the error associated with strain integration bellow the rosettes strain-gages is compensated by this closed formula. For the uniform uni-axial residual stress loading considered in this work ($\sigma_x = 0$ and $\sigma_y \neq 0$) and modeled by the Kirsch solution, $\sigma_1 = \sigma_y$ and $\sigma_2 = 0$. For a non-uniform residual stress loading, as the in-plane bending loading associated with Tuzi solution, the application of this methodology can result in larger error in the measurement of residual stresses. The application of the E-837 methodology to a situation with an in-plane bending loading in one direction (*x*, for example – Fig. 2), where σ_b is the maximum bending stress, and considering the *SG2* strain-gage rosette, results in $\sigma_1 = 1.324 \times 10^{-3}\ \sigma_b$ and $\sigma_2 = 5.433 \times 10^{-3}\ \sigma_b$.

6. CONCLUSIONS

Hole-drilling strain-gage rosette method is an experimental technique used to measure residual stresses near the surface of a mechanical component. Various types of rosettes are available with different geometric configurations and dimensions. The strain field high gradients observed near the hole, where the strain-gage rosette is bonded, makes the problem of choosing the optimal rosette position a critical one. In this work, a study of the effect of strain-gage rosette positioning is developed considering uniform and non-uniform residual stresses fields in a thin plate. The analytical analytic expressions developed for an elastic plate with a single hole by Kirsh and Tuzi are used to represent, respectively, a uniform and non-uniform (bending) residual stress fields. The simple analysis developed shows that the magnitude of strain-gage measurements can be increased by changing the strain-gages position or by using smaller strain-gages. Further studies must be developed considering different loadings, as a combination of uniform and linear (bending) residual stresses fields.

7. ACKNOWLEDGEMENTS

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