ON HIGH CYCLE FATIGUE DAMAGE

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Abstract. This work is concerned with the formulation of a thermodynamically consistent continuum model to high-cycle fatigue damage. A multifield description is adopted, in which damage processes are described by means of the introduction of a scalar damage variable and the corresponding force system. The damage kinetic equation, which is obtained from the balance of the aforementioned force system together with suitable constitutive information compatible with thermodynamics, accounts for non-local and rate effects as well as the influence of the strain state on damage resistance, amongst other things. An one-dimensional version of the model illustrating the influence of initial damage, loading frequency and different behavior under traction and compression on fatigue life and damage accumulation is presented.

Keywords. continuum mechanics, continuum damage mechanics, high-cycle fatigue damage, cumulative damage rules.

1. INTRODUCTION

The mechanical properties of materials deteriorate under the action of mechanical loads as the outcome of microscale damage processes involving breakage of atomic bonds. These microscale processes, which depend on the loading conditions, may involve both mechanisms of fracture and inelastic deformation. In the case of high-cycle fatigue, damage develops under repetitive loading and macroscopic inelastic strain can be neglected. Continuum models for problems involving degradation have been obtained within the framework of continuum damage mechanics, where a variable is introduced in order to represent the microstructural damage in a continuum sense (Lemaitre and Chaboche, 1990).

In this paper we employ the framework of continuum mechanics to formulate kinetic equations for high-cycle fatigue damage. A multifield description is adopted, in which damage processes are described by means of the introduction of a scalar damage variable and the corresponding force system. Damage kinetic equations are obtained from the balance of the aforementioned force system together with constitutive information compatible with thermodynamics. The constitutive theory comprises both constitutive equations and damage criteria providing conditions under which damage develops. The notions of damage driving force (damage energy release rate), damage resistance, loading/unloading appear naturally. For cyclic loads, a streamlined version of the resulting kinetic equation is written, after integration in one cycle, in a cycle based format. It generalizes most of the linear cumulative damage rules presented in the literature. It is discussed how loading sequence and frequency effects can be incorporated. For illustrative purpose, it is shown some simulations depicting the influence of initial damage and loading frequency on fatigue life and damage accumulation. This paper extends the work presented by the authors in Duda and Souza, 2003.

The usual notation of continuum mechanics is adopted in this work.

2. BASIC NOTIONS

Let $\mathcal{B}$ be a material body kinematically described by the displacement field $\mathbf{u}$ and the damage variable d. The damage variable varies from 0 (pristine material) to 1 (cracked-up material) and it is such $\dot{d} \geq 0$; i.e., damage is assumed to be irreversible.

The Principle of Virtual Power can be used to derive the standard force, standard moment and microforce balances, the latter dual to the damage variable. In particular, the microforce balance in local form may be written as

$$\operatorname{Div} \xi - \pi + \mu = 0, \tag{1}$$

where ξ is the microstress vector and π and μ are the internal and external microforce per unit volume, respectively. In the present context, the local form of dissipation inequality is

$$\dot{\psi} - \mathbf{T} \cdot \dot{\mathbf{E}} - \xi \cdot \dot{\mathbf{p}} - \pi \dot{d} \leq 0, \tag{2}$$

where ψ is the free energy density, $\mathbf{T}$ is the Cauchy stress tensor, $\mathbf{E}$ is the infinitesimal strain tensor and $\mathbf{p} := \nabla d$.

Equation (2) suggests constitutive equations for ψ, $\mathbf{T}$, ξ and π. Thus, we assume that

$$\psi = \hat{\psi}(\mathbf{e}, \mathbf{n}), \quad \mathbf{T} = \hat{\mathbf{T}}(\mathbf{e}, \mathbf{n}), \quad \xi = \hat{\xi}(\mathbf{e}, \mathbf{n}), \quad \pi = \hat{\pi}(\mathbf{e}, \mathbf{n}), \tag{3}$$

where $\mathbf{e} := (\mathbf{E}, d, \mathbf{p})$ and $\mathbf{n} := (\dot{\mathbf{E}}, \dot{d})$. The response function $\hat{\pi}$ is allowed to be singular at $\dot{d} = 0$, which means that π is constitutively indeterminate when $\dot{d} = 0$. This is crucial as will become clear in what follows.

Following the Coleman-Noll procedure and assuming that all the dissipation comes from damage processes, it can be shown that the response functions are restricted by

$$\hat{\psi}(\mathbf{e}, \mathbf{n}) = \hat{\psi}(\mathbf{e}), \quad \hat{\mathbf{T}} = \frac{\partial \hat{\psi}}{\partial \mathbf{E}}, \quad \hat{\xi} = \frac{\partial \hat{\psi}}{\partial \mathbf{p}}, \quad \hat{\pi}_d(\mathbf{e}, \mathbf{n}) \dot{d} \geq 0, \tag{4}$$

where $\hat{\pi}_d := \hat{\pi} - \dfrac{\partial \hat{\psi}}{\partial d}$ is the dissipative response.

Henceforth, we assume that:

$$\hat{\psi}(\mathbf{E}, d, \mathbf{p}) = (1 - d)\hat{\phi}(\mathbf{E}) + \hat{\varphi}(\mathbf{E}, d, \mathbf{p}) \quad \text{and} \quad \hat{\pi}_d(\mathbf{e}, \mathbf{n}) := \hat{a}(\mathbf{e}, \dot{\mathbf{E}}) + \hat{b}(\mathbf{e}, \mathbf{n}), \tag{5}$$

where: $\hat{\phi}$ is the strain energy response of the pristine material; $\hat{\varphi}$ is the damage energy response; $\hat{a}(\mathbf{e}, \dot{\mathbf{E}}) := \lim_{\epsilon \to 0} \hat{\pi}_d(\mathbf{e}, \dot{\mathbf{E}}, \epsilon) \geq 0$ and $\hat{b}(\mathbf{e}, \mathbf{n}) > 0$ such that the reduced dissipation inequality is satisfied.

By inserting the above assumptions into the microforce balance, with $\mu = 0$, it follows that

$$\hat{b}(\mathbf{e}, \mathbf{n}) = \tau - r \quad \text{if} \quad \dot{d} > 0, \tag{6}$$

where

$$\tau := \hat{\phi}(\mathbf{E}) \geq 0 \tag{7}$$

is the damage driving force and

$$r := -\text{Div}\frac{\partial \hat{\varphi}}{\partial \mathbf{p}}(\mathbf{e}) + \frac{\partial \hat{\varphi}}{\partial d}(\mathbf{e}) + \hat{a}(\mathbf{e}, \dot{\mathbf{E}}) \tag{8}$$

is the damage resistance. It is worth remarking that the damage resistance accounts for rate and nonlocal effects as well as for the influence of the strain state.

As $\hat{b} > 0$, equation (6) implies that

$$\tau > r \tag{9}$$

is a necessary condition for damage growth. Therefore, damage processes are frozen whenever $\tau \leq r$, which leads to the notion of elastic range.

3. FATIGUE DAMAGE

Now, we introduce sufficient conditions for which $\dot{d} \neq 0$. Besides (9), the supplementary condition that microstrucutral changes occur only for continued loading, and that no changes occur during unloading, is often assumed. In the present model, the loading or unloading situation is described by the variable $\tau = \dfrac{\partial \tau}{\partial \mathbf{E}} \cdot \dot{\mathbf{E}}$; i.e., $\dot{\tau} > 0$ defines loading whereas $\dot{\tau} \leq 0$ defines unloading. Therefore,

$$\dot{d} > 0 \quad \Longleftrightarrow \quad \tau > r \quad \text{and} \quad \dot{\tau} > 0, \tag{10}$$

which, together with (6), provide the fatigue damage kinetic equation

$$\hat{b}(\mathbf{e}, \mathbf{n}) = \langle \tau - r \rangle H(\dot{\tau}), \tag{11}$$

where $\langle \rangle$ is the Macauly bracket and H is the Heaviside function.

If we assume that $\hat{b}(\mathbf{e}, \mathbf{n}) = B(\mathbf{E}, d, \tau, \dot{d}, \dot{\tau})$, the above equation gives

$$B(\mathbf{E}, d, \tau, \dot{d}, \dot{\tau}) = \langle \tau - r \rangle H(\dot{\tau}). \tag{12}$$

The above equation is rate-independent if and only if the left hand side depends on $\dot{d}$ and $\dot{\tau}$ only through their ratio $\dfrac{\dot{d}}{\dot{\tau}}$.

3.1. Cumulative damage laws - one-dimensional case

Let ε the one-dimensional strain and E the Young modulus of the pristine material. The constitutive assumptions adopted in this section, along with their consequences, are the following:

- The strain energy of the pristine material is $\hat{\phi}(\varepsilon) = \frac{1}{2}E\varepsilon^2$ and the damage energy is equal to zero. These imply that $\tau = \frac{1}{2}E\varepsilon^2$, $\dot{\tau} = E\varepsilon^2\dot{\varepsilon}$, and $r = \hat{a}(\varepsilon, d)$;

- The dissipative response $\hat{\pi}_d$ is such that

$$\hat{a} = \begin{cases} R_T(d) \geq 0 & \text{if } \ \varepsilon \geq 0, \\ R_C(d) \geq 0 & \text{otherwise,} \end{cases}$$
$$B(\varepsilon, d, \tau, \dot{d}, \dot{\tau}) = (\tau - r)\frac{\dot{\overline{U(d)}}}{\dot{\overline{V(\tau)}} + \kappa}, \tag{13}$$

where R_T and R_C are the fatigue limits in traction and compression, $U'(d) > 0$, $V'(\tau) > 0$ and κ is a material parameter.

After taking $(13)_2$ into account, the kinetic equation for one-dimensional case is given by:

$$\dot{\overline{U(d)}} = H(\tau - r)\, H(\dot{\tau})\, (\dot{\overline{V(\tau)}} + \kappa). \tag{14}$$

It is worth remarking that contrary to the usual fatigue damage kinetic equations, which are written using number of cycles instead of time, (14) holds for any kind of fatigue loading, cyclic or not.

In order to recover some classical results, we consider a loading composed of well defined cycles, where each cycle has period T and two peaks $\tau_M(\tau\, maximum)$ and $\tau_m(\tau\, minimum)$. In this way, we can represent the evolution law in terms of the number of cycles N. This formulation is obtained by integration of (14) over one loading cycle.

Integrating (14) in one loading cycle we have:

$$U(d_N) - U(d_{N-1}) = \delta_N + \kappa T, \tag{15}$$

where

$$\delta_N := \delta(\tau_M, \tau_m, d_{N-1}) = V(\tau_M) - V(R_T(d_{N-1})) + V(\tau_m) - V(R_C(d_{N-1})).$$

Considering the loading frequency $f = \frac{1}{T}$, from (15) we obtain that:

$$U(d_N) = U(d_0) + \sum_{i=1}^{N}(\delta_i + \frac{\kappa}{f}). \tag{16}$$

If R_T and R_C are constants, δ_N is the same for all cycles ($\delta_N = \delta$), and the previous equation reduces to:

$$U(d_N) = U(d_0) + N(\delta + \frac{\kappa}{f}). \tag{17}$$

The number of cycles to failure N_f is reached when $d = d_c$ (critical damage), therefore, from (17) we have:

$$N_f = \frac{U(d_c) - U(d_0)}{\delta + \dfrac{\kappa}{f}}. \tag{18}$$

Using (18), relation (17) can be rewritten in terms of the relative number of cycles:

$$d_N = U^{-1}\left(U(d_0) + (U(d_c) - U(d_0))\frac{N}{N_f}\right). \tag{19}$$

To illustrate some effects we use functions $U(d)$ and $V(\tau)$ similars to the Paas et al. (1993) model, i.e.:

$$U(d) = \frac{d^{(1-a)}}{1-a} \quad \text{and} \quad V(\tau) = \frac{c}{(\gamma+1)}\left(\frac{\tau}{E}\right)^{(\gamma+1)}, \tag{20}$$

where a, γ and c are material parameters. It is also assumed for simplicity that the damage driving force τ is equal in traction and compression (τ_a), and that there are no fatigue limits ($R_T = R_C = 0$). In this case we have:

$$\delta = \frac{2c}{(\gamma+1)} \left(\frac{\tau_a}{E}\right)^{(\gamma+1)} = \frac{c}{2^\gamma(\gamma+1)} \varepsilon_a^{2(\gamma+1)}. \tag{21}$$

From Paas et al. (1993), we use the parameters corresponding to concrete: $E = 27000$ MPa, $c = 9.8\text{x}10^{-4}$, $\gamma = 0.8$, $a = 0.4$ From (18) we can observe the effect of frequency on the fatigue life. Figure 2 shows the influence of the parameter k in the fatigue life N_f, for no initial damage and critical damage $d_C = 1$.

From (19) we can obtain the behavior of the damage variable d_N as a function of the relative number of cycles $\frac{N}{N_f}$. Figure 3 shows the influence of the parameter a: a higher value of a results in an initially slower accumulation of damage, and a higher growth rate towards the end of the fatigue life. For $a = 0$ the so-called linear Palmgren-Miner damage rule is obtained. This results were obtained for no initial damage and critical damage $d_C = 1$. We can also observe in (19) the influence of the initial damage. This result is showed in Figure 4, for critical damage $d_C = 1$.

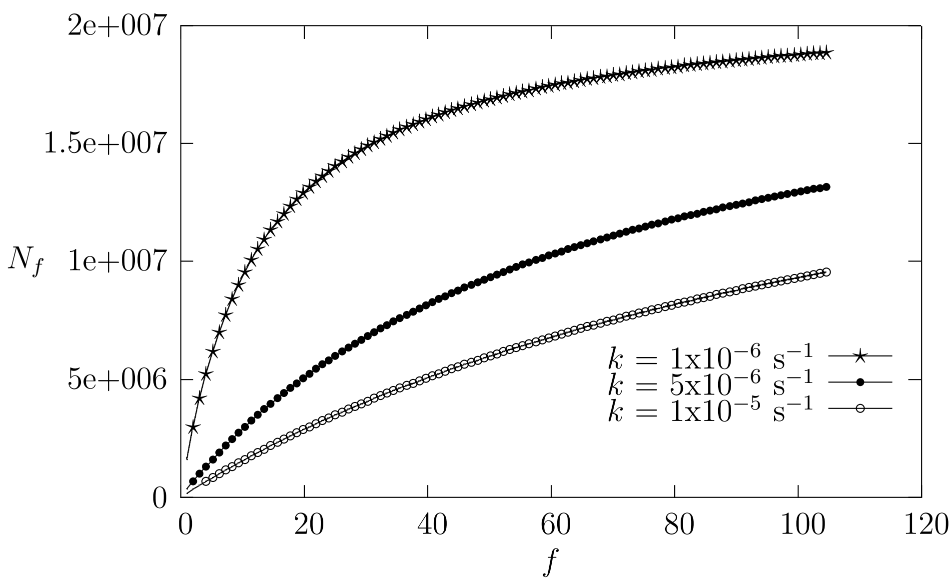

Figure 1: Influence of the parameter k

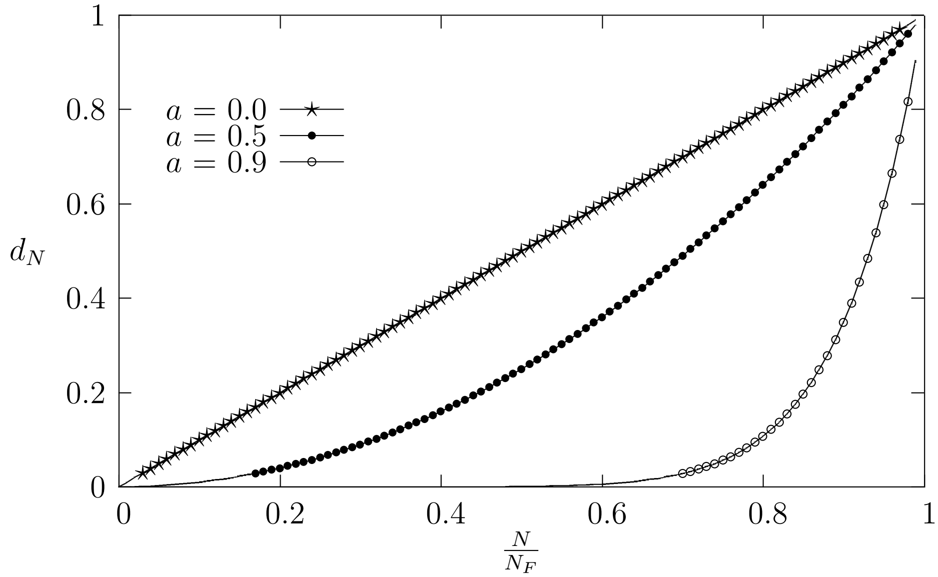

Figure 2: Influence of the parameter a

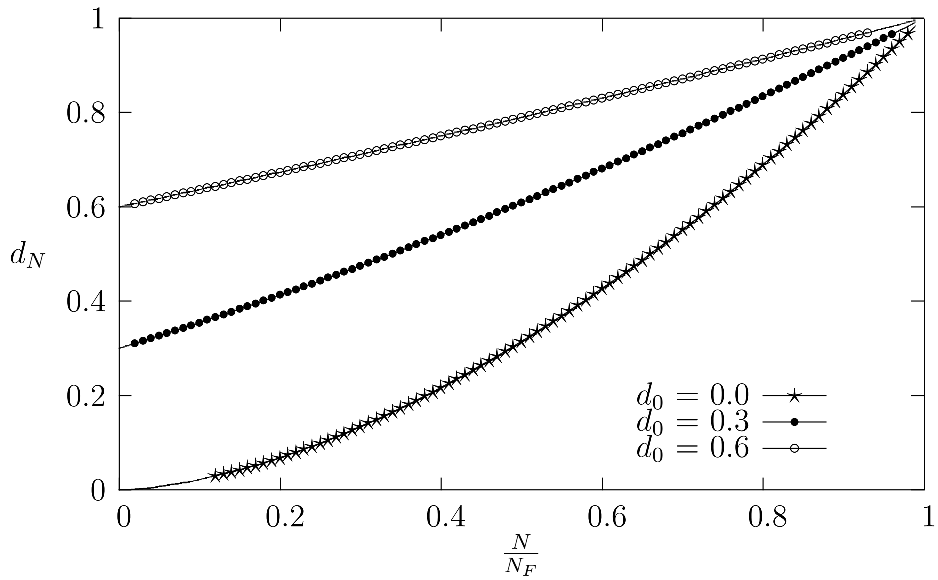

Figure 3: Influence of the initial damage d_0 in the actual damage d_N

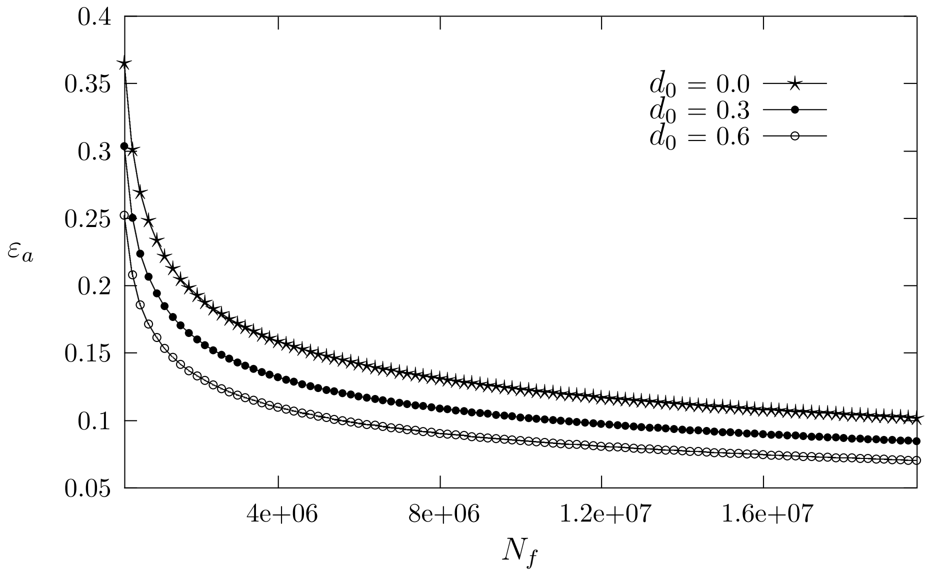

Figure 4: Influence of the initial damage d_0 in the fatigue life N_f

5. ACKNOWLEDGEMENTS

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