ON THE ROLE OF THE MAXIMUM PRINCIPAL STRESS UPON THE FATIGUE ENDURANCE UNDER MULTI-AXIAL LOADINGS

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Abstract: *Many high-cycle multi-axial fatigue endurance criteria consider, as measures of fatigue solicitation, a function of the shear stress amplitude and the maximum hydrostatic stress along the loading history. In this paper, we propose the maximum principal stress, rather than the maximum hydrostatic stress, as a measure of fatigue solicitation upon the micro-cracks embryonated by cyclic loadings. Assessment of the resulting criterion shows that it compares very well with experimental data published in the literature, with a conservative trend in situations where predictions against safety were obtained by considering the maximum hydrostatic stress.*

Keywords: *multi-axial high-cycle fatigue, fatigue endurance, maximum principal stress.*

1. INTRODUCTION

Complex cycling multi-axial non-proportional loading histories are very frequently imposed upon mechanical components under service conditions. The development of new, more accurate and if possible computationally cheaper multi-axial fatigue models capable to predict the durability of these structures is therefore extremely desirable. Many fatigue endurance criteria for situations involving multi-axial cyclic loadings — including those proposed by Crossland (1956), Sines (1959), Dang Van (1973), Papadopoulos (1987), Deperrois (1991), Bin Li et al. (2000), Zouain and Cruz (2002), amongst others — considers the hydrostatic stress as the macroscopic measure of the solicitation of the normal stresses upon micro-cracks. This choice is based on the fact that the hydrostatic stress is a quantity obtained by averaging the normal stress over all the planes passing through a given material point. Thus, the hydrostatic stress would represent the average solicitation upon micro-cracks randomly oriented in the medium.

In this paper, we claim that the worst situation — corresponding to an eventual embryo-crack oriented orthogonally to the maximum principal stress — instead of the average one, should be taken into account when describing the contribution of the normal stress to the fatigue damage. Thus, the maximum principal stress is considered instead of the maximum hydrostatic stress. Assessment of the criterion resulted from the present argument, together with a very simple measure of shear solicitation proposed by Mamiya and Araújo (2002), shows that it compares very well with experimental data published in the literature. Further, it is very simple to implement, making it very competitive with respect to those proposed, for instance, by Papadopoulos or by Zouain & Cruz.

The paper is organized as follows: measures for shear and normal solicitation to fatigue are proposed in section 2. The resulting criterion is assessed in section 3. Some concluding remarks are presented in section 4.

2. THE FATIGUE MODEL

Mechanical degradation due to fatigue under high number of loading cycles takes place at stress levels well below the yield limit. According to pioneering studies conducted by Ewin & Rosenhain (1900), high cycle fatigue damage can be associated with cyclic plastic deformations at the grain level, leading to the formation of persistent slip bands and later to the nucleation of micro-cracks, even if the material shows an essentially elastic behaviour at macroscopic level. On the other hand, if the material point manages to attain cyclic elastic behaviour at grain level, eventually after a number of initially plastic cyclic deformations, then fatigue failure is not expected to occur. Thus, since plasticity plays an important role on crack initiation, shear stresses must be considered as one of the driving forces of the fatigue process.

Another variable that must be considered is the normal stress acting upon embryo-cracks, which has been shown by Sines (1959) to affect the fatigue resistance. Its influence has been taken into account by many authors through an average of the normal stress acting upon all the planes passing through the material point. As remarked by Papadopoulos (1997) such average is equal to the hydrostatic stress.

Under such assumptions, many fatigue limit criteria can be written as:

$$f(\tau) + g(\sigma_h) \leq 0, \tag{1}$$

where f is a function of a measure τ of shear stress, while g is a function of the hydrostatic stress σ_h. Criteria proposed by Crossland (1956), Sines (1959), Dang Van (1973), Papadopoulos (1987), Deperrois (1991), Bin Li et al. (2000), Zouain and Cruz (2002), amongst others, fit into the representation expressed by inequality (1).

Although the present study is focused on the effect of the normal stresses upon the fatigue endurance, we introduce, for the sake of completeness, the measures of shear solicitation proposed by Crossland (1956), Papadopoulos (1997) and Mamiya and Araújo (2002). The Crossland criterion considers, as a measure of the shear solicitation to fatigue along the loading history, the $\sqrt{J_2}$ radius of the sphere circumscribing the stress path (after projection onto the deviatoric space):

$$f(\tau) := \sqrt{\frac{1}{2}||\mathbf{S} - \boldsymbol{\rho}||^2} \tag{2}$$

where the deviatoric component of the stress tensor $\boldsymbol{\sigma}$ is given by $\mathbf{S} := \boldsymbol{\sigma} - \frac{1}{3}(tr\,\boldsymbol{\sigma})\,\mathbf{I}$, while the multidimensional version $\boldsymbol{\rho}$ of the means stress is represented by the center of the sphere circumscribing the stress path. The fatigue criterion proposed by Papadopoulos relies on the argument that the accumulated plastic deformations at mesoscopic level, at each slip plane, are proportional to the resolved shear stress amplitude T_a. An average of this quantity within an elementary volume is given by:

$$f(\tau) := \sqrt{5}\sqrt{\frac{1}{8\pi^2}\int_{\varphi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{\psi=0}^{2\pi}(T_a(\varphi,\theta,\psi))^2\,d\psi\,\sin(\theta)\,d\theta\,d\varphi}. \tag{3}$$

The angle ψ covers all the gliding directions, while φ and θ define the orientation of the material plane inside the elementary volume. Consideration of the measure of shear solicitation proposed by Papadopoulos provides very good results when compared with experimental results for a wide range of materials and loading conditions. On the other hand, an disadvantage associated with his criterion is the fact that it requires quite lengthy and complicated calculations. Deperrois (1991) (and later Bin Li et al. (2000)) considers, as a measure of shear solicitation, the quantity:

$$f(\tau) := \sqrt{\sum_{i=1}^{5} \lambda_i^2}, \tag{4}$$

where $\lambda_i,\ i = 1, \ldots, 5$ are the semi-axes of the ellipsoid circumscribing the stress path (in the deviatoric space). Although the resulting predictions are very good in most of the situations when compared to experiments, practical determination of the circumscribing elliptic hull is difficult and has not been properly addressed by Deperrois nor by Bin Li et al. A simple result presented by Mamiya and Araújo (2002) precludes the determination of the actual elliptic hull in a wide range of situations. Indeed, whenever the circumscribing elliptic hull is a good approximation for the convex hull of the stress path, the shear stress amplitude $f(\tau)$ can be simply computed as:

$$f(\tau) := \sqrt{\sum_{i=1}^{5} a_i^2}, \tag{5}$$

where a_i, $i = 1, \ldots, 5$ are the distances (in the five-dimensional deviatoric stress space) of the faces of *any* rectangular prism circumscribing the stress path to its center. The measure was shown by Mamiya and Araújo (2002) to be invariant upon the choice of the circumscribing rectangular prism whenever the convex hull coincides with the elliptic one.

Tensile normal stresses contribute to the fatigue degradation by acting upon eventually existing embryo-cracks in the material (essentially in mode 1). Since Crossland in 1956, many fatigue endurance criteria consider the hydrostatic stress as the measure of the solicitation to fatigue produced by the normal stresses. This has been motivated by the argument that, as remarked by Papadopoulos (1997), the hydrostatic stress is basically the quantity obtained by averaging the normal stress over all the planes passing through a given material point. The averaging process of the normal stress could be associated with assumption of a homogeneous distribution of orientations of the embryo-cracks nucleated during the fatigue degradation process.

In this paper, we claim that the worst situation — which corresponds to considering the existence of an embryo-crack oriented orthogonally to the maximum principal stress (among the three eigenvalues of the stress tensor and along the stress path) — should be considered rather than the average solicitation given by the maximum hydrostatic stress. Indeed, failure should not be dictated by the mean solicitation, but rather by the critical one.

Based on the considerations developed along this section, we propose the following multiaxial high cycle fatigue endurance criterion:

$$\sqrt{\sum_{i=1}^{5} a_i^2} + \kappa\, \sigma_{p\,max} \leq \lambda, \tag{6}$$

where $a_i,\ i = 1, \ldots, 5$ are defined as in (5) and $\sigma_{p\,max}$ is the maximum principal stress among acting upon the material point along the loading history, while κ and λ are material parameters.

If f_{-1} and t_{-1} are the fatigue endurance limits under solicitation of alternate bending and alternate torsion, respectively, then the parameters κ and λ can be computed as:

$$\kappa = \frac{\sqrt{2}}{f_{-1} - t_{-1}} \left(t_{-1} - \frac{f_{-1}}{\sqrt{3}} \right) \qquad \text{and} \qquad \lambda = \sqrt{2}\, \frac{t_{-1} f_{-1}}{f_{-1} - t_{-1}} \left(1 - \frac{1}{\sqrt{3}} \right). \tag{7}$$

3. ASSESSMENT

Proportional and out-of-phase multi-axial fatigue experiments for a number of different materials were considered to assess the proposed criterion in predicting fatigue strength under a high number of cycles. The data collected are reported in Tables 1 to 4 and correspond to experiments on hard metals ($1,3 \leq f_{-1}/t_{-1} < \sqrt{3}$) involving biaxial stress states, where f_{-1} and t_{-1} are the fatigue limits under fully reversed bending and torsion, respectively. Data came from publications by Nishihara and Kawamoto (1945) (Table 1), Heidenreich et al. (1983) (Table 2), Lempp (1977) (Table 3) and Froustey and Lassere (1989) (Table 4). The following nomenclature was adopted in these Tables: the subscript a stands for the amplitude of stresses while m represents the mean value. As usual, σ and τ are normal and shear stresses while β contains information concerning phase difference. The stress values reported in each table correspond to the maximum combination of stresses that the specimen can stand without failing, up to a limit of 10^6 cycles.

To assess the quality of the results provided by our model, an error index I is defined as:

$$I = \frac{1}{\lambda} \left(\sqrt{\sum_{i=1}^{5} a_i^2} + \kappa\, \sigma_{p\,max} - \lambda \right) \times 100 \quad (\%), \tag{8}$$

which gives a measure of how close the prediction of the criterion is with respect to the experimental data. A negative I yields a non-conservative fatigue strength prediction since it indicates that the stress solicitation has not attained a critical value while the experimental data are representative of limiting situations. On the other hand, a positive I provides a conservative estimate while $I = 0$ means a perfect prediction for the observed fatigue strength.

Table 1 reports experimental data under in-phase and out-of-phase alternated bending and torsion conditions. Analysis of these data revealed that all the criteria considered show, in general, satisfactory predictions of fatigue strength, regardless of the phase angle. Exception was observed for the Crossland criterion in experiments 1-7 and 1-8, where the calculated error index were respectively -8.35% and -17.81%.

Results 2-1 to 2-6 from Table 2 are also associated with in-phase and out-of-phase alternated loadings producing normal and shear stresses. In this set of data, the Crossland criterion yielded quite poor predictions under out-of-phase loadings, while the other criteria rendered excellent results. Experiments 2-7 to 2-9 were carried out under alternated bending and repeated torsion, while experiments 2-10 to 2-12 considered repeated bending and alternated torsion. Under the presence of mean stresses, the proposed model produced more conservative results when compared with the remaining criteria. The same trend can be observed in the results reported in Tables 3 and 4. This fact is in agreement with the hypothesis that the worst situation — which corresponds to considering the existence of an embryo-crack oriented orthogonally to the maximum principal stress (among the three eigenvalues of the stress tensor and along all the stress path) — should be considered rather than the average solicitation given by the maximum hydrostatic stress. In summary, application of our model to the experimental data provided an error index which varied in the worst cases between −8.74% and 15.34% for all materials and loading conditions analyzed. The results provided by both Papadopoulos (1997) and by Mamiya & Araújo (2002) varied between −15.3% and 7.3% while the Crossland criterion provided significantly poorer predictions. In our model, a shift of the error index towards the conservative region can be clearly observed whenever a mean stress is present in the loading history.

Table 1 – Fatigue strength of hard steel (t_{-1}=196.2 MPa,f_{-1}=313.9 MPa): experimental data (Nishihara & Kawamoto (1945)) and predictions.

	σ_a(MPa)	σ_m(MPa)	τ_a(MPa)	τ_m(MPa)	β(°)	I^a(%)	I^b(%)	I^c(%)	I^d(%)
1-1	138.1	0	167.1	0	0	-2.27	-2.3	-2.28	-1.91
1-2	140.4	0	169.9	0	30	-2.60	-0.6	-0.64	-0.27
1-3	145.7	0	176.3	0	60	-3.61	3.1	3.10	3.49
1-4	150.2	0	181.7	0	90	-3.74	6.3	6.27	6.66
1-5	245.3	0	122.6	0	0	1.44	1.5	1.44	1.73
1-6	249.7	0	124.8	0	30	0.01	3.3	3.26	3.55
1-7	252.4	0	126.2	0	60	-8.35	4.4	4.39	4.69
1-8	258.0	0	129.0	0	90	-17.81	6.5	6.70	7.01
1-9	299.1	0	62.8	0	0	0.92	0.9	0.92	1.02
1-10	304.5	0	63.9	0	90	-2,99	2.7	2.74	2.83

^{a} Crossland, ^{b} Papadopoulos, ^{c} Mamiya & Araújo, ^{d} Current model

Table 2 – Fatigue strength of 34Cr4 (t_{-1}=256 MPa,f_{-1}=410 MPa) experimental data (Heidenreich et al. (1983)) and predictions.

	σ_a(MPa)	σ_m(MPa)	τ_a(MPa)	τ_m(MPa)	β(°)	I^a(%)	I^b(%)	I^c(%)	I^d(%)
2-1	314.0	0	157.0	0	0	-0.55	-0.6	-0.55	-0.27
2-2	315.0	0	158.0	0	60	-12.33	-0.1	-0.11	0.18
2-3	316.0	0	158.0	0	90	-22.93	0.1	0.08	0.37
2-4	315.0	0	158.0	0	120	-12.33	-0.1	-0.11	0.18
2-5	224.0	0	224.0	0	90	-8.38	5.2	5.15	5.55
2-6	380.0	0	95.0	0	90	-7.32	0.4	0.37	0.49
2-7	316.0	0	158.0	158.0	0	0.08	0.1	0.08	6.01
2-8	314.0	0	157.0	157.0	60	-12.69	-0.6	-0.54	5.34
2-9	315.0	0	158.0	158.0	90	-23.17	-0.1	-0.11	5.83
2-10	279.0	279.0	140.0	0	0	-6.38	-6.4	-6.38	-0.21
2-11	284.0	284.0	142.0	0	90	-25.5	-4.8	-4.83	1.45
2-12	212.0	212.0	212.0	0	90	-9.39	3.4	3.41	7.23

^{a} Crossland, ^{b} Papadopoulos, ^{c} Mamiya & Araújo, ^{d} Current model

Table 3 – Fatigue strength of 42CrMo4 (t_1=260 MPa,f_1=398 MPa): experimental data (Lempp (1977)) and predictions.

	σ_a(MPa)	σ_m(MPa)	τ_a(MPa)	τ_m(MPa)	$\beta(°)$	I^a(%)	I^b(%)	I^c(%)	I^d(%)
3-1	328.0	0	157.0	0	0	4.19	4.2	4.19	4.63
3-2	286.0	0	137.0	0	90	-28.14	-8.8	-9.13	-8.74
3-3	233 0	0	224.0	0	0	7.30	7.3	7.3	7.94
3-4	213.0	0	205.0	0	90	-14.94	-1.8	-1.84	-1.25
3-5	266.0	0	128.0	128.0	0	-15.34	-15.0	-15.3	-7.80
3-6	283.0	0	1360	136.0	90	-28.89	-9.6	-9.97	-1.97
3-7	333.0	0	160.0	160.0	180	5.92	5.8	5.92	15.34
3-8	280.0	280.0	134.0	0	0	-2.89	-2.7	-2.89	7.04
3-9	271.0	271.0	130.0	0	90	-23.99	-5.8	-5.93	3.67

^{a} Crossland, ^{b} Papdopoulos, ^{c} Mamiya & Araújo, ^{d} Current model

Table 4 – Fatigue strength of 30NCD16 (t_{-1}=410 MPa,f_{-1}=660 MPa): experimental data (Froustey & Lasserre (1989)) and predictions.

	σ_a(MPa)	σ_m(MPa)	τ_a(MPa)	τ_m(MPa)	$\beta(°)$	I^a(%)	I^b(%)	I^c(%)	I^d(%)
4-1	485.0	0	280.0	0	0	1.77	1.8	1.77	2.07
4-2	480.0	0	277.0	0	90	-27.27	0.7	0.70	1.00
4-3	480.0	300.0	277.0	0	0	3.91	3.9	3.91	7.63
4-4	480.0	300.0	277.0	0	45	-3.36	3.9	3.91	7.63
4-5	470.0	300.0	270.0	0	60	-10.93	1.6	1.60	5.32
4-6	473.0	300.0	273.0	0	90	-25.12	2.5	2.45	6.17
4-7	590.0	300.0	148.0	0	0	0.11	0.1	0.11	4.32
4-8	565.0	300.0	141.0	0	45	-7.23	-4.1	-4.07	0.14
4-9	540.0	300.0	135.0	0	90	-14.97	-8.1	-8.15	-3.94
4-10	211.0	300.0	365.0	0	0	-0.68	-0.7	-0.68	1.86

^{a} Crossland, ^{b} Papadopoulos, ^{c} Mamiya & Araújo, ^{d} Current model

4. CONCLUSIONS

A new multi-axial fatigue criterion which is very simple to implement has been proposed. Application of this criterion to a broad range of in-phase and out-of-phase loading conditions involving four different materials under multi-axial, in-phase and out-of-phase states of stress yielded very good predictions of fatigue endurance. The proposed criterion always provided more conservative endurance estimates than all the other criteria considered in the present study, whenever shear or normal mean stresses were present in the loading history. On the other hand, when such mean stresses were absent, the predictions were essentially the same for all criteria with exception of Crossland. A very interesting feature of the proposed model which should be stressed is the great simplicity of implementation of our criterion.

5. ACKNOWLEDGMENT

This project was supported by CNPq under contract 300345/03-0, by a PIBIC scholarship granted by CNPq/UnB and by Centrais Elétricas do Norte do Brasil S. A. – Eletronorte. These supports are gratefully acknowledged.

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