MOTION OF A GEOSTATIONARY ARTIFICIAL SATELLITE

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***Abstract.** A new technique is developed for determining the motion of an artificial satellite at an altitude of about six Earth radii, corresponding to an orbital period of 24 hours. The gravitational anomalies due to the axial asymmetry of the Earth give rise to a resonance condition between such anomalies and the motion in longitude of the satellite. The dominant part of the Hamiltonian Function is given by the Newtonian central field and includes the main influence of the Earth dynamical flattening. The disturbing function includes all major equatorial anomalies contributing to such resonance, except for luni-solar perturbations. By means of a canonical transformation, the original Hamiltonian Function is mapped into a new form corresponding to the classical ideal resonance problem. The existence of small divisors is avoided altogether, improving the convergence of the method of successive approximations. An alternate procedure, also including all major equatorial anomalies, is developed with the aid of Lagrange´s Planetary Equations, leading to the same result, to first approximation, of the canonical method first developed, despite the fact that for the particular problem under analysis the canonical method presents some numerical difficulties.*

***Keywords.** Resonance, Artificial Satellite, Geosynchronous Orbit*

1. INTRODUCTION

The problem of motion of a geostationary satellite has been treated by several authors, mainly due to the applications to communication satellites. In this context na ideal situation is considered, assuming a circular equatorial orbit, while in reality the orbit is not exactly circular nor the inclination is zero. Nevertheless, a first approximation to the problem is important in order to establish a nominal trajectory. In this respect, a major unsolved difficulty has been to define a nominal trajectory where many, if not all equatorial anomalies, are taken into account, contributing to a resonance with the equatorial bulge of the Earth. A better definition would imply an improved performance of the operations involved in station keeping.

It is well known that two major factors limit the life of a communication satellite, fuel available on board and degeneration of electronics due to cosmic radiation. A better definition of the best position for station keeping would certainly implies in less fuel consumption and a better definition of final orbit injection operation..

It is clear that today's computational methods and hardware are a strong support for a station keeping operation, but this does not invalidate a better knowledge of the nominal trajectory to be designed (Giacaglia, 2003).

The simple and classical approach given by Kaula (1966), where only two tesseral harmonic coefficients, C_{22} e S_{22}, are taken into account, is sufficient to indicate the presence of two centers and two saddle points for a geostationary satellite, on the equatorial plane. The influence of all other anomalies contributing to the 1:1 resonance with the 24-hour satellite are generally not taken into account, assuming a minor influence in the problem. Actually, considering values of all other harmonics it becomes clear that their cumulative effect may have a considerable influence on the station keeping operation.

The Hamiltonian of the problem has the form given by Eq. (1), where μ is the product of the universal gravitational constant and the mass of the Earth, R is the mean equatorial radius, ω_e is the earth diurnal rotation rate, $L^2=\mu a$, a the orbital radius of the satellite, λ its longitude reckoned from the first point of Aries and θ the Greenwich Hour Angle..

Coefficients C_{nm} and S_{nm} of the tesseral harmonics contribute to the 1:1 resonance with the equatorial bulge if $l - m = 2p$, $p = 0, 1, \ldots, [l/2\}$, and $l = 2, 4. 6, \ldots$ Legendre´s Polynomials reduce to numerical values corresponding to Kaula´s Inclination Functions (Kaula, op. cit.) for a zero inclination

$$H(a,\lambda-\theta)=\mu^2/2L^2+\mu^4C_{20}R^2/2L^6+\omega_e L+ \\ +\sum_{l=2}^{\infty}\mu(R^l/a^{l+1})\sum_{m=0}^{l}F_{lm(l-m)/2}\left[C_{lm}\cos m(\lambda-\theta)+S_{lm}\sin m(\lambda-\theta)\right]. \tag{1}$$

The adopted constants are:

R = 6 378 km, μ = 3,986 032 × 10^5 km^{3}s^{-2}, ω_e = 7,291 899 706 × 10^{-5}s^{-1}, a = 42 164,271 km, C_{20} = 1 082,7 × 10^{-6}, C_{22} = 1,540 × 10^{-6}, S_{22} = -0,870 × 10^{-6}, C_{33} = 0,078 × 10^{-6}, S_{33} = 0,226× 10^{-6}, C_{42} = 0,074 × 10^{-6}, S_{42} = 0,148 × 10^{-6}, C_{44} = -0,001 × 10^{-6}, S_{44} = 0,148 × 10^{-6}.

Other coefficients have not been considered, because the scope of this work is to show that it is possible to include any number of them, up to any degree of approximation. It is visible that, for instance, the value of these coefficients does not decrease very fast, although the numerical value of all other parts of the final coefficient of the Fourier Series as given by Eq. (1) is decreasing much faster.

The Hamiltonian given by Eq.(1) is written as

$$H(x,y)=H_0(x)+H_1(x,y) \tag{2}$$

where $x = L$ *and* $y = \lambda - \theta$, $H_0(x)$ corresponds two the first three terms of $H(x,y)$ and $H_1(x,y)$ to the trigonometric series which, including all possible resonances with a 24-hour satellite, assumes the form

$$H_1(x,y)=\sum_{k\geq 2}A_k(x)\cos ky+B_k(x)\sin ky \tag{3}$$

Including coefficients up to the fourth degree, one has the following values (Schutz, 2004)

$A_2=1{,}0571\times 10^{-7}(a/\mu)$, $B_2=-5{,}971\ 7\times 10^{-8}\ (a/\mu)$, $A_3=4{,}044\ 7\times 10^{-9}\ (a/\mu)$, $B_3=1{,}171\ 9\times 10^{-8}\ (a/\mu)$, $A_4 = -5{,}487\ 3\times 10^{-11}\ (a/\mu)$, $B_4 = 2{,}743\ 7\times 10^{-10}\ (a/\mu)$, $\mu/a = 9{,}453\ 577\ 396\ km^2s^{-2}$.

Retaining terms associated with the ellipticity of the equator, in the present work truncated at $l = 4$, the above series assumes the form

$$H_1(x,y)=\sum_{j\geq 1}Q_j(a)\cos 2(y-\lambda_{2j,2}) \tag{4}$$

where

$$Q_j=\frac{\mu R^{2j}}{a^{2j+1}}F_{2j,2,j-1}(0)\sqrt{C_{2j,2}^2+S_{2j,2}^2} \tag{5}$$

2. INTRODUCING A SMALL PARAMETER

The ideal resonance problem as defined by Garfinkel (1970) corresponds to the reduction of the general Hamiltonian $H_1(x,y)$ to the simple form where only the *cos2y* term appears. As was indicated by Giacaglia (1970, 2003) it is possible to take into account all resonant terms, not only those factoring the *cos2y* term, and reduce the Hamltonian to a form containing only such argument. Here it is shown that this is actually the case, that is, there is a canonical transformation to variables (u, v) such that the Hamiltonian is mapped into the simple form

$$H[x(u, v), y(u, v)] = P(u) + Q(u) \cos 2v + R(u) \sin 2v \tag{6}$$

The transformation (Brouwer, 1961; Giacaglia, 1972) is generated by a Hamilton – Jacobi Function $S(u,y)$ developed in terms of fractional powers of a small parameter which, in the general situation, where all resonant arguments are considered, is taken as the magnitude of the ratio of the dominant term in $H_1(x,y)$ and the Newtonian central field intensity, approximately 10^{-6}. This small parameter, taken exactly equal to this value is indicated by ε. Therefore the basic hypothesis is that

$$H(x,y) = H_0(x) + H_1(x,y),\ H_0(x) = O(1),\ H_1(x,y) = O(\varepsilon) \tag{7}$$

Since the satellite period is close to 24 hours, the difference between its mean motion n in longitude and the Earth rotation rate ω_e is taken to be of the order of the square root of the small parameter, that is, $n - \omega_e = O(\varepsilon^{1/2}) = O(10^{-3})$. Neglecting the tesseral harmonics disturbances, the differential equation for the angular variable y is given by the Hamiltonian

$$H_0(x) = \mu^2 / 2x^2 + \mu^4 C_{20} R^2 / 2x^6 + \omega_e x \tag{8}$$

so that

$$dy/dt = -dH_0(x)/dx = \mu^2/x^3 - \omega_e + O(10^{-3}) = n - \omega_e + O(10^{-3}) = O(10^{-3}) \tag{9}$$

2.1. The Canonical Transformation

The generating function is taken as

$$S(u,y) = uy + S_{1/2}(u,y) + S_1(u,y) + S_{3/2}(u,y) + \cdots \tag{10}$$

where the subscript shows the order of magnitude with respect to ε. The transformation is defined by

$$\begin{aligned} v &= \partial S/\partial u = y + \partial S_{1/2}/\partial u + \cdots \\ x &= \partial S/\partial y = u + \partial S_{1/2}/\partial y + \cdots \end{aligned} \tag{11}$$

The coefficients of the new Hamiltonian given in Eq.(7) are written as

$$P(u) = P_0 + P_{1/2} + P_1 + P_{3/2} + \cdots \tag{12}$$

and similar developments for $Q(u)$ e $R(u)$. By introducing Eq. (9) into Eq. (5) it is found that

$$P_0(u)=H_0(u),\ Q_0(u)=R_0(u)=0$$
$$P_{1/2}(u)=Q_{1/2}(u)=R_{1/2}(u)=0 \quad (13)$$

$$H_0'(u)S_{1/2,y}+\frac{1}{2}H_0''(u)S_{1/2,y}^2+H_1(u,y)=P_1(u)+Q_1(u)\cos 2y+R_1(u)\sin 2y \quad (14)$$

$$\left[H_0'(u)+H_0''(u)S_{1/2,y}\right]S_{1,y}+\frac{1}{6}H_0'''(u)S_{1/2,y}^3+H_1'(u)S_{1/2,y}=$$
$$=P_{3/2}(u)+Q_{3/2}(u)\cos 2y+R_{3/2}(u)\sin 2y \quad (15)$$

2.2. The Generating Function

The first approximation to the Generating Function is given by

$$S_{1/2,y}=-\frac{H_0'}{H_0''}\pm\left\{\left(\frac{H_0'}{H_0''}\right)^2+\frac{2}{H_0''}\left[P_1-(A_2-Q_1)\cos 2y-(B_2-R_1)\sin 2y\right]\right\}^{1/2} \quad (16)$$

where A_2 and B_2 are the dominant terms of the original Hamiltonian, factoring *cos2y* and *sin2y*. By defining

$$q=\left[(A_2-Q_1)^2+(B_2-R_1)^2\right],\ \cos 2\alpha_1=\frac{A_2-Q_1}{q^{1/2}},\ \sin 2\alpha_1=-\frac{B_2-R_1}{q^{1/2}}$$
$$r=\frac{H_0'}{H_0''},\ p^2=\frac{2q^{1/2}}{q^{1/2}+a^2H_0''+2P_1},\ b^2=r^2+(2P_1+q^{1/2})/H_0'',\ w=y+\pi/2-\alpha_1 \quad (17)$$
$$\Delta=\sqrt{1-p^2\sin^2 w}$$

the equation for $S_{1/2}$ is

$$S_{1/2,y}=r+b\sqrt{1-p^2\sin^2 w}=r+b\Delta \quad (18)$$

that is, considering that, in general p > 0,

$$S_{1/2}(u,y)=-ry\pm bE(p,w),\quad p<1$$
$$S_{1/2}(u,y)=-ry\pm b\left[pE(\frac{1}{p},z)-\frac{p^2-1}{p}F(\frac{1}{p},z)\right],\ p>1,\ \sin z=p\sin w \quad (19)$$

where $E(\kappa;\xi)$ and $F(\kappa;\xi)$ are the Elliptic Integrals of the Second and First kind, with amplitude ξ and modulus κ.

In the second of Eqs.(19), when $p>1$, it is clear that the angle w is bounded by the values given by $(-1/p)\le \sin w\le (1/p)$, so that, certainly, this case corresponds to librations around centers defined by $w=0$ or 180^o and x corresponding to a satellite´s period very close to 24 h.. Numerical values of C_{22} e

S_{22} give a value $\alpha_1 \cong 15^o$. The Earth relative longitudes, defined by $\alpha_1 \pm\pi/2$, correspond to 75° W and 105° E of Greenwich, and are the longitudes of the stable equilibrium points..

In order to avoid secular terms in the Generating Function, when $p < 1$, one should choose the unknown coefficients P_1 , Q_1 e R_1 in such a way that the secular term in the Elliptic Integral matches exactly the term ry. This gives a first condition for the definition of these unknown coefficients.

When $p < 1$, the Elliptic Integral of Second Kind may be written as:

$$E(p, w) = (2/\pi)E(p, \pi/2)w + \text{periodic function} \tag{20}$$

where $E(p, \pi/2)$ is the corresponding complete integral, that is,

$$(2/\pi)\, E(p, \pi/2) = F(-1/2,1/2;1;\, p) = 1-\sum_{n=1}^{\infty}\left[\frac{(2n-1)!!}{2^n n!}\right]^2 \frac{p^{2n}}{2n-1} \tag{21}$$

where F *is* the Hypergeometric Function with modulus p.

When $p > 1$, the elliptic integrals $E(1/p,w)$ and $K(1/p, w)$ also have a secular term in w but since this angle is restricted the linear part is limited in value.

In the libration case, p < 1, the solution is

$$S_{1/2}(u,w) = -2(b/\pi)E(p,\pi/2)w + bE(p,w) \tag{22}$$

Carrying the solution up to the next order (one part in a million) presents no difficulties and, after defining

$$\begin{aligned}
S_{3/2} &= A_3' sin3\alpha_1 - B_3' \cos 3\alpha_1 \\
C_{3/2} &= A_3' \cos 3\alpha_1 + B_3' sin3\alpha_1 \\
D_{3/2} &= -Q_{3/2} sin2\alpha_1 + R_{3/2} \cos 2\alpha_1 \\
G_{3/2} &= Q_{3/2} \cos 2\alpha_1 + R_{3/2} sin2\alpha_1 \\
B_0 &= -H_0''/6 \\
\Delta &= \sqrt{1-p^2 sin^2 w}
\end{aligned} \tag{23}$$

one finds, for $p < 1$,

$$\begin{aligned}
C_{1/2}S_1(u,y) &= 3r^2 bB_0(\alpha_1 - \pi/2) + B_0 b\left[3r^2 + b^2(1-p^2/2)\right]w + \frac{1}{4}b^3 B_0\, psin2w + \\
&+ \frac{1}{3}bS_{3/2} sin3w - \frac{1}{3}bC_{3/2}\cos 3w + \frac{2}{k^2}D_{3/2}\Delta + \frac{2r}{p^2}S_{3/2}\Delta sinw - \frac{2r}{p^2}C_{3/2}\Delta\cos w + \\
&+ \frac{r}{p^3}\left[(1-p^2)S_{3/2} + (2-p^2)C_{3/2}\right]ln(p\cos w + \Delta) - \frac{3r}{k}S_{3/2} arcsin(psinw) + \\
&+ \left(B_0 r^3 + P_{3/2} + \frac{2-p^2}{p^2}G_{3/2}\right)F(w,p) + \left(3rb^2 B_0 - \frac{2}{p^2}G_{3/2}\right)E(w,p)
\end{aligned} \tag{24}$$

Equation (24) involves Elliptic Integrals of the first and second kind, as well as periodic functions such as the natural log of a periodic function, the arcsin of a periodic function and the function Δ isolately or combined with with periodic functions. It should be noted that the order of magnitude is affected by the fact that the derivative with respect to the variable $x = \sqrt{\mu a}$ which is very large for a geostationary satellite, produce eventually unrealistic results when only literal developments, without actual verification of the magnitude of terms, are considered, as is the case of works by Romanowicz (1975) and Morando (1963).There is no singular term since for $p < 1$, $p \cos w + \Delta > 0$ for all values of w. Furthermore all functions involved are periodic, except for the Elliptic Integrals.

2.3. Coordinate transformation

In the new variables the Hamiltonian is mapped into

$$K(u,y) = P(u) + Q(u)\cos 2w + R(u)\sin 2w \tag{25}$$

and the canonical transformation is defined by the Generating Function

$$S(u,y) = uy + S_{1/2}(u,y) + S_1(u,y) + O(\varepsilon^{3/2}) \tag{26}$$

that is

$$\begin{aligned} v &= \partial S/\partial u = y + \partial S_{1/2}/\partial u + \partial S_1/\partial u + O(\varepsilon^{3/2}) \\ x &= \partial S/\partial y = u + \partial S_{1/2}/\partial y + \partial S_1/\partial y + O(\varepsilon^{3/2}) \end{aligned} \tag{28}$$

The implicit form of these transformations can be avoided making use of a method introduced by Hori (1976) where the transformation is obtained by Lie Series. But the scope of this work is to demonstrate the possibility of obtaining a new Hamiltonian maintaining the form of the Ideal resonance problem. It should be noted that $O(\varepsilon^{1/2})$ corresponds, in this problem to 10^{-3} (one part in a thousand) and $O(\varepsilon)$ corresponds to 10^{-6} (one part in a million) and the generating function was developed up to this order of approximation, although any derivative with respect to x, as noted before, may introduce some change in the order of magnitude of the terms involved in the development.

3. DIRECT SOLUTION

In order to show that the solution obtained by means of a canonical transformation matches with a first order variation of parameters technique. Lagrange´s Planetary Equations (Brouwer, op. cit.) are used next. The equation for the perturbation in semi-major axis of the satellite is

$$\dot{a} = \frac{2}{na}\frac{\partial H_1}{\partial \lambda} = -\frac{4}{na}\sum_{j\geq 1} Q_j(a)\sin 2(y - \lambda_{2j,2}) \tag{29}$$

where the disturbing function has been given in Eq. (4). Taking into account that $y = \lambda - \theta$ and that, by Kepler Harmonic Equation, $n^2a^3 = \mu$, it follows that

$$\ddot{y} = \ddot{\lambda} = \frac{\partial n}{\partial a}\dot{a} = -\frac{3}{2}\frac{n}{a}\dot{a} = \frac{6}{a^2}\sum jQ_j \sin 2(y - \lambda_{2j,2}) \tag{30}$$

By defining $\psi = y - \lambda_2 - \pi/2$, integration of Eq. (30) gives

$$\dot{\psi}^2 = C + \frac{24}{a^2} K_2^2 \cos 2\psi \tag{31}$$

where C is an integration constant corresponding to the square of the angular velocity $\dot{\psi}$ when the angle ψ is equal to $\pi/4$. The longitude λ_2 is given by

$$K_2^2 \cos 2\lambda_2 = \sum_{j=1}^{\infty} Q_j(a) \cos \lambda_{2j,2}$$
$$K_2^2 \sin 2\lambda_2 = \sum_{j=1}^{\infty} Q_j(a) \sin \lambda_{2j,2} \tag{32}$$

$$K_2^4 = \left(\sum_{j=1}^{\infty} Q_j(a) \cos \lambda_{2j,2} \right)^2 + \left(\sum_{j=1}^{\infty} Q_j(a) \sin \lambda_{2j,2} \right)^2 \tag{33}$$

Equation (31) yields

$$\dot{\psi}^2 = \left(C + \frac{24K_2^2}{a^2} \right) \left(1 - k^2 \sin^2 \psi \right) \tag{34}$$

where the parameter k is defined by

$$k^2 = \frac{48K_2^2}{Ca^2 + 24K_2^2} \tag{35}$$

Two situations may occur:

a) $k < 1$: in this case, corresponding to $C > 24K_2^2 / a^2$, one has libration about two equilibrium points, two potential wells in the Earth potential field at the equator, corresponding to 0^o or 180^o values of the variable ψ, that is, when the longitude of the satellite referred to Greenwich, is at 90^o or at 270^o from the equatorial bulge, defined by λ_2 (about 15^o) and approximately equal to 75^o *W* or 105^o *E* of Greenwich. A 24-hour satellite at these locations has no drift, if the equator were an exact elliptic shape. At theoretical right angles of these locations, one has two saddle points, two picks in the Earth potential field at the equator. A 24-hour satellite at these locations will drift away very fast and will move toward the longitudes corresponding to one of the two stable equilibrium points. It will reach these points with considerable speed, about 1o per day, giving rise to an overshooting and, therefore, to a self exited oscillation, to be damped by activating the satellite energy systems. It is clear that the angle ψ will oscillate between values such that $1/k \geq \sin\psi \geq -1/k$. From Eq. (35) it follows that the equilibrium points (centers) correspond to a value of the constant C given by $C = -24K_2^2 / a^2$, a certainly negative value. When $k > 1$, the solution of the problem is given by

$$F(k,\psi) = \left(\sqrt{C + 24K_2^2 / a^2}\right) t \tag{36}$$

and the libration period is given by $T = \left(4F(k,\pi/2) / \sqrt{C + 24K_2^2 / a^2}\right)$.

b) $k > 1$: in this case, corresponding to $C < 24K_2^2 / a^2$, a positive value, one has circulation around the Earth equator, with a period greater or smaller than 24 hours. It should be noted that a small fraction of 1 hour is enough to put the satellite in this situation. The value $C = 0$, is a possible case, although not general. Another possibility is to have a negative value of C, in between 0 and $-24K_2^2 / a^2$, excluding this limiting value. When $k > 1$ the solution is

$$\frac{1}{k} F\left(\frac{1}{k}, arcsin(k\, sin\psi)\right) = \left(\sqrt{C + 24K_2^2 / a^2}\right) t \tag{37}$$

and the circulation period is given by $T = \left(4F(\frac{1}{k}, \pi/2) / k\sqrt{C + 24K_2^2 / a^2}\right)$.

As k approaches a unit value, this period approaches an infinite value, corresponding to asymptotic orbits toward saddle points. These orbits separate the stability regions of libration around centers and the circulation regions around both centers.

It is seen that both methods developed here lead to the same conclusion, to the first order of approximation. On the other hand, the method used in the previous chapter allows an improvement of the solution, in principle to any order, although beyond the order corresponding to 10^{-6} (one part in a million) the mathematics becomes extremely complex and the presence of the variable x in the denominator may distort the classical equations used in low satellites theories and in the motion of asteroids (Brouwer, op. cit.).

4. CONCLUSIONS

The canonical method developed in Chapter 2 has a great potential for solutions of high order of approximation, although as noted before care should be taken with the fact that x is a very large quantity due to the high altitude of the satellite and derivatives with respect to this variable may mask the order of magnitude of the approximations, something certainly true above the firs order. Up to the first order of approximation (one part in a thousand) the solution gives the same answer as the direct use of Lagrange´s Planetary Equations for the variation of the elliptic element of the osculating orbit.

A better definition of the equilibrium points of the problem will lead to a more realistic values of the longitudes, corresponding two the wells in the Earth potential field. It is known (Morgan, 1989) that observations indicate the longitudes 107^o E and 79^o W as the actual locations of those longitudes, one over Sri Lanka, in the Indian Ocean and the other over the Pacific Ocean, off the coast of Equador, slightly different from the locations indicated by just considering the second degree and order tesseral harmonics of the Earth potential field. These observed longitudes do not correspond to opposite points on the equatorial plane of the Earth, a consequence of the influence of high order tesseral harmonics and the presence of the Sun and of the Moon. These are important gravitational forces affecting the orbit of a communication satellite, producing additional drift in longitude and also in latitude, shifting the satellite, periodically from the equatorial plane. It is amazing that a simple difference of about 100 m around the equator is responsible for such a strong effect on a satellite, orbiting at 35786 km altitude above the Earth equator. It is also clear that the equator of the Earth is not an ellipse, even to a first approximation.

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