

# EXPERIMENTAL DYNAMIC MODEL VALIDATION TO A ONE LINK FLEXIBLE MANIPULATOR

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**Abstract.** *Active control synthesis to manipulators robots with flexible links is still a problem with a hard solution due to two main reasons: significant differences between the structural dynamic model and the physic plant; presence of non-linearity associated to the gear motor actuators internal friction. The last one can be avoided by using actuators without a gear box (direct-drive). However, the knowledge of a realistic structural dynamic model is imperative to a good performance of active control laws of structural vibration. It was developed, in this work, one manipulator robot with only one link, flexible, with the immediate objective to validate structural dynamical modeling formalisms. A modeling formalism published in 2002 was tested experimentally with success, proving it to be quite efficient and validating the modeling formalism.*

**Keywords:** *Flexible robots, dynamic modeling, experimental validation, control.*

## 1. INTRODUCTION

This work deals with dynamic modeling for flexible structures, specifically the case of manipulator robots with flexible links. To validate experimentally the dynamic modeling approaches, it was constructed a manipulator robot with one flexible link, equipped with four sensors to compare the experimental and simulation results. The tested dynamic modeling approach was proposed in Machado et al. (2002). As will be seen later, this model presented very good results compared to experimental results obtained in open loop.

Active control for flexible structures was studied by Maizza-Neto (1974), being one of the first works in this research domain. In the eighteen years, many works were published on control law for flexible robots, including some experimental works by Schmitz (1985). Gomes and Chrétien (1992) showed that the real problem was associated with the nonlinear friction acting inside the motor gear actuator: the active control torque necessary to attenuate the vibration of a flexible link is usually smaller than the dry friction (torque dead zone) and there will be no motion at the actuator's output axis, causing the compensation mechanism not to work. Soares (1997) used the modal analytic approach to model a manipulator with one flexible link. This approach is very precise, as demonstrated in Pereira, (1999), but it presents difficulty to deal with measures of state variables Gervini et. al. (2001). This approach also imposes many difficulties to obtain kinematics

models of the manipulator Gervini et. al. (2001) and motivated the utilization of the discrete modeling approach in the present work.

## 2. EXPERIMENTAL SETUP

It was developed a robot manipulator with one flexible link to obtain experimental results. Fig. 1 shows a block diagram of the system.

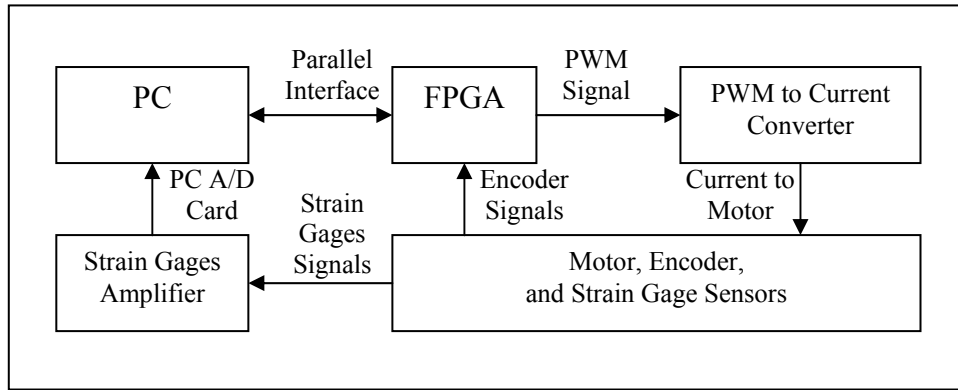


Figure 1. Block diagram of the system.

The manipulator is composed by an actuator harmonic-drive (motor with gear), an encoder fixed on the rotor side of the actuator and three strain gauge sensors, placed in the surface of the flexible link along its surface (Fig. 2).

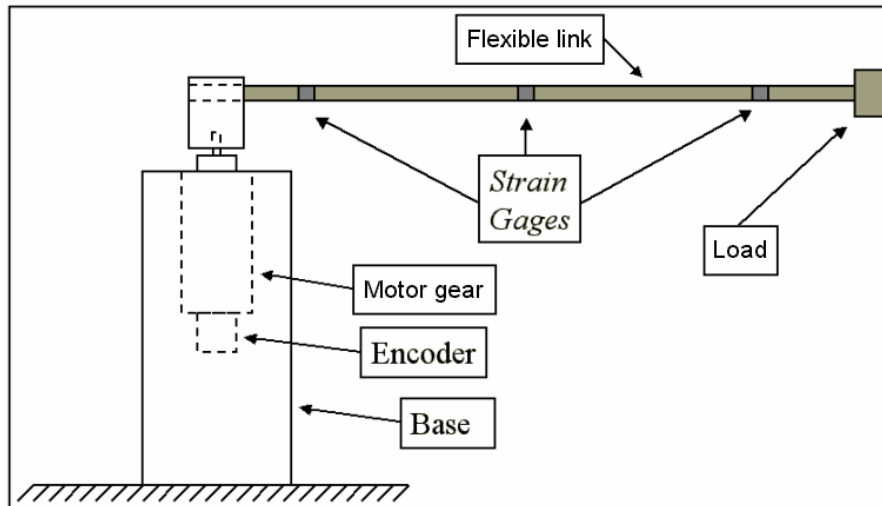


Figure 2. Manipulator scheme.

In Fig. 3 one can see a picture of the flexible link robot manipulator. The actuator employed was a PSA-08-100 model, made by Harmonic Drive Technologies. In Table 1 are showed some actuator parameters. It was implemented a current control since we have a direct linear relationship between current applied in the motor windings and the generated torque.

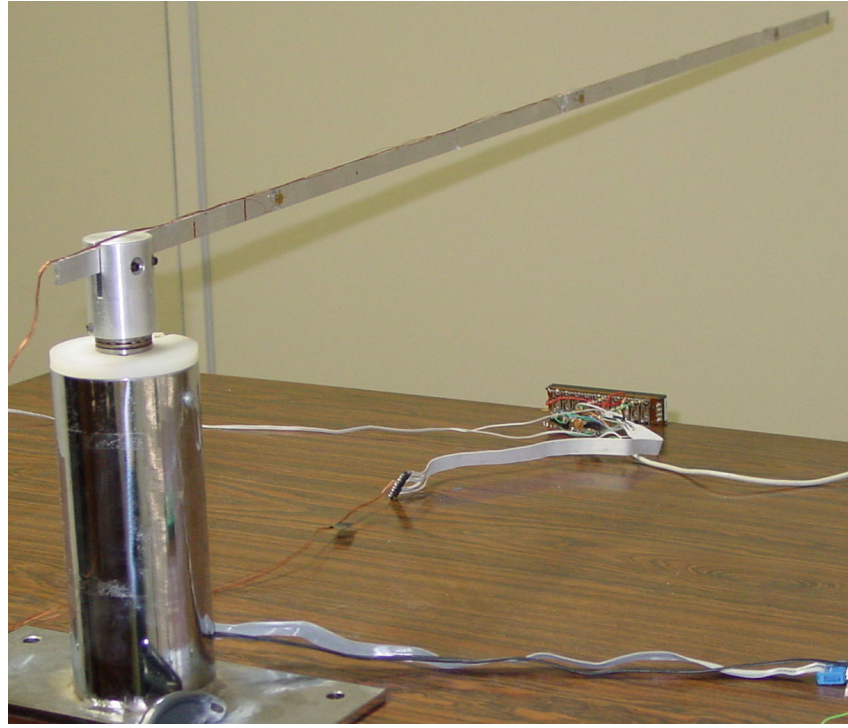


Figure 3. Flexible link manipulator robot.

Table 1. Relevant actuator parameters for this work.

Specification	Value	Unit
Gear ratio	1/100	$\text{Rev}_{\text{output}}/\text{Rev}_{\text{rotor}}$
Maximum torque	1.9	Nm
Nominal voltage	12	V
Nominal courant	1.22	A
Torque / Courant constant	1.76	Nm/A
Encoder resolution	500	Cycles/Rev (rotor)
	50,000	Cycles/Rev (output)



Figure 4. Harmonic-drive actuator employed in this work

### 3. DYNAMIC MODEL

In order to improve the precision of the global model (actuator and structural model), it is very important to consider the actuator dynamic model. This is particularly true to robot with flexible links, due to the nonlinear friction acting inside actuator with gear boxes. The global dynamic

modeling will be presentation of the in two distinct parts: one specific to the actuator and another to the structural model.

### 3.1. Actuator Dynamic Model

A drive joint actuator can be considered as an element of transfer motion with a certain elasticity, as can be seen in Fig. 5, where  $I_r$  is the rotor inertia,  $K$  is the internal elastic constant, and  $I_s$  is the load inertia.  $\tau_m$ ,  $\theta_r$  and  $\theta_s$  are the motor torque, rotor angular position, and load angular position, respectively.

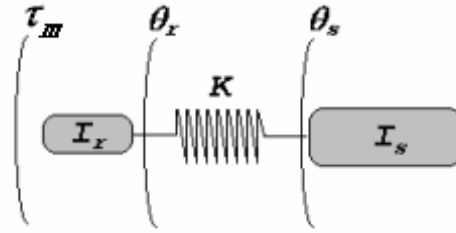


Figure 5. Drive joint with one flexible mode.

The differential equation that represents the drive joint dynamic can be obtained in the form

$$\begin{aligned} I_r \ddot{\theta}_r + \frac{K}{n^2} (\theta_r - n\theta_s) &= \tau_m - \tau_{at} \\ I_s \ddot{\theta}_s - K \left( \frac{\theta_r}{n} - \theta_s \right) &= 0 \end{aligned}, \quad (1)$$

where  $\tau_{at}$  corresponds to the nonlinear friction torque, considered here as entirely on the motor side and  $n$  is the gear ratio. When a rigid approximation is considered, the dynamic model has 1 DOF and  $\theta_r/n = \theta_s = \theta$ . Hence the system (1) reduces to

$$(n^2 I_r + I_s) \ddot{\theta} = n(\tau_m - \tau_{at}) \Rightarrow I \ddot{\theta} = T_m - T_{at}, \quad (2)$$

where  $I$ ,  $T_m$ , and  $T_{at}$  are the rotor inertia, motor torque and friction torque with physical units on the load side, respectively. The simulation results presented in this work were performed with the nonlinear friction model proposed by Gomes and Rosa (2003). This is a very realistic friction model and it reproduces well the stick-slip phenomenon.

### 3.2. Structural Dynamic Model

The modeling procedure presented here is named lumped mass approach, which represents the continuous flexibility of a structure by means of small rigid links united by fictitious joints (Gomes and Chrétien, 1992).

These joints are positioned in the structure as displayed in Figure 6. For the case of one fictitious joint, the structure is divided into two parts with same length (rigid elements) and the joint is placed between them; for the model with two fictitious joints, each joint is positioned in the middle of each rigid element in which the structure was divided in the previous case and for  $n$  fictitious joints, the joints are positioned in the middle of each rigid element of the case of  $n-1$  fictitious joints.

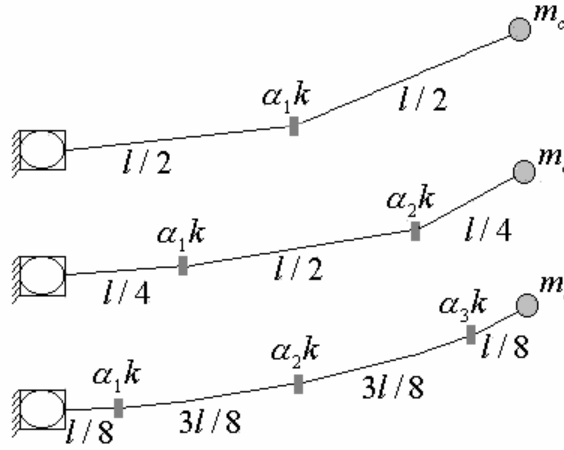


Figure 6. Flexible structures and its discrete approach for the cases of one, two and three fictitious joints.

After the positioning of the fictitious joints and supposing  $l$  as the length of the structure, the mass of each rigid element is known (concentrated in their respective mass centers). Also, the mass of the terminal load and the elastic constants of each of the fictitious joints are known.  $\theta_i$  is the angle between the direction  $x$  and the respective rigid element of number  $i$ . The kinetics and potential energies are obtained and consequently the Lagrangean of the system. Through the Euler-Lagrange equations and considering small angular deformations in the fictitious joints, the dynamic model obtained for the flexible structure is:

$$[I]\ddot{\theta} + [C_{at}]\dot{\theta} + [K_{el}]\theta = B'T_m, \quad (3)$$

where  $n$  is the number of fictitious joints,  $[I]_{n+1,n+1}$  the inertia matrix,  $[K_{el}]_{n+1,n+1}$  the matrix of elastic constant,  $[C_{at}]_{n+1,n+1}$  the matrix of viscous friction and  $B' = [1 \ 0 \ \dots \ 0]^T$ . The parameters  $\alpha_i$  are chosen in a way that the modes frequency of the model coincides with the frequency of the analytical flexible modes. Machado et. al. (2002) presents an algorithm to obtain the elements of the matrix of the equation (3).

Considering three fictitious joints, the state vector has eight variables:

$$\bar{x} = [\theta_r \ \theta_1 \ \theta_2 \ \theta_3 \ \dot{\theta}_r \ \dot{\theta}_1 \ \dot{\theta}_2 \ \dot{\theta}_3]$$

where  $\theta_r$  is the rotor angular position and  $\theta_1$ ,  $\theta_2$  and  $\theta_3$  are angular positions in the three fictitious joints, related to the inertial frame. The dynamic model may be written in the followed state form:

$$\dot{\bar{x}} = A\bar{x} + BT_m \quad (4)$$

The matrix  $A$  and  $B$  can be obtained easily in the form:

$$A = \begin{bmatrix} [0]_{4 \times 4} & [1]_{4 \times 4} \\ -[I]^{-1}[K_{el}]_{4 \times 4} & -[I]^{-1}[C_{at}]_{4 \times 4} \end{bmatrix}; \quad B = \begin{bmatrix} [0]_{4 \times 1} \\ [I]^{-1}B'_{4 \times 1} \end{bmatrix}.$$

Considering a linear approximation to the actuator (only viscous friction), the matrix  $A$  will have constant coefficients. Table 2 presents a pole and a zero configuration in this case, to the collocated transfer function  $\theta_r(s)/T_m(s)$ . However, for the simulation purpose, it was considered in this work

the nonlinear friction model proposed in Gomes and Rosa (2003). In this case,  $[C_{at}]$  matrix is a variable state dependent and then,  $A$  matrix is also a function of the state variables.

It is important to note that the adopted dynamic model uses state variables related to an inertial frame. As the strain gauges sensors provides the angular deviation in the fictitious joints, the state variables may be extracted from the strain gauges signals in the form:

$$\begin{aligned}\theta_r &\Rightarrow \text{encoder signal (rotor angular position);} \\ \theta_1 &= \theta_r + e_1; \\ \theta_2 &= \theta_1 + e_2; \\ \theta_3 &= \theta_2 + e_3;\end{aligned},$$

where  $e_1, e_2$  and  $e_3$  are the signals from the strain gauges 1, 2 and 3, respectively.

Table 2. Open loop results.

Poles	$0; -1.78; -2.55 \pm 23.6i; -0.87 \pm 78.724i; -4.352 \pm 249.99i$
Zeros	$-0.0057 \pm 12.1i; -0.658 \pm 76.19i; -4.169 \pm 247.89i$

#### 4. EXPERIMENTAL RESULTS AND SIMULATION

Many simulations and experimental results were obtained under the same conditions (all of them in open loop) with the objective to test the precision of the dynamic model. Fig. 8 shows a result obtained with the application of a torque pulse in open loop, with period of 0.05s and with amplitude of 50% of the maximum available torque. After this open loop excitation, the motor torque was set to zero. It can be verified that the global manipulator model (actuator and structural dynamics) reproduces the experimental result in which the two firsts vibration modes are visible. These modes have the frequencies equal to the imaginary parts of the zeros (see Table 2). Physically, the actuator is in stick mode and the arm has a behavior as it would be fixed in the actuator extremity and free in the load extremity (cantilever modes).

Fig. 9 shows a simulation and an experimental result, both in open loop, obtained as in Fig. 8, with a initial excitation with a constant torque for 0.5s. For this experiment the model showed to be realistic, reproducing the experimental result which shows the physical phenomenon explained in the previous case: at almost 0.5s after motor torque has fallen to null, rotor velocity falls to zero and the rotor stays at stick mode. The manipulator has a behavior as fixed in one extremity and free in the other one. The load extremity remains vibrating with frequencies equal to the frequency of the zeros (see Tab. 1).

#### 5. CONCLUSIONS

This work was dedicated to validate experimentally a dynamic modeling formalism to flexible structures, published in 2002. The actuator dynamic model (published in 2003) was also experimentally validated. One link flexible manipulator was built, with a reasonable instrumentation level to guarantee the comparison between experimental and simulation results. The main conclusions obtained from this work are:

- The developed structural model, based on discrete formalism, has a simple formulation and showed a predictive and a realistic behavior;
- The actuator dynamic model well reproduces the friction nonlinear behavior (stick-slip modes);
- This paper showed that, at stick mode, the actuator is locked and the flexible structure vibrates with the frequencies of the zeros of the collocated transfer function.

Finally, it was showed that the dynamic model must agree with the experimental results at poles and at zeros frequencies. Future research would use the manipulator model validated experimentally, including the actuator and structure dynamics to propose and to implement robust state feedback control laws.

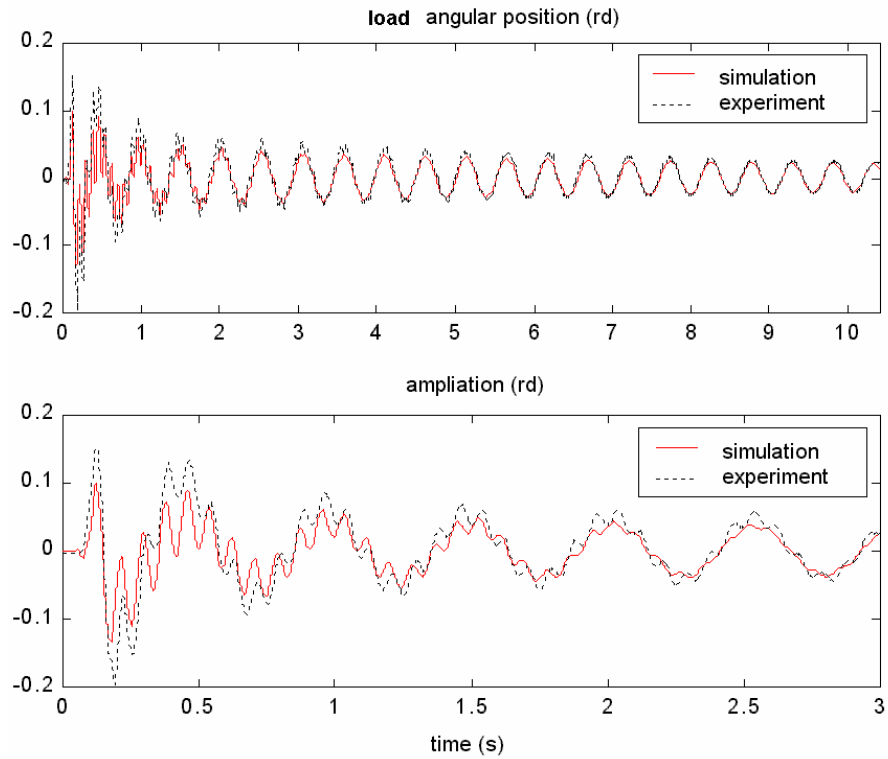


Figure 8. Simulation and experiment compare (open loop, 0.5 s torque pulse period).

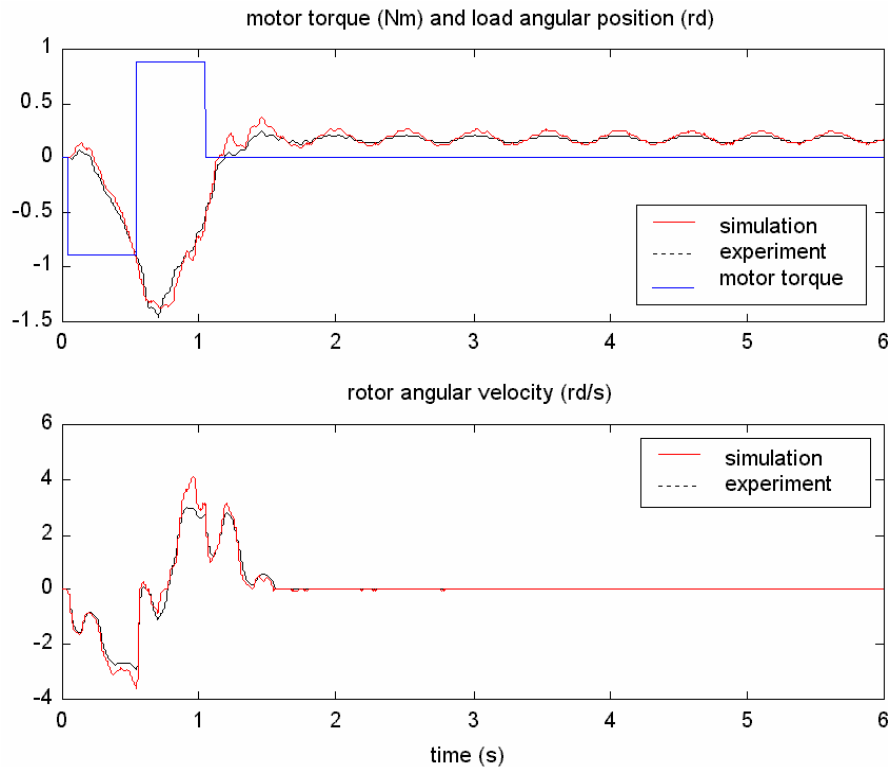


Figure 9. Simulation and experiment comparison (open loop, 0.5 s torque pulse period).

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