ACTIVE VIBRATION ATTENUATION IN TRUSS STRUCTURES USING LQR CONTROL AND STATE OBSERVER

Ricardo Carvalhal
Paulista State University – UNESP – Ilha Solteira
Av. Brasil, 56, CEP: 15385-000, Ilha Solteira, SP, Brazil.
ricardoc@dem.feis.unesp.br

Vicente Lopes Júnior
Paulista State University – UNESP – Ilha Solteira
Av. Brasil, 56, CEP: 15385-000, Ilha Solteira, SP, Brazil.
vicente@dem.feis.unesp.br

Abstract. *Many control strategies and techniques have been used in active vibrations control, as for instance, LQG control, H_2 and H_∞ methodologies. Usually, these methodologies involve distributed sensors and actuators and a control law to minimize a selected objective function. In special, vibrations control in truss structures has great practical interest. Light structures are, also, usually lightly damped, which can cause large amplitude vibration for any disturbances. Therefore, the main proposal of this paper is to use a modal LQR feedback control applied in light truss structures, in order to attenuate the two first vibration modes. Others advantages of this methodology can be perceived for reducing and simplifying the control systems with complex dynamics. The control force can be obtained by actives members, as PZT wafer stacks substituting bars, that accomplish an axial force. The piezoelectric effect of actuator is ignored in the proposed mathematical model. The analytical model is obtained using finite elements methods and classical modal analysis. The paper concludes with a numerical application.*

Keywords: *Active Vibrations Control, Intelligent Truss Structure, LQR Control, State Observer.*

1. INTRODUCTION

Vibrations control in truss structures has great practical interest, mainly in modern structures of huge space vehicles and aircrafts. Two demands essences are requested in designs of such structures. The first one is the excellent dynamic behavior, in order to guarantee the stability of the structure and high precision pointing. The second one is the necessity of obtaining light structures, in order to reduce the cost. However, these two requirements are contradictory, because light structures have low degrees of internal damping, which hinder the accuracy requirements, Yan et al. (2002).

These difficulties can be overcome by applying piezoelectric materials, Brennan et al. (1994). Several researchers have proved that piezoelectric material can be effective in active vibration control. In truss structures the control force can be accomplished by piezoelectric active members, known as “PZT wafer stacks”, that are mechanically linked in series producing an axial force in the bar that are positioned, Fig. (1).

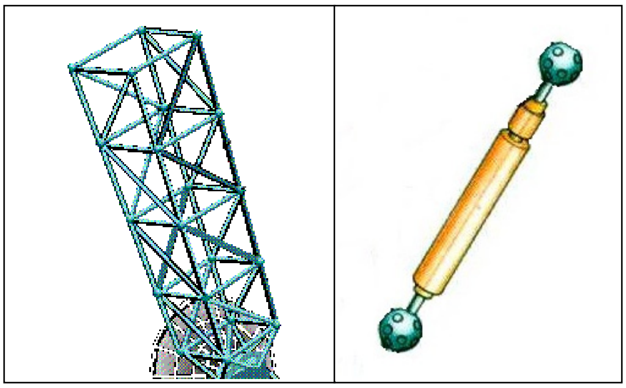


Figure 1. Truss structure example and Piezelectric actuator stacks in detail

Many strategies and approaches have been used to model and to design control flexible structures, as for instance: Shibuta et al. (1992) present the control of a truss structure using LQG/LTR, while Liu et al. (2000) use IMSC in a truss with 96 bars.

2. TRUSS STRUCTURE MODELING

A flexible structure in nodal coordinates is represented by the following second-order matrix differential equation:

$$\begin{aligned} M\ddot{q} + D\dot{q} + Kq &= B_0 u \\ y &= C_{oq} q + C_{ov} \dot{q} \end{aligned} \tag{1}$$

where q is the n x 1 displacement vector, u is the s x 1 input vector, y is the r x 1 output vector, M, D, and K are mass, damping and stiffness matrices, of order n, B_0 is the n x s input matrix , C_{oq} and C_{ov} are r x n output displacement and velocity matrices, respectively. Let n be the number of degrees of freedom of the system, r the number of outputs, and s the number of inputs.

For control system analysis and design purposes, it is convenient to represent the flexible structure equations in a state-space form:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx. \end{aligned} \tag{2}$$

By defining the state vector $x = \{q \quad \dot{q}\}^T$,in which the first term component is the system displacement, and the second component is the velocity. In this case, after elementary manipulations, one obtains the following state-space representation:

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ M^{-1}B_0 \end{bmatrix}, \quad C = \begin{bmatrix} C_{oq} & C_{ov} \end{bmatrix} \tag{3}$$

where A is a N x N, B is a N x s, e C is r x N. The dimension of the state model N is twice the number of degrees of freedom of the system n.

The above equation, in state-space representation, is not a modal state representation, Gawronsky, 1998. The modal state-space representation has a triple (A_m, B_m, C_m) characterized by the block-diagonal matrix, A_m, and the related input and output matrices:

$$A_m = blockdiag\begin{bmatrix} -\zeta_i\omega_i & \omega_i \\ -\omega_i & -\zeta_i\omega_i \end{bmatrix}, \quad B_m = \begin{bmatrix} B_{m1} \\ B_{m2} \\ \vdots \\ B_{mn} \end{bmatrix}, \quad C_m = \begin{bmatrix} C_{m1} & C_{m2} & \cdots & C_{mn} \end{bmatrix} \tag{4}$$

where i = 1, 2, ..., n, and A_{mi}, B_{mi}, and C_{mi} are blocks 2 x 2, 2 x s, e r x 2, respectively.

The state x of the modal representation consist of n independent components, x_i, that represent a state of each mode. The ith state component is $x_i = \{q_{mi} \quad q_{moi}\}^T$, where q_{mi} and $\dot{q}_{mi}$ are the ith modal displacement and velocity, and $q_{moi} = \zeta_i q_{mi} + \dot{q}_{mi}/\omega_i$.

Then, the mathematical model of a flexible structure in the modal state representation, can be given by:

$$\begin{aligned} \dot{x} &= A_m x + B_m u \\ y &= C_m x \end{aligned} \tag{5}$$

It's proposed to control only a limited number of lower modes. Hence, the modal state vector x can be particioned as $x = \{x_c \quad x_r\}^T$, where x_c is the controlled modal state vector e x_r is the uncontrolled or residual modal state vector.

So, the state triple (A_m, B_m, C_m) can be partitioned accordingly:

$$A_m = \begin{bmatrix} A_c & 0 \\ 0 & A_r \end{bmatrix}, \qquad B_m = \begin{bmatrix} B_c \\ B_r \end{bmatrix}, \qquad C_m = \begin{bmatrix} C_c & C_r \end{bmatrix} \tag{6}$$

in which the notation is obvious.

3. CONTROL LAW DESIGN

Feedback control laws have various applications in many dynamics systems. The key idea of a feedback control law is to utilize the measurement of the current state of a system, and to use the measured signal to construct an actuator signal.

In order to design the state feedback optimal controller, an objective function is a quadratic functional of the plant states and control inputs. With a linear system, the quadratic objective function can be expressed as:

$$J = \frac{1}{2}\int_0^\infty \left(x_c^T Q x_c + u_c^T R u_c\right) dt \tag{7}$$

where Q is a symmetric semi-definite matrix with regard to the state vector and R is a symmetric positive-definite matrix with respect to the control input. The upper limit of the performance index is ∞, which implies that we are interested in the steady-state behavior of the system. The minimization of J with respect to the control input u_c is known as the linear quadratic regulator (LQR). The control law to minimize is given as:

$$u_c = -G_c x_c \tag{8}$$

where $G_c = R^{-1} B_c^T P$, is a steady state gain matrix for an asymptotically stable closed-loop system, and P is the positive-definite solution of the algebraic Riccati equation:

$$PA_c + A_c^T P + Q - PB_c R^{-1} B_c^{-1} P = 0 \tag{9}$$

This control law is called full state feedback since it uses all state variables, x_c.

4. CONTROL DESIGN USING STATE OBSERVERS

Quite often, it is not practical to measure the full state vector, but only certain combinations of the state. The relation between the measurement vector and the state vector can be given by Eq. (5). The assumption is made here that the system is observable, which implies that the initial state can be deduced from the outputs within a finite time period.

A modal observer for the state triple (A_c, B_c, C_c) is assumed to have the form:

$$\dot{\hat{x}}_c = (A_c - B_c G_c)\hat{x}_c + K_c C_c [x_c - \hat{x}_c] \tag{10}$$

where $\hat{x}_c$ is the estimated controlled modal state and K_c is the observer gain matrix. The error vector is introduced as $e_c = \hat{x}_c - x_c$. Because the dimension of the state observer vector is the same as the dimension of the state vector x of the $\hat{x}_c$ original system, an observer of this type is said to be a full-order observer, Meirovitch, 1990.

In implementing the feedback control we must use the estimated state, since the actual state x_c is not available. Hence, Eq. (8) must be replaced by:

$$u_c = -G_c \hat{x}_c \tag{11}$$

The controlled modal state, residual modal state and error vectors can be written in the matrix form:

$$\begin{bmatrix} \dot{x}_c \\ \dot{x}_r \\ \dot{e}_c \end{bmatrix} = \begin{bmatrix} A_c - B_c G_c & 0 & -B_c G_c \\ -B_r G_c & A_r & -B_r G_c \\ 0 & 0 & A_c - K_c C_c \end{bmatrix} \begin{bmatrix} x_c \\ x_r \\ e_c \end{bmatrix} \tag{12}$$

The term ($-B_r G_c$) is responsible for the excitation of the residuals modes by the control forces and it is known as *control spillover*.

5. NUMERICAL APPLICATION

In the numerical application is proposed the control design of a truss plane structure with 16 bars, Fig. (2). At the principle, each truss member can be replaced by an active member. The accelerometer is collocated at node 6, in the y direction. The nodes 1 and 10 are clamped. The materials properties and dimensions are given in Tab. (1). The tubes are made of steel with diameter of 11.3mm. It is considered that the damping is proportional to the stiffness matrix (D=10^{-6}.K). The aim of these numerical investigations is to demonstrate the applicability and efficiency of the theory shown in the previous sections. The state-space structural model is obtained through a program developed in Matlab® using FEM.

Table 1. Material properties and dimensions of the truss structure

Properties	Values
Young's Modulus (N/m^2)	2.0 x 10^{11}
Mass Density (kg/m^3)	7850
Cross Section Area (m^2)	0.0001
L (m)	0.15

Each node has two degrees of freedom, translation in x and y direction, so, the truss structure has 16 active dofs, and the model in the form of states space results in order of 32. The control design to attenuate the vibration considers only the first two modes (4 states) of the structure, and the fourteen remaining modes (28 states) are considered as residual modes in the model. The actuator positions are shown in Fig. (2).

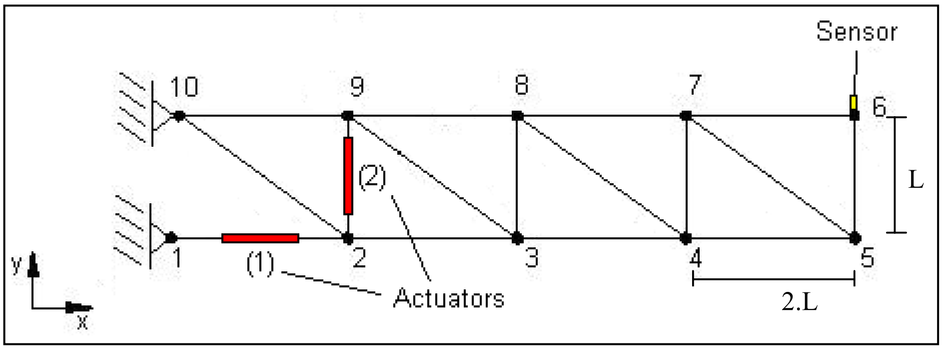
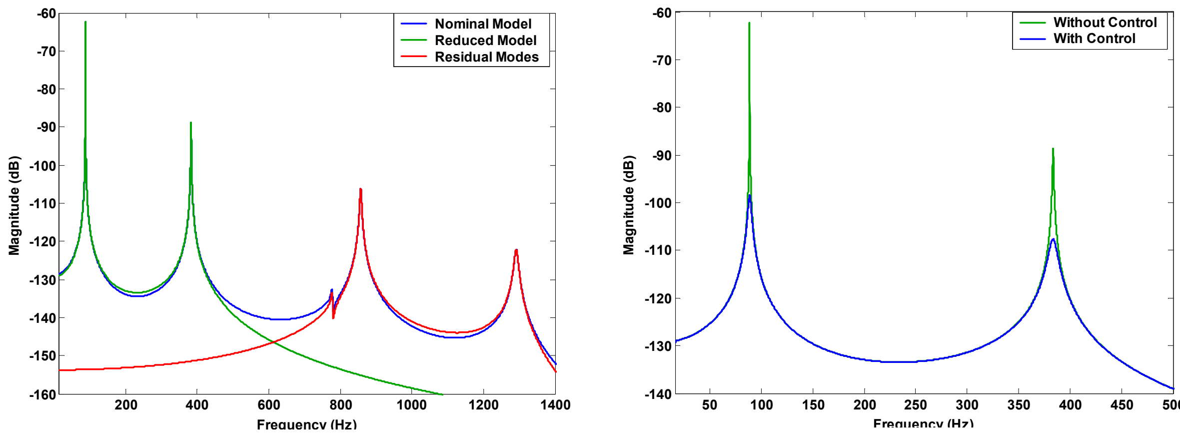


Figure 2. Truss structure showing the actuator and sensor location

In the following example, it is assumed that the piezelectric actuator effect is negligible, so, the dynamic properties of the truss structure do not change considerably. Consequently, the eigenvector only have to be calculated once. Figure (3a) shows the transfer functions for the nominal system, the reduced model, and the residual modes. Figure (3b) shows the transfer function of the first two modes for the closed and open-loop system.



(a) (b)

Figure 3. (a) Magnitudes of the transfer function of the nominal model, reduced and residuals modes; (b) Magnitudes of the closed and open-loop transfer functions

The reduced model (A_c, B_c, C_c) was given by the following system:

$$A_c = 10^2 \begin{bmatrix} -0.002 & 5.560 & 0 & 0 \\ -5.560 & -0.002 & 0 & 0 \\ 0 & 0 & -0.029 & 24.094 \\ 0 & 0 & -24.094 & -0.029 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0.0451 & 0.0037 \\ -0.0000 & -0.0000 \\ -0.0995 & -0.0261 \\ 0.0001 & 0.0000 \end{bmatrix}, \quad C_c = \begin{bmatrix} -0.0000 \\ -0.0018 \\ -0.0000 \\ -0.0004 \end{bmatrix}^T$$

The following procedure for computing the optimal gain matrix G_c of the ideal controler was done. First, it was calculated P by Eq. (9), the weighting factor of respectively the actuator energy (Q) and the output (R) in the cost function to be minimized are unknown, so it was chosen:

$$Q = 10^3 \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix} \qquad R = 10^{-3} \begin{bmatrix} 54 & 0 \\ 0 & 9 \end{bmatrix}$$

And the optimal gain matrix G_c was calculated by $G_c = R^{-1} B_c^T P$. The result was:

$$G_c = \begin{bmatrix} 415.3 & -2.2 & -322.9 & 4.9 \\ 204.6 & 0.5 & -509.1 & 4.5 \end{bmatrix}$$

The determination of the observer gain matrix K_c is a dual problem to the LQR, so some correspondences were made to use Eq. (9), it is shown in Tab. (2).

Table 2. Duality between LQR and State Observer

LQR	A_c	B_c	Q	R
State Observer	A_c^T	C_c^T	W	V

To estimate the observer gain matrix, it was chosen:

$$W = 10^3 \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix} \qquad V = 1$$

The observer gain matrix K_c was calculated by $K_c = \mathrm{P} C_c^T V^{-1}$. The result was:

$$K_c = 10^{-1} \begin{bmatrix} -0.069 & -2.678 & -0.004 & -3.026 \end{bmatrix}$$

The attenuation of flexible motion of the structure depends on the pole mobility to the left-hand side of the complex plane. Therefore, if a particular pair of poles is moved easily, the respective states are easy to control and estimate. On the other hand, if a particular pair of poles is difficult to move, the respective states are difficult to control and estimate. Figure (4) shows the pole location for the nominal system, controlled, and residual modes.

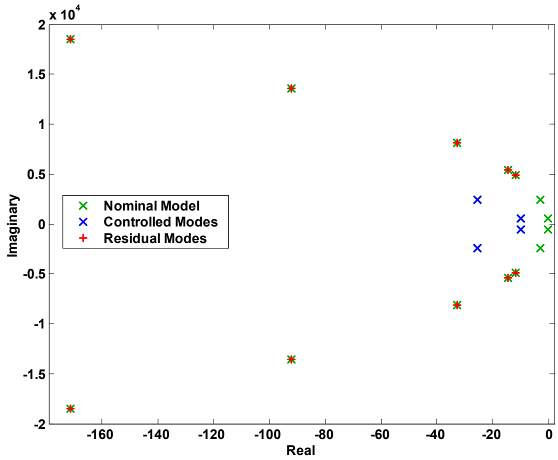


Figure 4. Poles of the nominal model, controlled and residuals modes

The closed loop natural frequencies and damping ratio for controlling the first two modes are shown in Tab. (3) which are compared with open loops ones. When the truss structure is controlled, the damping ratio of the first controlled mode is improved from 0.028% to 1.78%, and the damping ratio of the second controlled mode increases from 0.120% to 1.06%. The damping ratio of other uncontrolled modes is nearly the same as those when the control system is open loop.

Table 3. Control results

Mode number		1	2	3	4
Open loop	Freq. (Hz)	88.5	383.5	778.1	857.0
	Damping ratio (%)	0.028	0.120	0.244	0.269
	Poles ($.10^3$)	-0.0 ±0.556i	-0.003 ±2.409i	-0.012 ±4.889i	-0.014 ±5.385i
Closed loop	Freq. (Hz)	88.5	383.5	778.1	857.0
	Damping ratio (%)	1.78	1.06	0.244	0.269
	Poles ($.10^3$)	-0.01 ±0.556i	-0.026 ±2.409i	-0.012 ±4.889i	-0.014 ±5.385i

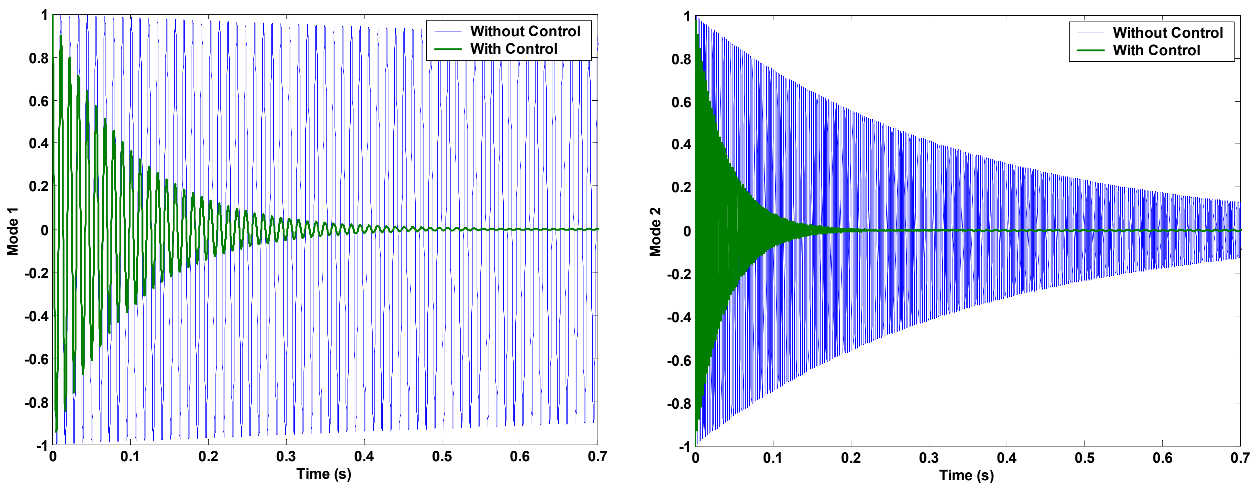


Figure 5. First and second controlled modes

The response of the controlled modes in the time domain is shown in Fig. (5). While, the response, in time domain, of the residuals modes are shown in Fig. (6). It is clear that the largest contribution to the response of the truss comes from the lowest modes.

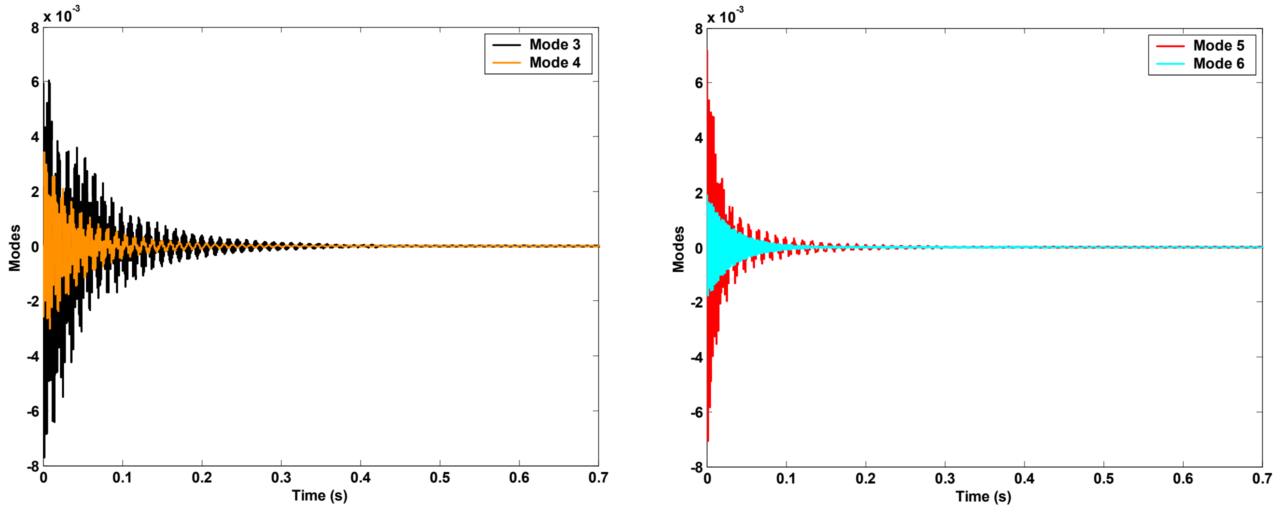


Figure 6. First four residuals modes

Control spillover does exist, as can be seen from Fig. (6), but it is reasonably small and tends to disappear as the controlled modes decay. In this regard, note that Fig. (5) and (6) use different scales for the modal amplitudes.

Figure (7) shows the modal force performed by actuator 1 and 2. The results for both, open and closed-loop, system was obtained assuming the initial condition:

$$x(0) = [\ 1 \quad 0 \quad -1 \quad 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ ]^T.$$

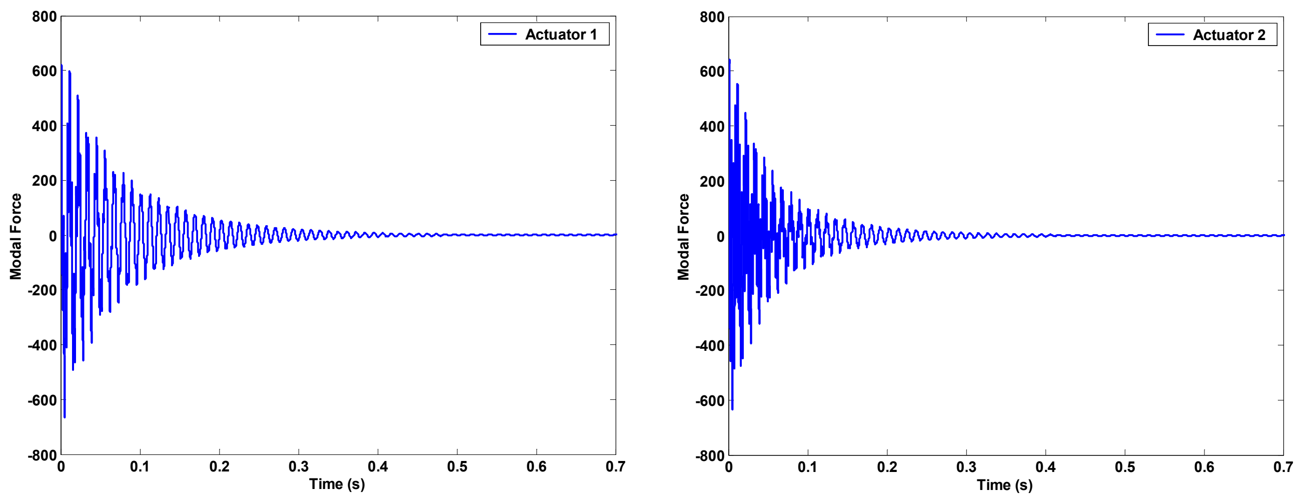


Figure 7. The actuator modal forces

When the system is controlled, the open loop value of the first mode attenuation reaches 37 dB. While the second mode an attenuation of 16 dB is achieved. Figure (5) and (6) illustrate that the transient reaction is attenuated quickly (approximately 0.6 s).

6. CONCLUSION

When designing adaptive structures, the location of actuators must be chosen properly, because they influence the efficiency of the control forces and the stability of the whole system.

An LQR feedback control strategy and a state observer were used actively to control the first two modes of a truss structure. The applicability of this theory was shown through an example in a plane structure with 16 bars. Active suppression of a truss plane structure with an applied disturbance force was reached by using adaptive control approach in modal space. The control force, in this example, excited the residual modes, and it caused spillover. However, the amplitude was small and tends to disappear.

7. ACKNOWLEDGEMENTS

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