FLOW AND HEAT TRANSFER ANALYSIS IN A CAVITY PARTIALLY FILLED WITH HEAT GENERATING POROUS MEDIA

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Abstract. *In this work the flow and heat transfer characteristics of a cavity partially filled with a heat generating porous medium are analyzed. The geometry considered is a porous block placed at the cavity's center with a uniform volumetric heat source in the porous matrix. The streamlines and isotherms are obtained for several combinations of porosity, permeability and volume ratio of the porous medium. The cavities are insulated at the top and bottom and cooled at the sides isothermally. The Nusselt numbers are obtained on the heat exchanging walls. The local thermal equilibrium hypothesis is utilized although it may not be valid in all the cases considered.*

Keywords: *Porous Medium, Heat Transfer, Numerical Methods.*

1. INTRODUCTION

A great many problems in engineering and science are concerned with the solution of flow and heat transport equations in porous media as, e.g., heat exchangers, combustion in porous matrices, grain storage, to mention just a few.

The problem of storage in silos, where the grains fill up the whole storage volume available, has been studied in detail by several researchers as Jimenez-Islas et al. (1999). Nevertheless, when these grains are stored in a large storage building, packed in bags and laid on platforms, the problem of a partially filled cavity arises. The geometry analyzed in this article is related to this later case. Heat generation is assumed to occur in the porous medium and a uniform side-wall temperature boundary condition is considered. The top and bottom walls are considered to be insulated. The flow is assumed to be laminar and a set of parameters such as porosity, permeability and volume ratio of the porous block, are varied to assess their influence on the average Nusselt number. Figure 1 shows the geometry considered with the porous block placed at the cavity's center.

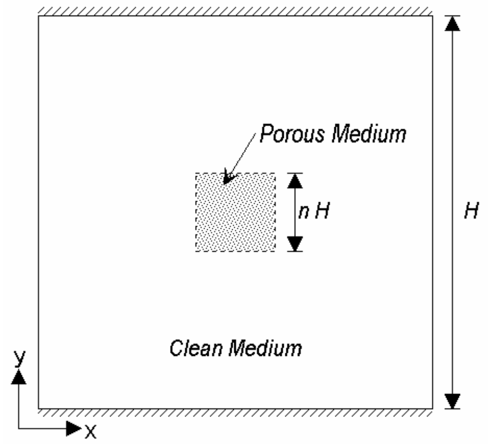


Figure 1- Cavity Geometry

2. MATHEMATICAL MODEL

The general equations for an incompressible laminar flow in a rigid, homogeneous and saturated porous matrix are given as (Pedras e de Lemos (1999a), Rocamora and de Lemos (2000)):

Continuity equation -

$$\nabla \cdot \mathbf{u}_D = 0 \tag{1}$$

Momentum equation -

$$r_f \left[\frac{\partial \mathbf{u}_D}{\partial t} + \nabla \cdot \left(\frac{\mathbf{u}_D \mathbf{u}_D}{f} \right) \right] = -\nabla \left(f \langle p \rangle^i \right) + m \nabla^2 \mathbf{u}_D + f r_m \mathbf{g} - \left[\frac{m f}{K} \mathbf{u}_D + \frac{c_F f r_f |\mathbf{u}_D| \mathbf{u}_D}{\sqrt{K}} \right] \tag{2}$$

Energy equation -

$$\left\{ \left(r c_p \right)_f f + \left(r c_p \right)_s (1-f) \right\} \frac{\partial \langle T \rangle^i}{\partial t} + \left(r c_p \right)_f \nabla \cdot \left(\mathbf{u}_D \langle T \rangle^i \right) = \nabla \cdot \left\{ \underline{\underline{K}}_{eff} \cdot \nabla \langle T \rangle^i \right\} + S \tag{3}$$

where the effective conductivity tensor, $\underline{\underline{K}}_{eff}$, is given by:

$$\underline{\underline{K}}_{eff} = \left[f k_f + (1-f) k_s \right] \underline{\underline{I}} + \underline{\underline{K}}_{tor} + \underline{\underline{K}}_{disp} \tag{4}$$

Note that in Equations (1) to (3), making $f \rightarrow 1$ and $K \rightarrow \infty$ the equations for a clean medium are recovered, allowing the use of the same set of equations to treat both the porous medium and the clean medium with the following interface conditions:

$$\mathbf{u}_D \big|_{f \neq 1} = \mathbf{u}_D \big|_{f=1} \tag{5}$$

$$\langle p \rangle^i \big|_{f \neq 1} = \langle p \rangle^i \big|_{f=1} \tag{6}$$

$$f^{-1} \frac{\partial u_{D//}}{\partial n} \bigg|_{f \neq 1} = \frac{\partial u_{D//}}{\partial n} \bigg|_{f=1} \tag{7}$$

$$\langle T \rangle^i \big|_{f \neq 1} = \langle T \rangle^i \big|_{f=1} \tag{8}$$

$$\mathbf{n}\cdot\left(\underline{\underline{K}}_{eff}\cdot\nabla\langle T\rangle^{i}\right)\Big|_{f\neq 1}=k_f\,\frac{\partial\langle T\rangle^{i}}{\partial n}\bigg|_{f=1} \tag{9}$$

Also, in the energy equation, Eq. (3), the thermal equilibrium hypothesis was used such that $\langle\overline{T}\rangle^{i}_{f,s}=\langle\overline{T}\rangle^{v}$, and the tortuosity and dispersion conductivity tensors were neglected.

For the buoyancy term Eq. (2), The Boussinesq approximation is used, so that:

$$r_m = r_f\left(1-b\langle T\rangle^{i}\right) \tag{10}$$

where b is the volumetric expansion coefficient for the fluid.

3. NUMERICAL METHOD

The set of equations described above are solved using the finite volume method for the spatial discretization together with the SIMPLE method of Patankar (1980) for the pressure-velocity coupling. The resulting algebraic equation system is solved using the SIP algorithm. In all the cases analyzed, a uniform (50x50) grid was used and the results were considered converged when the algebraic equations' residues were $\leq 10^{-5}$ for the steady-state situation.

4. RESULTS

Five sets of results were obtained, one for each parameter variation. The parameters considered were porosity, f, permeability, K, and volume ratio, VR, of the porous medium inside the cavity. The cases analyzed are summarized in Table 1 below. In all the cases a uniform volumetric heat source inside the porous medium was used (S =10^3 W/m^3), and the heat exchanging walls are considered to be at a uniform temperature (T_w =35 ^{0}C). The average Nusselt number was obtained considering the average temperature inside the cavity as:

$$T_{avg}=\frac{\int_{V_c} T\left|\mathbf{u}_D\right| dV}{\int_{V_c}\left|\mathbf{u}_D\right| dV} \tag{11}$$

so that the local heat transfer coefficient, h_{local}, is obtained as:

$$h_{local}=\frac{q_w^{''}}{\left(T_{avg}-T_w\right)} \tag{12}$$

where $q_w^{''}$ is the local heat flux and T_w is the wall temperature. The average heat transfer coefficient is then calculated as:

$$h_{avg}=\frac{1}{H}\int_0^H h_{local}\Big|_{x=0}\,dy \tag{13}$$

Finally, the average Nusselt number is obtained as:

$$Nu_{avg}=\frac{2\,h_{avg}\,H}{k_f} \tag{14}$$

The fluid properties are shown in Table 2. Cases 6-10 are taken as reference for comparison with all other cases. At this point it should be mentioned that obtaining non-dimensional numbers such as the Raylaigh and Darcy's numbers for this kind of cavity is a complex task since there are no standard definitions of properties for hybrid media, *i.e.*, media composed by porous and clean regions.

Table 1 - Summary of the cases considered.

Case	K (m^2)	f	$VR = n^2$	Nu_{avg}
1	10^{-5}	0.8	0.04	38.2
2	10^{-5}	0.8	0.16	53.3
3	10-^{5}	0.8	0.36	68.2
4	10^{-5}	0.8	0.64	86.1
5	10^{-5}	0.8	1.00	62.2
6	10^{-4}	0.8	0.04	38.0
7	10^{-4}	0.8	0.16	58.0
8	10^{-4}	0.8	0.36	80.9
9	10^{-4}	0.8	0.64	100.6
10	10^{-4}	0.8	1.00	110.0
11	10^{-3}	0.8	0.04	39.2
12	10^{-3}	0.8	0.16	61.9
13	10^{-3}	0.8	0.36	84.3
14	10^{-3}	0.8	0.64	104.6
15	10^{-3}	0.8	1.00	147.0
16	10^{-4}	0.7	0.04	38.3
17	10^{-4}	0.7	0.16	58.5
18	10^{-4}	0.7	0.36	81.2
19	10^{-4}	0.7	0.64	100.5
20	10^{-4}	0.7	1.00	121.6
21	10^{-4}	0.9	0.04	37.7
22	10^{-4}	0.9	0.16	57.4
23	10^{-4}	0.9	0.36	80.5
24	10^{-4}	0.9	0.64	100.6
25	10^{-4}	0.9	1.00	95.3

Table 2 - Fluid properties

r (Kg/m3)	1.0
m (N·s/m2)	10^{-3}
Pr	1.0
c_p (J/Kg·K)	10^{3}
b (1/K)	0.1

Figure 2 shows the behavior of the average Nusselt number versus the volume ratio for the set of porosities and permeabilities considered. This figure summarizes the whole set of results obtained and allows us to draw some conclusions. Firstly, one can observe that the porosity variation has little effect on the average Nusselt number for the range analyzed, except when VR approaches 1, *i.e.*, when the cavity is totally filled by the porous medium. One can also notice that the permeability has a more pronounced effect, specially as VR increases. The higher the permeability, the higher the average Nusselt number. For very small VR's, all the cases behave essentially the same way, *i.e.*, the porous medium has very little effect on the average Nusselt number due to the huge clean region available for the flow. Figure 3 shows the streamlines for three cases where the permeability is varied. It can be noticed that the flow pattern changes considerably from the lower permeability to the higher.

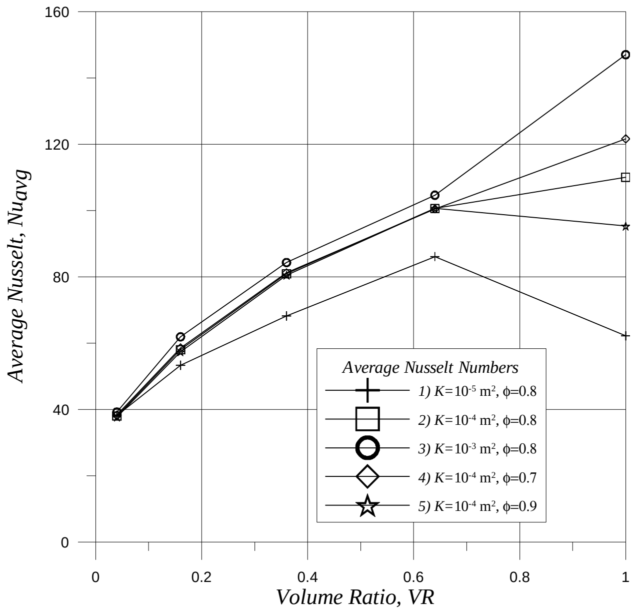
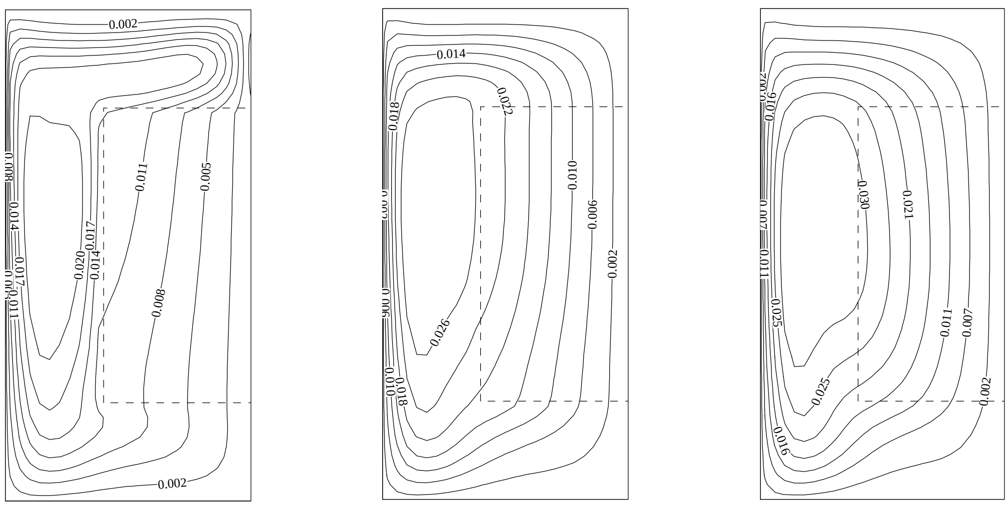


Figure 2 - Average Nusselt number versus Volume Ratio.



a) *b*) *c*)

Figure 3 - Streamlines for cases *a*) 3, *b*) 8 and *c*) 13.

As the porous medium becomes more permeable, the circulation increases, thus increasing the average Nusselt number.

It is also important to mention that in all the cases analyzed the Darcy, Forchheimer and Brinkman terms are taken into account so that the no-slip condition for the flow at the walls is always satisfied when the cavity is completely filled with the porous medium (*VR* =1).

5. CONCLUSIONS

In this work the flow and heat transfer characteristics for a cavity partially filled with a porous block was studied. Relevant parameters that influence them were varied and their effect on the average Nusselt number was assessed. It was found that the permeability of the porous matrix is the most important parameter for this kind of flow. Also, the lack of non-dimensional parameters suitable to the characterization of hybrid media, *i.e.*, media composed by porous and clean regions, was identified and shall be the object of further investigation.

6. ACKNOWLEDGEMENTS

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