INVESTIGATION OF LAMINAR NATURAL CONVECTION IN INCLINED CAVITIES FILLED WITH POROUS MATERIAL

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Abstract. *Detailed numerical computations for 2-D steady-state laminar natural convection within inclined cavities totally filled with saturated porous medium are numerically analyzed using the finite volume method in a generalized coordinate system. The inclined walls are maintained at constant but different temperatures, while the horizontal walls are kept insulated. Governing equations are written in terms of primitive variables and are recast into a general form. Flow and heat transfer characteristics, (streamlines, isotherms and average Nusselt number), are investigated for a wide range of values of Rayleigh numbers and inclination angles. The present solutions are compared with the known results from the open literature. It was found that these results are in satisfactory agreement. Analyses of important environmental and engineering flows can benefit from the derivations herein and, ultimately, it is expected that additional research on this new subject be stimulated by the work here presented.*

Keywords*: Porous Media, Natural Convection, Numerical Simulation.*

1. INTRODUCTION

The investigation of buoyancy driven flows through porous media has several applications in many fields of science, technology and environment. Heat exchangers, grain storage, optimal design of furnaces and solar collectors, crystal growth in liquids, packed-bed catalytic reactors, nuclear reactor safety, food processing and underground spread of pollutants are just some applications of this subject.

Traditionally, modeling of macroscopic transport for incompressible flows in porous media has been based on the volume-average methodology for either heat (Hsu & Cheng (1990)) or mass transfer (Bear (1972),Whitaker (1966),Whitaker (1967)). If time fluctuations of the flow properties are also considered, in addition to spatial deviations, there are two possible methodologies to follow in order to obtain macroscopic equations: *a*) application of time-average operator followed by volume-averaging (Masuoka & Takatsu (1996), Kuwahara & Nakayama (1998), Nakayama & Kuwahara (1999)), or *b*) use of volume-averaging before time-averaging is applied (Lee & Howell (1987), Antohe & Lage (1997), Getachewa et al (2000)). However, both sets of macroscopic mass transport equations are equivalent when examined under the recently established *double decomposition* concept (Pedras & de Lemos (2000), Pedras & de Lemos (2001a), Pedras & de Lemos (2001b), Pedras & de Lemos (2001c)). This methodology, initially developed for the flow variables, has been extended to heat transfer in porous media where both time fluctuations and spatial deviations were considered for velocity and temperature (Rocamora & de Lemos (2000)) and (De Lemos & Rocamora (2002)). A general classification of all proposed models for turbulent flow and heat transfer in porous media has been recently published (De Lemos & Pedras (2001)). Extension of the double-decomposition theory for treating turbulent natural convection, (De Lemos & Braga (2003)), and mass transfer, (De Lemos & Mesquita (2003)), in saturated rigid porous media has also been recently documented.

The thermal convection in porous media has been studied extensively in recent years. Underground spread of pollutants, grain storage, food processing, geothermal systems, oil extraction, store of nuclear waste material and solar power collectors are just some applications of this theme. The monographs of Nield & Bejan (1992) and Ingham & Pop (1998) fully document natural convection in porous media.

The case of free convection in a rectangular cavity heated on a side and cooled at the opposing side is an important problem in thermal convection in porous media. Walker & Homsy (1978), Bejan (1979), Prasad & Kulacki (1984), Beckermann et al. (1986), Gross et al (1986) and Manole & Lage (1992) have contributed with some important results to this problem.

The recent work of Baytas & Pop (1999), concerned a numerical study of the steady free convection flow in rectangular and oblique cavities filled with homogeneous porous media using a nonlinear axis transformation. The Darcy momentum and energy equations are solved numerically using the (ADI) method.

Motivated by the foregoing work, this paper presents results for laminar natural convection in oblique cavities totally filled with a porous material, heated from the left and cooled from the opposing side. The other two walls are kept insulated.

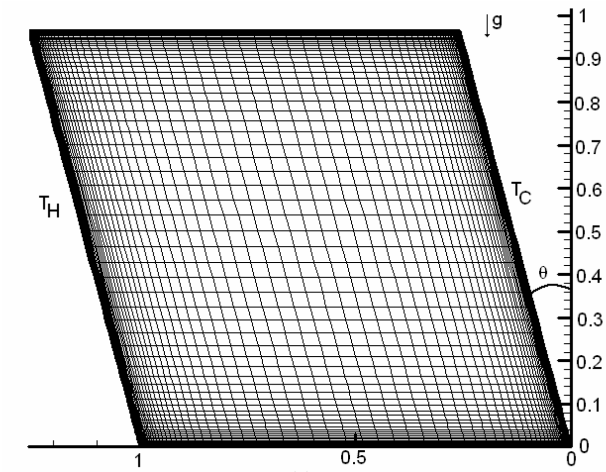

Figure 1 – Geometry and grid under consideration

2. GEOMETRY AND BOUNDARY CONDITIONS

The problem considered is showed schematically in Fig. 1 and refers to an oblique cavity with width L=1 m completely filled with porous medium. The cavity is isothermally heated from the left, T_H, and cooled from the opposing side, T_C. The other two walls are insulated. The porous medium is considered to be rigid and satured by an incompressible fluid. The Ra_m is the dimensionless parameter used for porous media and it is defined as, $Ra_m = Ra_f Da$, with $a_{eff} = k_{eff} / (rc_p)_f$ and the particle diameter is given by $D_p = \sqrt{\dfrac{144K(1-f)^2}{f^3}}$.

3. GOVERNING EQUATIONS

The equations used herein are fully developed in the work of Pedras & de Lemos (2001a), De Lemos & Rocamora (2002) and De Lemos & Braga (2003).

Thus, for steady-state laminar natural convection, the macroscopic equations for continuity, momentum and temperature take the form:

$$\nabla \mathbf{u}_D = 0 \tag{1}$$

$$r\left[\nabla\left(\frac{\mathbf{u}_D\mathbf{u}_D}{f}\right)\right] = -\nabla\left(f\langle p\rangle^i\right) + m\nabla^2\mathbf{u}_D - \left[\frac{mf}{K}\mathbf{u}_D + \frac{c_F f r|\mathbf{u}_D|\mathbf{u}_D}{\sqrt{K}}\right] - r b_f \mathbf{g} f\left(\langle T\rangle^i - T_{ref}\right) \tag{2}$$

$$\left(r c_p\right)_f \nabla\left(\mathbf{u}_D\langle T\rangle^i\right) = \nabla\left\{\left[k_f f + k_s(1-f)\right]\nabla\langle T\rangle^i\right\} \tag{3}$$

where $\mathbf{u}_D$ is the Darcy velocity, r is the density of the fluid, p is the total pressure and m is the dynamic viscosity. The gravity acceleration vector is defined by $\mathbf{g}$ and b_f is the macroscopic thermal expansion coefficient. $\langle T\rangle^i$ and T_{ref} are the macroscopic and the reference temperatures respectively. The thermal conductivity for the fluid and solid are labeled k_f and k_s respectively. Finally, c_p is the specific heat and f is the porosity, K is the permeability and c_F is the Forchheimer coefficient.

3. NUMERICAL METHOD AND COMPUTATIONAL DETAILS

The numerical method employed for discretizing the governing equations is the control-volume approach with a generalized grid. A hybrid scheme, Upwind Differencing Scheme (UDS) and Central Differencing Scheme (CDS), is used for interpolating the convective fluxes. The well-established SIMPLE algorithm (Patankar & Spalding (1972)) is followed for handling the pressure-velocity coupling. Individual algebraic equation sets were solved by the SIP procedure of Stone (1968).

The present results were performed with ϕ=0.8 and Da=10^{-7}. The Prandtl number and the conductivity ratio between the solid and fluid phases are assumed to be a unit. Runs for laminar flow were performed with an 80x80 control volumes in a stretched grid like shown in Fig. 1. Although not shown here, several runs were performed with others meshes in order to guarantee grid independence. These runs showed that the 80x80 CV´s stretched mesh is refined enough to capture the thin boundary layers that appear along the vertical surfaces giving a percentual error in relation with other with 110x110 CV´s stretched mesh less than 1%. The convergence process is terminated when the residual error is equal to 10^{-5}.

4. RESULTS AND DISCUSSION

When other parameters, e.g., (Porosity, Prandtl number, conductivity ratio between the fluid and solid matrix) are fixed the available literature shows that for the non-Darcy region, (Merrikh & Mohamad (2002), Braga & de Lemos (2004)), fluid flow and heat transfer depend on the fluid Rayleigh number, Ra_f, and the Darcy number, Da

In the work of Braga & de Lemos (2004) it was observed that for a fixed Ra_m the lower the permeability, the higher the average Nusselt number at the hot wall. It is evident that different combinations of Ra_f and Da yields different heat transfer results. The increasing of the fluid Rayleigh number increases the natural convection inside the enclosure. Since the Ra_m is fixed, a higher fluid Rayleigh number is associated with a less permeable media (i.e. lower Darcy number).

Figure 2 shows the isotherms and streamlines for Da=10^{-7} and Ra_m=10^3. The isotherms are almost stratified for the range of inclination angles analyzed. It is clearly observed from Fig. 2 (left) that the isotherms are equally spaced between the maximum and minimum temperature. It seems that the inclination does not affect the behavior of the isotherms significantly and only a little distortion due the inclination is observed. The streamlines, on the other hand, show a small dependence with the angle variation. For the range of inclination angles analyzed, the higher the

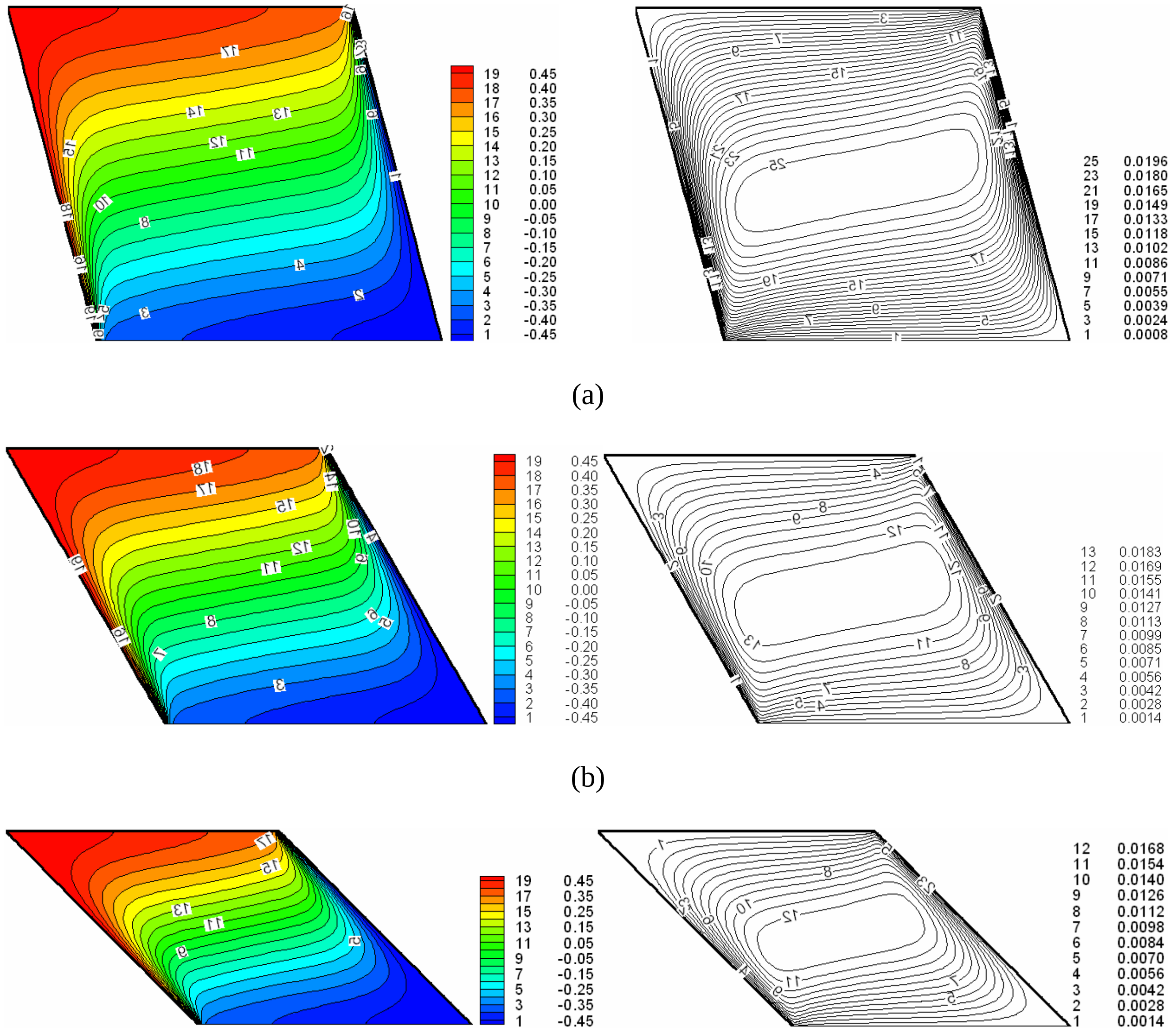

Figure 2 - Isotherms and streamlines for Ra_m=10^3, Da=10^{-7}; (a) q=15°, (b) q=30°, (c) q=45°.

inclination angle, the lower the overall values of recirculation intensity. This is probably due to the area reduction with the increase of the inclination angle minimizing the convective transport.

Figure 3 shows the isotherms and streamlines of the work of Baytas & Pop (1999) for Ra_m=10^3. It is important to emphasize that the present values of the streamlines must be divided by (a_{eff}=10^{-3}) to be compared with those from Baytas & Pop (1999). It is clearly seen from the Fig. 2 and 3 the good agreement between the results.

Figure 4 shows the isotherms and streamlines for Da=10^{-7} and Ra_m=10^4. Here, the isotherms are stratified for all inclination angles considered and the streamlines are now more intense with the increase of the Ra_m. Figure 5 shows the isotherms and streamlines of the work of Baytas & Pop (1999) for Ra_m=10^4. Here again, the good agreement between the Fig. 4 and 5 is also observed.

Figure 6 shows the behavior of the averaged Nusselt number with q for 10^3=Ra_m=10^4. It is clearly observed from Fig. 6 that the averaged Nusselt number increases as Ra_m is increased. However, the overall values of $\overline{Nu}$ for higher inclination angles are smaller than those for lower inclinations. As mentioned, the increase of the inclination angle decreases the recirculation intensity which, in turn, decreases the averaged Nusselt number for a fixed Ra_m. For clockwise inclination angles the decrease is even higher due to the influence of the heated wall that acts like an obstacle to the ascendant buoyant flow.

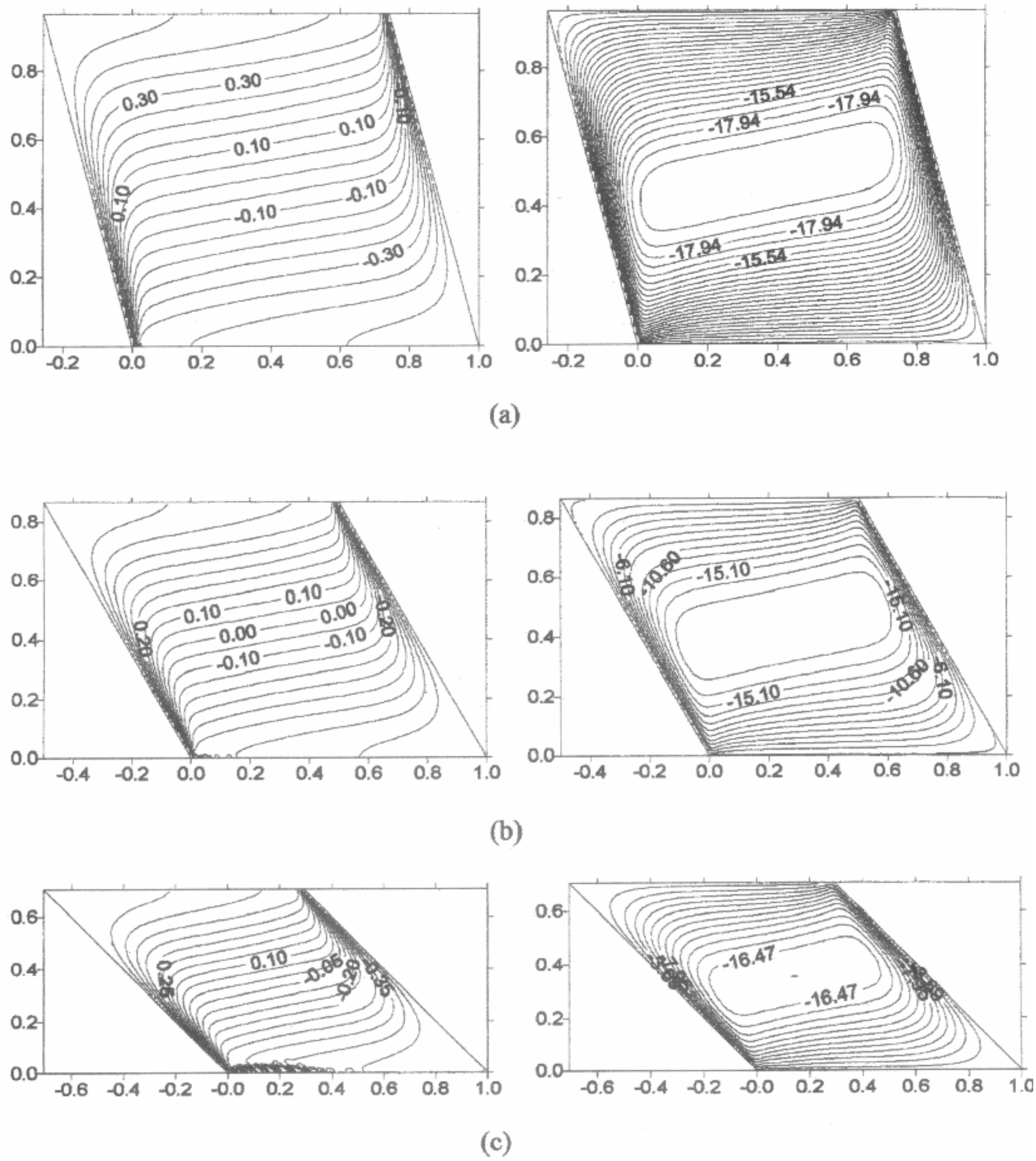

Figure 3 - Isotherms and streamlines for $Ra_m=10^3$,(a) q=15°, (b) q=30°, (c) q=45°, (Baytas & Pop (1999)).

5. CONCLUSION

This paper presented computations for simulation of laminar natural convection in oblique cavities fulfilled with porous material. The cavity is heated from the left side and cooled from the opposing side. The results yielded generally satisfactory agreement with previous works in the available literature. It was observed that the overall values of $\overline{Nu}$ for higher inclination angles are smaller than those for lower inclinations. Further, for the range of inclination angles analyzed, the higher the inclination angle, the lower the overall values of recirculation intensity.

Ultimately, it is expected that additional research on this new subject be stimulated by the work here presented.

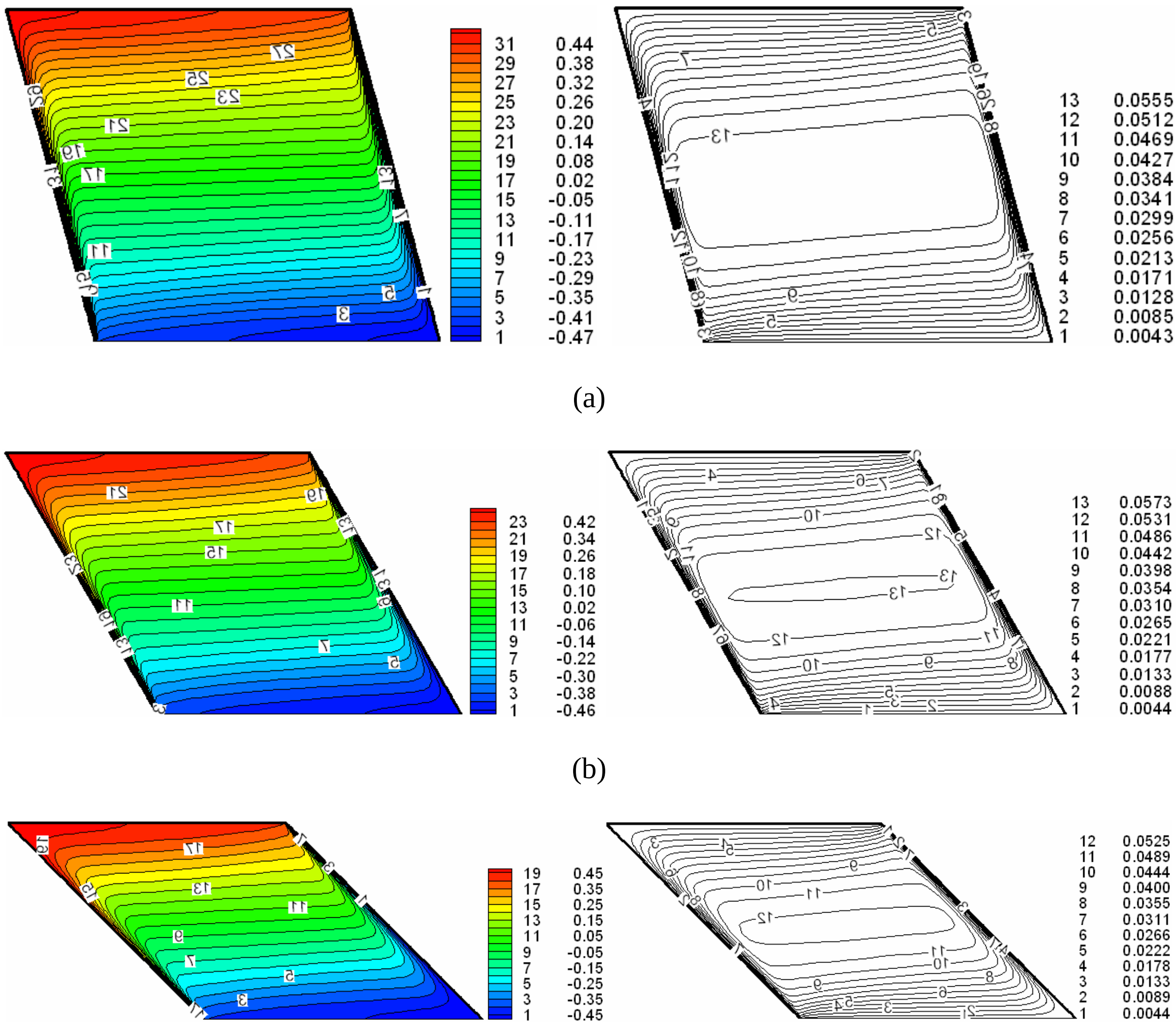

Figure 4 - Isotherms and streamlines for Ra_m=10^4, Da=10^{-7}; (a) q=15°, (b) q=30°, (c) q=45°.

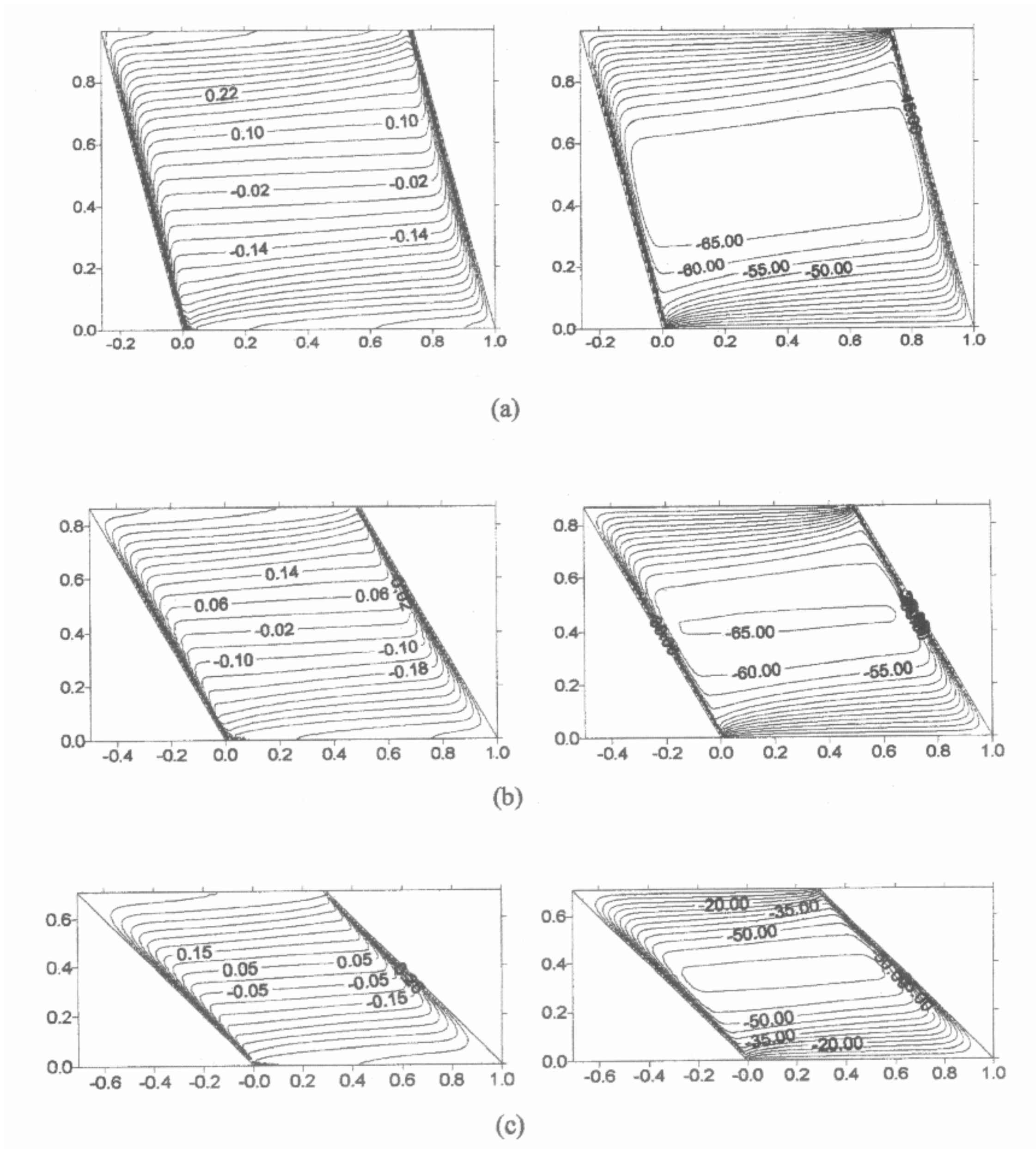

Figure 5 - Isotherms and streamlines for $Ra_m=10^4$; (a) *q*=15°, (b) *q*=30°, (c) *q*=45°, (Baytas & Pop (1999)).

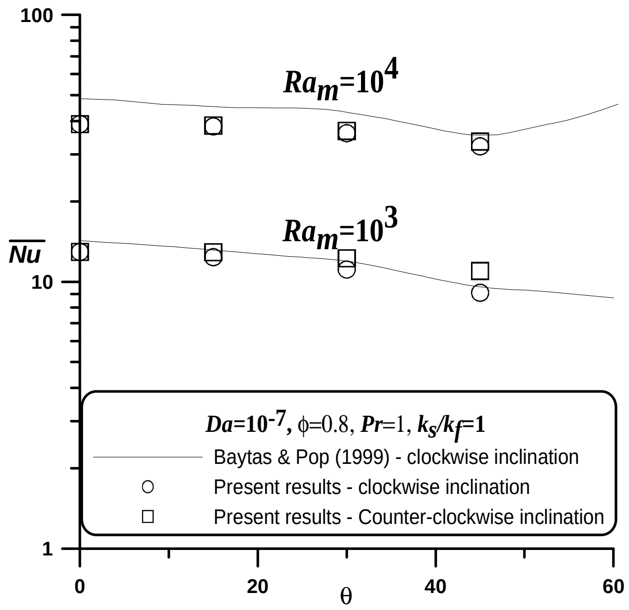

Figure 6 - Variation of averaged Nusselt number on the hot wall with *q*

6. ACKNOWLEDGEMENTS

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