TURBULENT FLOW IN A CHANNEL WITH SOLID AND POROUS FINS

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***Abstract**. In this work, numerical solutions are presented for turbulent flow in a channel containing fins made with porous material. The condition of spatially periodic cell is applied longitudinally along the channel. A macroscopic two-equation turbulence model is employed in both the porous region and the clear fluid. The equations of mass continuity, momentum and turbulence transport equations are written for an elementary representative volume yielding a set of equations valid for the entire computational domain. Results are presented for the velocity field as a function of Reynolds, porosity and permeability of the fins. Pressure drop along the channel is compared with the case of solid material.*

***Keywords**: numerical solution, porous media, turbulence.*

1. INTRODUÇÃO

Turbulent fluid flow in channels containing porous and solid obstacles is found in a number of engineering equipment such as shell-and-tube heat exchangers, chemical reactors, etc. Several other applications are found in the chemical and petroleum industries, leading to a great interest by many research groups in order to mathematically model and realistic describe this type of flow Hwang (1997), Yang and Hwang (2003) and Ko and Anand (2003).

In the works Rocamora and de Lemos (2000a-c) presented numerical solutions for laminar and turbulent flow in hybrid (clear/porous) media. Those works did not consider a stress jump at the interface between the porous media and the clear fluid. In the literature, for laminar flow, Ochoa and Whitaker (1995a-b) proposed an adjustable coefficient for modeling the stress jump at the interface. Kuznetsov (1996a-b) and Kuznetsov (1997) presented analytical solutions for velocity profiles in channel partially filled with porous material, taking into consideration such a jump condition. Recently, Silva and de Lemos (2002) presented numerical solutions for this same geometry, also considering the jump at the interface. For turbulent flow in permeable structures, de Lemos and Pedras (2000a)-b) and de Lemos and Pedras (2001) developed a macroscopic two-equation turbulence model.

More recently, laminar flow in channel containing porous fins was investigated in Tofaneli and de Lemos (2002a), where the effects of the inlet Reynolds number and jump coefficients were considered. There, use was made of the numerical methodology proposed in Silva and de Lemos (2002). Later, Tofaneli and de Lemos (2002b-c) investigated the influence of porosity and permeability on the flow pattern in the channel.

The objective of this work is to extent the results in Tofaneli and de Lemos (2002c); de Lemos and Tofaneli (2003a) considering now turbulent flow. Here, the model of Tofaneli and de Lemos (2002b), de Lemos and Pedras (2001) is applied.

2. MATHEMATICAL MODEL

2.1- Geometry

The flows here investigated are schematically shown in Figure 1.

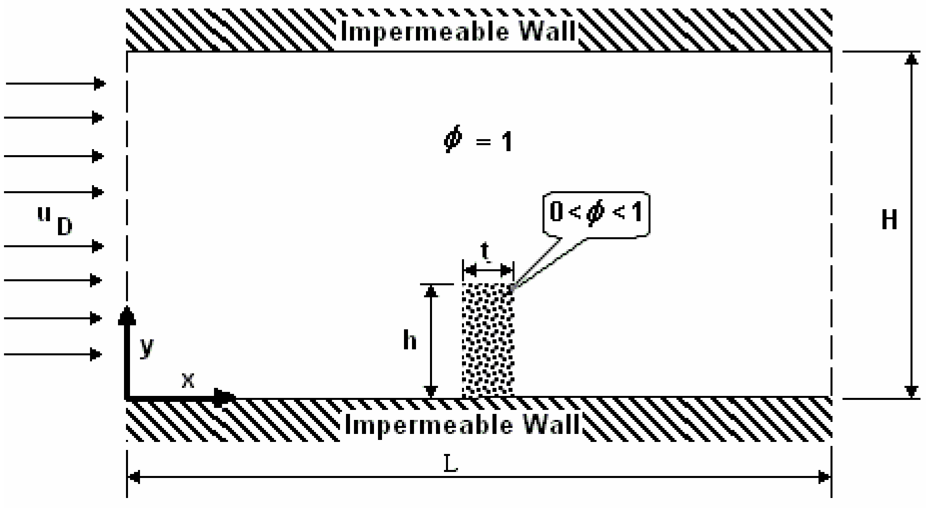


Figure 1: Channel baffles in one of the plates with a fin.

2.2- Governing Equations

The mathematical model here employed has its origin in the works of Pedras and de Lemos (2000), Pedras and de Lemos (2001a-b), Pedras and de Lemos (2003). The implementation of the jump condition at the interface was considered in Silva and de Lemos (2002) based on the theory proposed in Ochoa and Whitaker (1995a-b). Therefore, these equations will be here just reproduced and details about their derivations can be obtained in the mentioned works. These equations are:

$$\nabla \cdot \bar{\mathbf{u}}_D = 0 \tag{1}$$

where, $\bar{\mathbf{u}}_D$ is the average surface velocity ('seepage' or Darcy velocity). Equation (1) represents the macroscopic continuity equation for an incompressible fluid. The Macroscopic momentum equation is given as:

$$\left[\nabla\cdot\left(r\,\frac{\bar{\mathbf{u}}_D\bar{\mathbf{u}}_D}{f}\right)\right] = -\nabla(f\langle \bar{p}\rangle^i) + m\nabla^2\bar{\mathbf{u}}_D + \nabla\cdot(-r f\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i) - \left[\frac{mf}{K}\bar{\mathbf{u}}_D + \frac{c_F f r|\bar{\mathbf{u}}_D|\,\bar{\mathbf{u}}_D}{\sqrt{K}}\right] \tag{2}$$

where the last two terms in Eq. (2), represent the Darcy-Forchheimer contribution. The symbol K is the porous medium permeability, c_F is the form drag coefficient (Forchheimer coefficient), $\langle p\rangle^i$ is the intrinsic average pressure of the fluid, r is the fluid density, m represents the fluid viscosity and f is the porosity of the porous medium. The macroscopic Reynolds stress $-r f\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i$ is given as,

$$-r f\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i = m_{t_f}\, 2\langle\overline{\mathbf{D}}\rangle^{v} - \frac{2}{3} f\, r \langle k\rangle^i \mathbf{I} \tag{3}$$

where

$$\langle\overline{\mathbf{D}}\rangle^{v} = \frac{1}{2}\left[\nabla\left(f\langle\overline{\mathbf{u}}\rangle^i\right) + \left[\nabla\left(f\langle\overline{\mathbf{u}}\rangle^i\right)\right]^T\right] \tag{4}$$

is the macroscopic deformation tensor, $\langle k\rangle^i = \langle\overline{\mathbf{u}'\cdot\mathbf{u}'}\rangle^i / 2$ is the intrinsic turbulent kinetic energy, k and m_{t_f}, is the turbulent viscosity which is modeled in Rocamora and de Lemos (2000a) similarly to the case of clear flow, in the form,

$$m_{t_f} = r\, c_m \frac{\langle k\rangle^{i\,2}}{\langle e\rangle^i}$$

Transport equations for $\langle k\rangle^i$ and its dissipations rate $\langle e\rangle^i = m\langle\overline{\nabla\mathbf{u}' : (\nabla\mathbf{u}')^T}\rangle^i / r$ are proposed in Tofaneli and de Lemos (2002c) as:

$$r\left[\frac{\partial}{\partial t}\left(f\langle k\rangle^i\right) + \nabla\cdot\left(\overline{\mathbf{u}}_D\langle k\rangle^i\right)\right] = \nabla\cdot\left[\left(m + \frac{m_{t_f}}{s_k}\right)\nabla\left(f\langle k\rangle^i\right)\right] + P^i + G^i - r f\langle e\rangle^i \tag{5}$$

$$r\left[\frac{\partial}{\partial t}\left(f\langle e\rangle^i\right) + \nabla\cdot\left(\overline{\mathbf{u}}_D\langle e\rangle^i\right)\right] = \nabla\cdot\left[\left(m + \frac{m_{t_f}}{s_e}\right)\nabla\left(f\langle e\rangle^i\right)\right] + \frac{\langle e\rangle^i}{\langle k\rangle^i}\left[c_1 P^i + c_2 G^i - c_2 r f\langle e\rangle^i\right] \tag{6}$$

where c_1, c_2 and c_k are model constants, $P^i = -r\langle\overline{\mathbf{u}'\mathbf{u}'}\rangle^i : \nabla\overline{\mathbf{u}}_D$ and $G^i = c_k\, r \frac{f\langle k\rangle^i |\overline{\mathbf{u}}_D|}{\sqrt{K}}$ are generation rates of $\langle k\rangle^i$ due to gradients of $\overline{\mathbf{u}}_D$ and due to the action of the porous matrix. At the interface, the conditions of continuity of velocity, pressure, turbulent kinetic energy k and its dissipation rate e, in addition to their respective diffusive fluxes, are given by,

$$\overline{\mathbf{u}}_D\Big|_{0<f<1} = \overline{\mathbf{u}}_D\Big|_{f=1} \tag{7}$$

$$\langle \overline{p} \rangle^i\Big|_{0<f<1} = \langle \overline{p} \rangle^i\Big|_{f=1} \tag{8}$$

$$\langle k \rangle^v\Big|_{0<f<1} = \langle k \rangle^v\Big|_{f=1} \tag{9}$$

$$(m + \frac{m_{t_f}}{s_k})\frac{\partial \langle k \rangle^v}{\partial y}\Bigg|_{0<f<1} = (m + \frac{m_t}{s_k})\frac{\partial \langle k \rangle^v}{\partial y}\Bigg|_{f=1} \tag{10}$$

$$\langle e \rangle^v\Big|_{0<f<1} = \langle e \rangle^v\Big|_{f=1} \tag{11}$$

$$(m + \frac{m_{t_f}}{s_e})\frac{\partial \langle e \rangle^v}{\partial y}\Bigg|_{0<f<1} = (m + \frac{m_t}{s_e})\frac{\partial \langle e \rangle^v}{\partial y}\Bigg|_{f=1} \tag{12}$$

The jump condition at the interface is given by,

$$\left(m_{ef} + m_{t_f}\right)\frac{\partial \overline{u}_{D_p}}{\partial y}\Bigg|_{0<f<1} - \left(m + m_t\right)\frac{\partial \overline{u}_{D_p}}{\partial y}\Bigg|_{f=1} = \left(m + m_t\right)\frac{b}{\sqrt{K}}\overline{u}_{D_i}\Big|_{\text{interface}} \tag{13}$$

The non-slip condition for velocity is applied to all of the four walls.

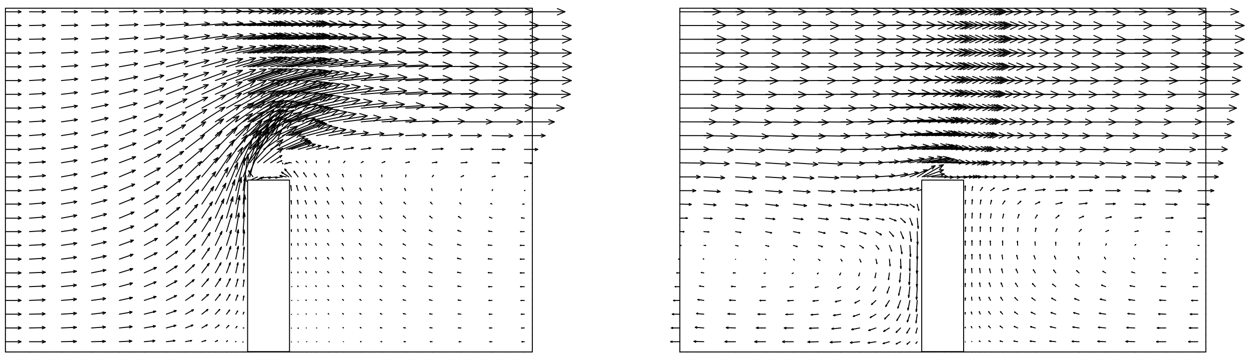
3. NUMERICAL MODEL

The numerical method utilized to solve the flow equations is the Finite Volume method applied to a boundary-fitted coordinate system, can be seen in Pedras and de Lemos (2001a). Equations (1)-(2) subjected to boundary and interface conditions Eq. (7)-(13), were discretized in a two-dimensional control volume involving both clear and porous media. The numerical method used in the resolution of the equations above was the SIMPLE algorithm of Patankar (1980). The interface is positioned to coincide with the border between two control volumes, generating, in such a way, only volumes of the types 'totally porous' or 'totally clear'. The flow equations are then resolved in the porous and clear domains, being respected the interface conditions mentioned earlier. In the implementation herein, a system of generalized coordinates was used although all simulation to be shown employed only Cartesian Coordinates. Nevertheless, the use of a general system $h-e$ for discretizing the equations was found to be adequate for future simulations. Details of the numerical implementation can be seen in Silva and de Lemos (2002) and Pedras and de Lemos (2001a). Here, all computations were carried out until normalized residues of the algebraic equations were brought down to 10^{-5}, where the residue was defined as the difference between the right and left sides of the discretized equations. The permeability of the porous way in all of the cases was esteemed being respected the correlation proposed for Kameyama, Yamashita and Nakayama (1998) for half porous, in the way,

$$K=\frac{f^3}{144(1-f)^2}D_p^2 \tag{14}$$

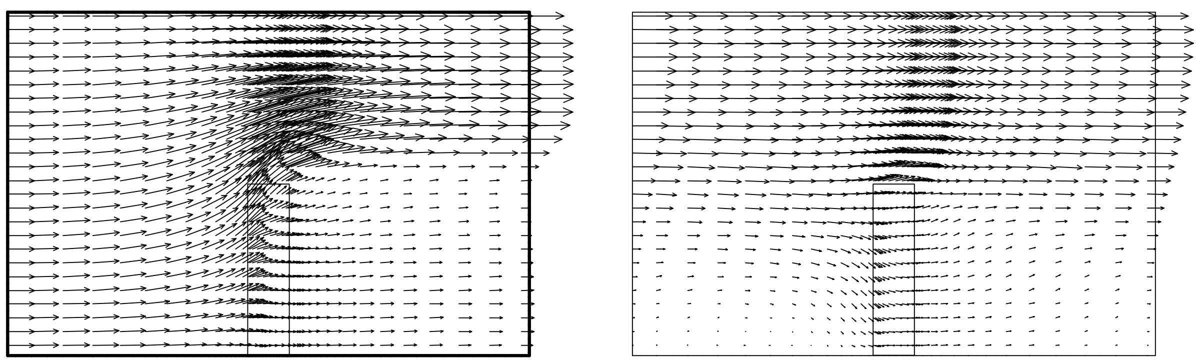
4. RESULTS AND DISCUSSION

Figure 2 and Figure 3 show velocity fields obtained with uniform velocity at entrance a) as well as for a periodic cell b). For solid fins, Figure 2, we can observe that there is a recirculation zone right after the obstacle, whereas for the porous fin such recirculation doesn't occur (Figure 3).



a) b)

Figure 2: Velocity field for solid fin, $\mathrm{Re}_H = 30{,}000$: a) uniform profile at the entrance; b) periodic channel.



a) b)

Figure 3: Velocity field for $\mathrm{Re}_H = 30{,}000$: a) uniform profile at the entrance; b) periodic channel $\left(f=0.9; K=5\times10^{-7}\,m^2\right)$.

Figure 4 and Figure 5 show the distribution of turbulent kinetic energy for solid and porous fins respectively. For the porous fin case, the largest level of turbulence intensity is inside and around the fin. However for the solid fin case, it at the top of the fin. For solid fins, the largest velocity gradients, must responsable for turbulence generation, are located at the top of the fins. On the other hand, for porous baffles, generation of k occurs inside the fins.

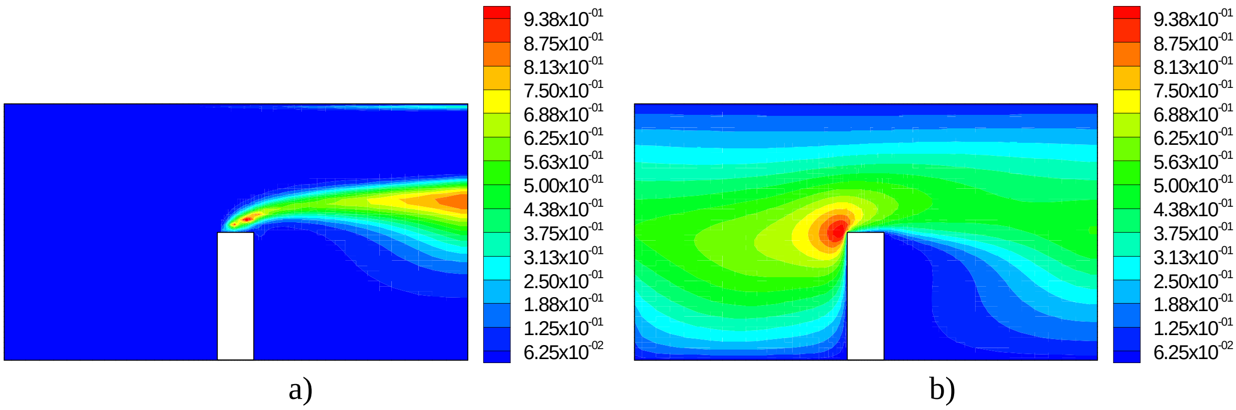


Figure 4: Distribution of $k_+ = \dfrac{k - k_{\min}}{k - k_{\max}}$ for solid fin ($\mathrm{Re}_H = 30{,}000$): a) uniform profile at the entrance, b) periodic channel.

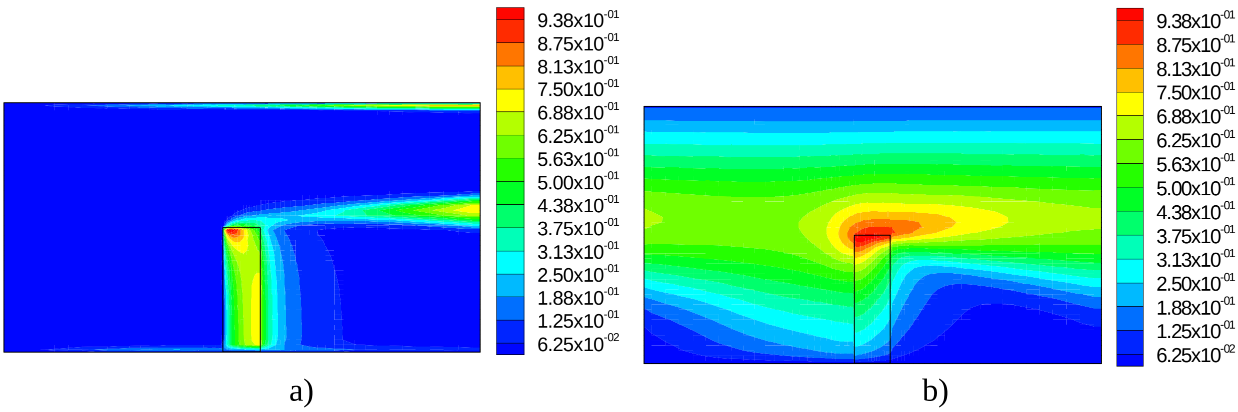


Figure 5: Distribution of $k_+ = \dfrac{k - k_{\min}}{k - k_{\max}}$ for ($\mathrm{Re}_H = 30{,}000$, $K = 5\times10^{-7}\,m^2$, $f = 0.9$): a) porous fin, uniform profile, b) porous fin, periodic channel.

Tables (1) and (2) show calculated pressure drop in a periodic section using the high Reynolds $k-e$ model. The average pressure drop along the periodic section is numerically obtained as:

$$\overline{\Delta P} = \frac{1}{A_t}\int_{A_t}(p_{in} - p_{ex})dy \qquad (15)$$

The percent difference in relation to the solid fin case is calculated as:

$$e_P = \left(\overline{\Delta P}\Big|_{poroso} - \overline{\Delta P}\Big|_{solid}\right)\times 100 \Big/ \overline{\Delta P}\Big|_{solid} \qquad (16)$$

where the subscripts *in* and *ex* indicate inlet and outlet areas, respectively.

Table (1) shows the results for the pressure loss for the case of uniform flow at the inlet of the cell in Figure 1. It can be noticed that using porous fins implies in less energy losses (less head loss) when compared to the solid fin case.

Table (2) shows the results for the pressure loss for the condition of periodic flow, that is, the follow condition at the entrance of the channel is the same as in the exit. As it can be seen, the pressure drop is lower the channel in the condition of periodic drainage, than for the case where the profile of uniform velocity is imposed at the channel entrance. However, the pressure drop difference between porous and solid fins is larger for the periodic case.

Table (1): Percent pressure loss in relation to solid fins for turbulent flow and profile of uniform speed, grid=210×50, $\mathrm{Re}_H = 30{,}000$.

f	K/H^2	$\varepsilon_P = \dfrac{\overline{\Delta P_p} - \overline{\Delta P_s}}{\overline{\Delta P_s}} \times 100$	Calculated *DP*
Solid			223.98 *
0.6	4×10^{-5}	-5.57	211.49
	2×10^{-4}	-20.55	177.94
0.7	4×10^{-5}	-4.56	213.75
	2×10^{-4}	-19.30	180.74
0.8	4×10^{-5}	-3.33	216.51
	2×10^{-4}	-17.66	184.41
0.9	4×10^{-5}	-1.65	220.27
	2×10^{-4}	-15.48	189.29

* Reference value given in N/m^2 for calculating e_P for porous fins

Table (2): Percent pressure loss in relation to solid fins for turbulent flow in periodic channel, grid=70×50, $\mathrm{Re}_H = 30{,}000$.

f	K/H^2	$\varepsilon_P = \dfrac{\overline{\Delta P_p} - \overline{\Delta P_s}}{\overline{\Delta P_s}} \times 100$	Calculated *DP*
Solid			46.61 *
0.6	4×10^{-5}	-34.82	30.38
	2×10^{-4}	-48.65	23.93
0.7	4×10^{-5}	-33.40	31.04
	2×10^{-4}	-46.77	24.81
0.8	4×10^{-5}	-31.76	31.80
	2×10^{-4}	-44.51	25.86
0.9	4×10^{-5}	-28.07	33.52
	2×10^{-4}	-40.00	27.96

* Reference value given in N/m^2 for calculating e_P for porous fins

5. CONCLUDING REMARKS

This work presented results for the numerical solution of turbulent flow in a channel containing porous obstructions. The effect of the permeability of the porous material on the flow pattern was considered. Also taking into consideration was the jump condition of the shear stress at the interface. Discretization of the governing equations used the finite volume method of and the set of algebraic equations was solved by the SIMPLE method. Results indicated an prominent effect of the jump condition coefficient modifying the flow patter within the periodic cell.

6. ACKNOWLEDGMENTS

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