HYBRID ANALYSIS OF THE MAGNETOHYDRODYNAMIC FLOW AND HEAT TRANSFER WHITIN A PARALLEL-PLATES CHANNEL

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Abstract. *In the present work, the magnetohydrodynamic flow and heat transfer (MHD) of a Newtonian, electrically conducting, viscous and incompressible fluid inside a parallel-plates channel is studied through the so-called Generalized Integral Transform Technique (GITT). The main goal is to investigate the effects on the flow of the external magnetic field, which is perpendicular to the flow, as well as to analyze the variation of the fluid viscosity with temperature through this hybrid technique. In order to study as many physical possibilities as possible maintaining, however, a simple mathematical formulation for the problem, two kinds of analyses are considered. The first, which evidences the transient regime, assumes that flow is sustained by a constant pressure gradient only; whereas the second, the steady-state situation, considers both a constant pressure gradient and a movement of the upper plate, as well as the action of an inflow and an outflow perpendicular to the plates (porous plates). Results for velocity and temperature fields are obtained within the governing parameters, namely, pressure gradient, suction velocity, upper plate velocity and Hartmann numbers, for typical situations. A convergence analysis is also performed showing the consistency of the results. Finally, the results obtained by the present approach are compared with literature published data, showing excellent agreement.*

Keywords: *Magnetohydrodynamic (MHD), Internal Forced Convection, Integral Transform (GITT)*

1. INTRODUCTION

The magnetohydrodynamic flow and heat transfer (MHD) of a viscous, electrically conducting fluid presents important industrial applications, mainly, in the petroleum industry, nuclear reactors engineering and, more recently, in the field of the metallurgy. Generators, pumps, accelerators and hydromagnetic flowmeters are examples of such applications. Beginning in the twenties, the numerical studies about magnetohydrodynamics reappeared strongly in the sixties of the past century until the present days.

The MHD flow of a viscous, electrically conducting fluid inside a parallel-plates channel has been considered by many researchers for various situations (Tao, 1960; Nigam and Singh, 1960; Alpher, 1961 and Sutton and Sherman, 1965). The simplest analysis involving this geometry was the study of the thermally developing flow considering the Hartman (1937) profile for the velocity field (Nigam and Singh, 1960). Implied in this was the hypothesis of small temperature differences so that the velocity field could be decoupled from the temperature one, i.e., these studies were based on constant physical properties. However, in order to make more realistic predictions the variations of these properties would be taken into account. The first work that took into account variable properties on the entrance flow in a channel was conducted by Klemp et al. (1990). Attia and Kotb (1996), considering an exponential variation of the viscosity, porous plates, as well as the movement of the upper plate, studied the steady flow and heat transfer. Isothermal and non-steady MHD flow of blood through porous channel was studied by Rao and Rao (1988), considering constant physical properties. In their study, Rao and Rao (1988) considered blood as a Newtonian fluid. Later, Attia (1999) included the effect of an exponential variation of the viscosity with temperature on the unsteady flow.

In all analyzed cases the results were obtained through application of purely numerical methods, or analytical solutions were obtained for limiting situations. The present work reproduces such studies through the so-called GITT (Generalized Integral Transform Technique), a hybrid numerical-analytical spectral based approach (Cotta, 1993, Cotta, 1998 and Santos et al., 2001). In order to investigate as many physical possibilities as possible maintaining a simple mathematical formulation for the problem, two kinds of analyses are considered. The first one, which evidences the transient regime, assumes that the flow is only sustained by a constant pressure gradient; whereas the second, the steady-state situation, considers a constant pressure gradient and a mobile upper plate, as well as action of an inflow perpendicular to the plates (porous plates).

2. MATHEMATICAL FORMULATION

A schematic representation of the MHD parallel-plate channel is illustrated in Fig. 1. It consists of two parallel and electrically insulated porous or non-porous plates separated by a distance h, maintained at constant and different temperatures. Within this channel flows a Newtonian, viscous, incompressible and electrically conducting fluid, which is initially at rest and submitted to a perpendicular, uniform and constant magnetic field. Viscous dissipation is also taken into account.

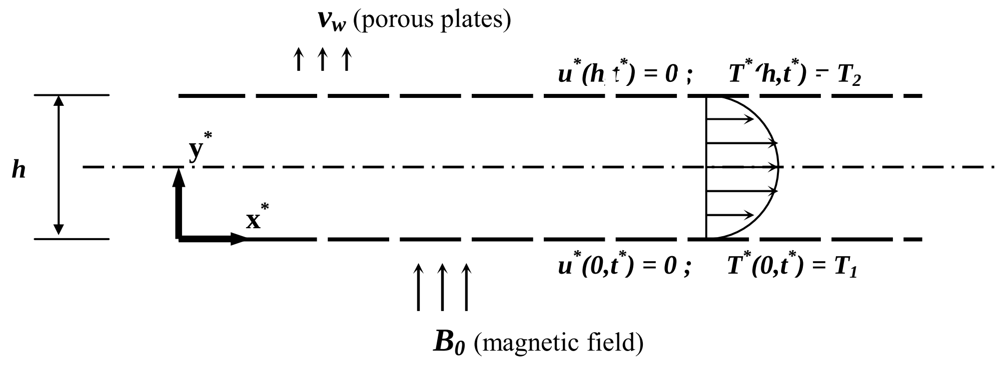

Figure 1. Schematic representation of the analyzed problem

Under the above hypothesis, the dimensionless mathematical formulation for the one-dimensional transient MHD problem is given as:

$$\frac{\partial \hat{u}}{\partial t} + R_v \frac{\partial \hat{u}}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left[\mu(T)\frac{\partial \hat{u}}{\partial y}\right] - Ha^2\,\hat{u} \tag{1}$$

$$\frac{\partial T}{\partial t} + R_v \frac{\partial T}{\partial y} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 T}{\partial y^2} + Ec\,\mu(T)\left(\frac{\partial \hat{u}}{\partial y}\right)^2 + Ec\,Ha^2\,\hat{u}^2 \tag{2}$$

Subjected to initial and boundary conditions:

$$\left.\begin{aligned}\hat{u}(y,0)=0\\ T(y,0)=0\end{aligned}\right\} ;\quad t=0,\quad 0<y<1 \tag{3,4}$$

$$\left.\begin{aligned}\hat{u}(0,t)=0\\ T(0,t)=0\end{aligned}\right\} ;\quad y=0,\quad t>0 \;;\qquad \left.\begin{aligned}\hat{u}(1,t)=R_u\\ T(1,t)=1\end{aligned}\right\} ;\quad y=1,\quad t>0 \tag{5-8}$$

In the above formulation, the following dimensionless groups were employed:

$$y=\frac{y^*}{h}\;;\quad x=\frac{x^*}{h}\;;\quad \hat{u}=\frac{u^*h}{\nu_0}\;;\quad T=\frac{T^*-T_1}{T_2-T_1}\;;\quad t=\frac{t^*\nu_0}{h^2}\;;$$

$$R_u=\frac{u_1h}{\nu_0}\;;\quad R_v=\frac{v_wh}{\nu_0}\;;\quad P=\frac{h^2}{\rho\nu_0{}^2}P^*\;;\quad G=-\frac{\partial P}{\partial x}\;;\quad \mu=\frac{\mu^*}{\mu_0}\;; \tag{9}$$

$$\Pr=\frac{\mu_0c_p}{k}\;;\quad Ha=B_0\,h\sqrt{\sigma/\mu_0}\;;\quad Ec=\frac{\nu_0^2}{h^2c_p(T_2-T_1)}$$

where h is the distance between plates, y is the transversal coordinate, x is the longitudinal coordinate, T_1 and T_2 are the temperatures of the lower and upper plates, respectively, u_1 is the upper plate velocity, v_w is the inflow velocity and B_0 is the external applied magnetic field. The physical properties ρ, the fluid density, c_p, the specific heat, k, the fluid conductivity, μ_0, the dynamic viscosity, ν_0, the cinematic viscosity and σ, the electrical conductivity are evaluated at the reference initial/lower plate temperature, $T_0=T_1$. With the dimensionless groups adopted the following dimensionless parameters were introduced, namely, Ru, Rv, Pr, Ec and Ha; the Reynolds, Prandtl, Eckert and Hartaman numbers, respectively.

As employed in previous references, the following exponential variation was adopted for the dynamic viscosity:

$$\mu(t)=e^{-aT} \tag{10}$$

where a is the viscosity parameter defined as

$$a=\ln\left(\mu_1/\mu_2\right)\qquad \begin{aligned}&\mu_1 \text{ is the dynamic viscosity evaluated at } T=T_1\\ &\mu_2 \text{ is the dynamic viscosity evaluated at } T=T_2\end{aligned} \tag{11}$$

3. SOLUTION METHODOLOGY

3.1. Filtering Process

In order to employ the GITT approach in its more efficient form, it is necessary to homogenize the boundary conditions in the direction to be integral transformed, in this case the y direction, through a filtering process. This is accomplished by splitting up the analyzed potentials as:

$$\hat{u}(y,t)\;=\;u(y,t)+u_F(y)\;;\qquad T(y,t)\;=\;\theta(y,t)+T_F(y) \tag{12,13}$$

For simplicity, in order to avoid computationally involved mathematical expressions, the filters employed in the present analysis are the solutions of the steady-state version of the original problems, making $Rv=0$ and $Ec=0$:

$$u_F(y) = \begin{cases} \dfrac{G}{2}\left(y - y^2\right) + R_u y & ; \quad Ha = 0 \\ \dfrac{G\left[1 - \cosh(Ha\, y)\right] + \left[G\left(\cosh(Ha) - 1\right) + Ha^2 R_u\right] \dfrac{\sinh(Ha\, y)}{\sinh(Ha)}}{Ha^2} & ; \quad Ha \neq 0 \end{cases} \tag{14}$$

$$T_F(y) = y \tag{15}$$

Therefore, the governing equations of the MHD flow with heat transfer become:

$$\frac{\partial u}{\partial t} + R_v\left[\frac{\partial u}{\partial y} + \frac{du_F}{dy}\right] = -\frac{d^2 u_F}{dy^2} + \frac{\partial}{\partial y}\left[\mu(T)\left(\frac{\partial u}{\partial y} + \frac{du_F}{dy}\right)\right] - Ha^2 u \tag{16}$$

$$\frac{\partial \theta}{\partial t} + R_v\left[\frac{\partial \theta}{\partial y} + \frac{dT_F}{dy}\right] = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2} + Ec\, \mu(t)\left[\frac{\partial u}{\partial y} + \frac{du_F}{dy}\right]^2 + Ec\, Ha^2\left[u + u_F\right] \tag{17}$$

Subjected to the initial and boundary conditions:

$$\left.\begin{aligned} u(y,0) &= -u_F(y) \\ \theta(y,0) &= -T_F(y) \end{aligned}\right\} ; \quad t = 0, \quad 0 < y < 1 \tag{18,19}$$

$$\left.\begin{aligned} u(0,t) &= 0 \\ \theta(0,t) &= 0 \end{aligned}\right\} ; \quad y = 0, \quad t > 0 \; ; \qquad \left.\begin{aligned} u(1,t) &= 0 \\ \theta(1,t) &= 0 \end{aligned}\right\} ; \quad y = 1, \quad t > 0 \tag{20-23}$$

3.2. Integral Transformation

Following the basic steps in the integral transform approach, appropriate eigenvalue problems that permit the integral transformation process must be chosen. For these classes of problems, they can be easily found in Cotta (1993), Cotta (1998) and Santos et al. (2001). According to such eigenvalue problems, the following integral-transform pairs are developed:

$$u(y,t) = \sum_{i=1}^{\infty} \tilde{\Omega}_i(y)\, \overline{u}_i(t) \; ; \qquad \overline{u}_i(t) = \int_0^1 \tilde{\Omega}_i(y)\, u(y,t)\, dy \tag{24,25}$$

$$\theta(y,t) = \sum_{i=1}^{\infty} \tilde{\tau}_i(y)\, \overline{\theta}_i(t) \; ; \qquad \overline{\theta}_i(t) = \int_0^1 \tilde{\tau}_i(y)\, \theta(y,t)\, dy \tag{26,27}$$

To perform the integral transformation, Eqs. (16) and (18) should be multiplied by the velocity eigenfunction, $\tilde{\Omega}_i(y)$ and Eqs. (17) and (19) multiplied by the temperature eigenfunction, $\tilde{\tau}_i(y)$; after that they are integrated over the domain of solution [0,1]. After employing the inversion formulae, Eqs. (24) and (26), and boundary conditions, Eqs. (20) to (23), the resultant transformed equations are written as:

$$\frac{d\overline{u}_i}{dt} = -Ha^2\, \overline{u}_i - B_i \quad ; \qquad \overline{u}_i(0) = -\overline{f}_i \quad , \quad i = 1,\ 2,\ ...,\ \infty \tag{26,27}$$

$$\frac{d\overline{\theta}_i}{dt} = -\frac{\beta_i^2}{\Pr}\overline{\theta}_i + S_i \quad ; \qquad \overline{\theta}_i(0) = -\overline{g}_i \quad , \quad i = 1,\ 2,\ ...,\ \infty \tag{28,29}$$

where the coefficients are defined as:

$$B_i = \int_0^1 \left\{ \tilde{\Omega}_i \left[R_v \left(\frac{\partial u}{\partial y} + \frac{du_F}{dy} \right) + \frac{d^2 u_F}{dy^2} \right] + \tilde{\Omega}_i' \, \mu(T) \left(\frac{\partial u}{\partial y} + \frac{du_F}{dy} \right) \right\} dy \quad (30)$$

$$\overline{f}_i = \int_0^1 \tilde{\Omega}_i \, u_F \, dy \quad (31)$$

$$S_i = \int_0^1 \tilde{\tau}_i \left\{ Ec \left[\mu(T) \left(\frac{\partial u}{\partial y} + \frac{du_F}{dy} \right)^2 + Ha^2 \left(u + u_F \right)^2 \right] - R_v \left(\frac{\partial \theta}{\partial y} + \frac{dT_F}{dy} \right) \right\} dy \quad (32)$$

$$\overline{g}_i = \int_0^1 \tilde{\Omega}_i \, u_F \, dy \quad (33)$$

Therefore, the integral transformation process eliminates the transversal coordinate, y, and offers an ordinary differential system for the transformed potentials in the time variable. The infinity system, Eqs. (26) to (29), should be truncated to sufficiently large finite orders, NU and NT (for the velocity and temperature expansion, respectively), in order to achieve numerical results to within a user prescribed accuracy target. This is attained through well-established subroutines for initial value problems such as DIVPAG from the IMSL package (IMSL, 1987). Once these transformed potentials have been numerically evaluated at any time, *t*, the related potentials for the velocity and temperature are analytically recovered, by recalling their inversion formulae.

4. RESULTS AND DISCUSSION

A Fortran code was built and implemented on an Atlhon XP2000 computer imposing a relative error criterion of 10^{-6} for subroutine DIVPAG (IMSL, 1987), i.e., an error control in the sixth significant digit for all transformed potentials is searched. Results for the main potentials are illustrated and critically compared against previously reported numerical results, for various combinations of the dimensionless parameters *Ha*, *Ru*, *Rv*, *a*, *G*, *Pr* and *Ec*. Convergence behaviors for velocity and temperature fields are showed for different time and transversal coordinate. All figures presented here are illustrate by employing $N=NU=NT=200$ in the eigenfunction expansions.

Tables (1) and (2) illustrate the convergence behavior of the centerline velocity and centerline temperature, respectively, at two time instants for different values of the viscosity parameter, *a*, Hartmann number, *Ha*, and $Pr=1$ and $Ec=0.050875$. The situations are showed considering that the upper plate is fixed ($R_u=0$), i.e., the flow is only sustained by a negative gradient pressure, $G=40$. In addition, neither inflow nor outflow through the plates is permitted to occur ($R_v=0$).

Table 1. Convergence behavior of the centerline velocity, at $t=0.5$ and $t\rightarrow\infty$, for different values of the viscosity parameter, *a*, and Hartman number, *Ha*. ($R_u=0$, $R_v=0$, $G=40$, $Pr=1$ and $Ec=0.050875$)

u(0, 0.5)									
	Ha=0			*Ha=1*			*Ha=2*		
N	*a= - 0.5*	*a=0*	*a=0.5*	*a= - 0.5*	*a=0*	*a=0.5*	*a= - 0.5*	*a=0*	*a=0.5*
10	1.566	1.7486	1.898	1.491	1.662	1.801	1.296	1.435	1.549
50	1.566	1.7486	1.898	1.491	1.662	1.801	1.297	1.435	1.549
300	1.566	1.7486	1.898	1.491	1.662	1.801	1.297	1.435	1.549
U(0, ∞)									
	Ha=0			*Ha=1*			*Ha=2*		
N	*a= - 0.5*	*a=0*	*a=0.5*	*a= - 0.5*	*a=0*	*a=0.5*	*a= - 0.5*	*a=0*	*A= 0.5*
10	1.755	2.500	4.081	1.637	2.264	3.425	1.362	1.760	2.317
50	1.754	2.500	4.086	1.637	2.264	3.428	1.362	1.760	2.318
300	1.754	2.500	4.086	1.637	2.264	3.428	1.362	1.760	2.318

Table 2. Convergence behavior of the centerline temperature, at $t=0.5$ and $t\to\infty$, for different values of the viscosity parameter, a, and Hartman number, Ha. ($R_u=0$, $R_v=0$, $G=40$, $Pr=1$ and $Ec=0.050875$)

	$T(0, 0.5)$								
	$Ha=0$			$Ha=1$			$Ha=2$		
N	$a= - 0.5$	$a=0$	$a=0.5$	$a= - 0.5$	$a=0$	$a=0.5$	$a= - 0.5$	$a=0$	$a=0.5$
10	0.3804	0.3720	0.3655	0.3935	0.3888	0.3856	0.4212	0.4242	0.4277
50	0.3798	0.3713	0.3646	0.3930	0.3882	0.3848	0.4208	0.4236	0.4271
300	0.3798	0.3713	0.3646	0.3930	0.3882	0.3848	0.4208	0.4236	0.4271
	$T(0, \infty)$								
	$Ha=0$			$Ha=1$			$Ha=2$		
N	$a= - 0.5$	$a=0$	$a=0.5$	$a= - 0.5$	$a=0$	$a=0.5$	$a= - 0.5$	$a=0$	$a=0.5$
10	0.7990	0.9252	1.232	0.8091	0.9417	1.234	0.8170	0.9388	1.142
50	0.7979	0.9240	1.232	0.8082	0.9407	1.234	0.8163	0.9381	1.141
300	0.7979	0.9240	1.232	0.8081	0.9407	1.234	0.8163	0.9381	1.141

According to Tabs. (1) and (2) above both potentials showed an extremely fast convergence, mainly for the centerline velocity since, as one can see, low orders in the eigenfunction expansions are required. This behavior can be easily explained since, for the range of values of parameters R_v and a, the solution of the original problem is practically obtained by the filtering solution.

Table 3 shows the convergence behavior of the steady-state centerline velocity for different values of the viscosity parameter, a, and Hartmann number, Ha. Now, the upper plate is given a horizontal velocity, $R_u=1$, and a positive pressure gradient is applied in the horizontal direction, $G= -5$, while the suction parameter R_v is still set equal to zero.

Table 3. Convergence behavior of the steady-state velocity at the centerline, $u(0, \infty)$, for different values of the viscosity parameter, a, and Hartmann number, Ha. ($R_u=1$, $R_v=0$, $G= -5$, $Pr=1$ and $Ec=1$)

	$U(0, \infty)$					
	$a = 0.0$			$a = 0.5$		
N	$Ha = 0$	$Ha = 2$	$Ha = 10$	$Ha = 0$	$Ha = 2$	$Ha = 10$
10	- 0.1250	- 0.1159	- 0.04259	- 0.4341	- 0.3071	- 0.04941
50	- 0.1250	- 0.1159	- 0.04259	- 0.4343	- 0.3072	- 0.04804
300	- 0.1250	- 0.1159	- 0.04259	- 0.4343	- 0.3072	- 0.04803

As previously commented, the filtering solution showed itself an excellent procedure, since it recovers the original potential (no terms are required in the velocity expansion) for large values of time, and $R_v=0$ and $a=0$. For cases where the viscosity parameter differs from zero, i.e., when the momentum and energy equation are fully coupled, a noticeable effect in the improvement of the convergence is still verified, even for high values of the Hartmann number. A comparison between the fully converged results provided by the present approach, available in Tab. 3 for $a=0$, and the numerical results of Attia and Kotb (1996) is visualized in Fig. 2a showing the influence of the Hartmann number on the flow.

The same trends verified in Tab. 3, for convergence behavior, are also observable in Tab. 4, which illustrates the influence of the inflow/outflow parameter and of the positive pressure gradient over the overall convergence.

Table 4. Convergence behavior of the steady-state velocity at the centerline, $u(0, \infty)$, for different values of the suction parameter, R_v. ($a=0$, $Ha=0$, $R_u=1$, $G= -5$, $Pr=1$ and $Ec=1$)

	$u(0, \infty)$			
N	$R_v=0$	$R_v=1$	$R_v=5$	$R_v=10$
10	- 0.1250	- 0.2349	- 0.3496	- 0.2430
50	- 0.1250	- 0.2348	- 0.3483	- 0.2400
300	- 0.1250	- 0.2348	- 0.3483	- 0.2400

Additionally, Fig. 2b, which illustrates the converged results from Tab. 4, offers a comparison between the present results and those of Attia and Kotb (1996), showing the interesting intrinsic MHD behavior, namely, increasing the inflow parameter (up till 5) increase the reversed flow depth, since the gradient pressure is positive, but for higher inflow parameter ($R_v=10$) a reduction in the recirculation flow depth is clearly noticed.

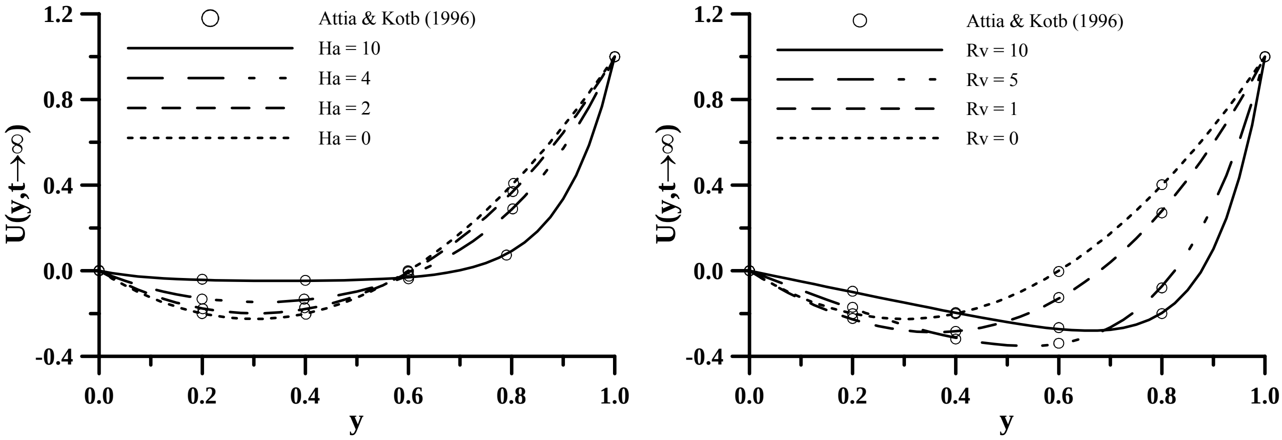

Figure 2. Influence of (a) Hartmann and (b) inflow parameters on the steady-state centerline velocity

Figures 3a and 3b bring comparisons between the present results and those of Attia (1999) for the steady-state velocity and temperature profiles, respectively, for different values of viscosity parameter, *a*. The following values for the dimensionless parameters were employed $Ha=0.5$, $R_u=0$, $R_v=0$, $G=40$, $Pr=1$ and $Ec=0.050875$. The asymmetry effect introduced in the steady-state velocity profile by considering the viscosity as a temperature dependent variable is clearly visualized in Fig. 3a. Some lost of adherence between results provide by two approaches are also observed, mainly for high values of viscosity parameter, *a*.

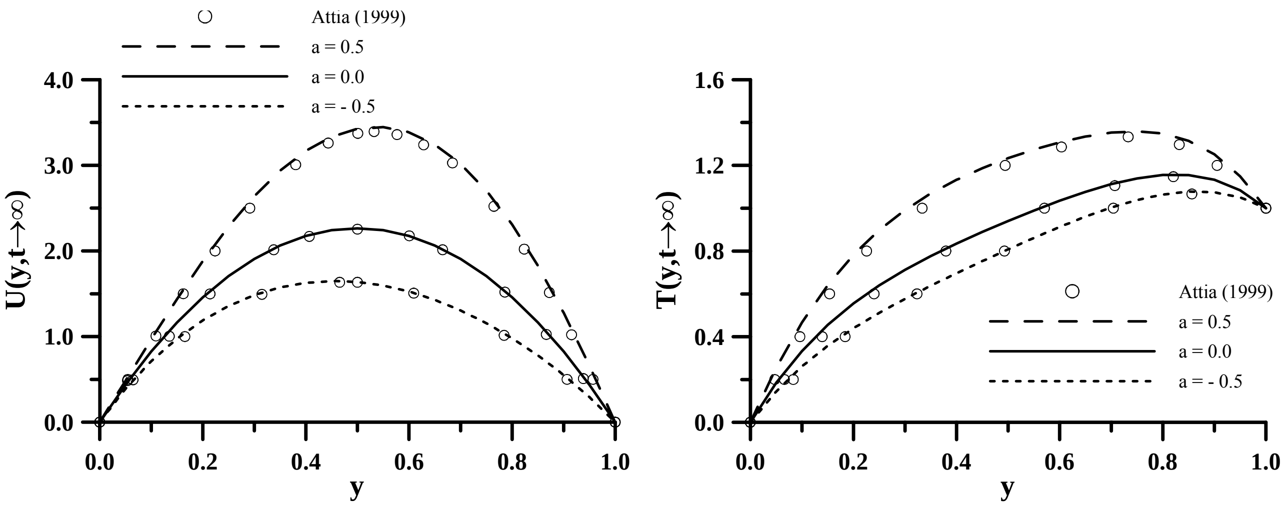

Figure 3. Influence of the viscosity parameter, *a*, on the steady-state
(a) centerline velocity and (b) centerline temperature

Tables 5 and 6 illustrate the time variation of the centerline velocity and centerline temperature, respectively, through a comparison of the present converged results against previous reported numerical results by Attia (1999). Simulations were performed by employing different values of the viscosity parameter, *a*, and two values of Hartman number ($Ha=0$ and $Ha=1,0$). Neither movement of the upper plate is permitted to occur nor inflow/outflow through the plates. Flow of a typical fluid ($Pr=1$ and $Ec=0.050875$) is sustained by a negative pressure gradient, $G=40$.

Table 5. The time variation of the centerline velocity, $u(0,t)$, for different values of the viscosity parameter, a, and Hartman number, Ha. ($R_u=0$, $R_v=0$, $G=40$, $Pr=1$ and $Ec=0.050875$)

Ha=0										
	a= -0.5		*a= -0.2*		*a= -0.1*		*a=0*		*a=0.5*	*a=1.0*
T	Present	Attia (1999)	Present	Attia (1999)	Present	Attia (1999)	Present	Attia (1999)	Present	Present
0.5	1.566	1.593	1.679	1.714	1.715	1.752	1.749	1.789	1.898	2.012
1.0	1.792	1.797	2.082	2.091	2.182	2.192	2.281	2.294	2.769	3.205
2.0	1.762	1.765	2.158	2.160	2.313	2.315	2.481	2.483	3.528	4.801
3.0	1.755	1.758	2.154	2.156	2.317	2.318	2.498	2.498	3.832	6.022
4.0	1.754	1.758	2.154	2.156	2.316	2.318	2.500	2.499	3.967	7.112
5.0	1.754	---	2.154	---	2.316	---	2.500	---	4.030	8.168
10.0	1.754	---	2.154	---	2.316	---	2.500	---	4.085	14.42
∞	1.754	---	2.154	---	2.316	---	2.500	---	4.086	∞
Ha=1										
	a= -0.5		*a= -0.2*		*a= -0.1*		*a=0*		*a=0.5*	*a=1.0*
t	Present	Attia (1999)	Present	Attia (1999)	Present	Attia (1999)	Present	Attia (1999)	Present	Present
0.5	1.297	1.313	1.383	1.405	1.410	1.433	1.435	1.461	1.549	1.634
1.0	1.393	1.397	1.578	1.583	1.640	1.646	1.702	1.709	2.007	2.278
2.0	1.366	1.369	1.589	1.592	1.671	1.674	1.758	1.760	2.255	2.827
3.0	1.362	1.366	1.587	1.589	1.671	1.673	1.760	1.762	2.304	3.041
4.0	1.362	1.366	1.587	1.589	1.670	1.673	1.760	1.762	2.315	3.130
5.0	1.362	---	1.587	---	1.670	---	1.760	---	2.317	3.169
10.0	1.362	---	1.587	---	1.670	---	1.760	---	2.318	3.198
∞	1.362	---	1.587	---	1.670	---	1.760	---	2.318	3.199

Table 6. The time variation of the centerline temperature, $T(0,t)$, for different values of the viscosity parameter, a, and Hartman number, Ha. ($R_u=0$, $R_v=0$, $G=5$, $Pr=1$ and $Ec=1$)

Ha = 0											
	a= -0.5		*a= -0.2*	*a= -0.1*		*a=0*		*a=0.5*		*a=1.0*	
t	Present	Attia (1999)	Present	Present	Attia (1999)	Present	Attia (1999)	Present	Attia (1999)	Present	Attia (1999)
0.5	0.3798	0.395	0.3745	0.3728	0.388	0.3713	0.386	0.3646	0.379	0.3596	0.374
1.0	0.6566	0.665	0.6616	0.6627	0.673	0.6635	0.675	0.6633	0.676	0.6559	0.669
2.0	0.7907	0.793	0.8431	0.8628	0.866	0.8834	0.887	0.9880	0.993	1.055	1.060
3.0	0.7978	0.799	0.8619	0.8887	0.890	0.9189	0.920	1.122	1.122	1.329	1.333
4.0	0.7979	0.799	0.8631	0.8911	0.893	0.9234	0.925	1.181	1.178	1.549	1.548
5.0	0.7979	0.799	0.8631	0.8913	0.893	0.9239	0.925	1.208	1.203	1.743	1.738
10.0	0.7979	---	0.8631	0.8913	---	0.9240	---	1.231	---	2.642	---
∞	0.7979	---	0.8631	0.8913	---	0.9240	---	1.232	---	∞	---
Ha = 1											
	a= -0.5		*a= -0.2*	*a= - 0.1*		*a=0*		*a=0.5*		*a=1.0*	
t	Present	Attia (1999)	Present	Present	Attia (1999)	Present	Attia (1999)	Present	Attia (1999)	Present	Attia (1999)
0.5	0.4208	0.436	0.4224	0.4230	0.439	0.4236	0.439	0.4271	0.443	0.4305	0.447
1.0	0.6983	0.704	0.7229	0.7314	0.737	0.7400	0.746	0.7831	0.789	0.8231	0.829
2.0	0.8098	0.809	0.8697	0.8926	0.889	0.9171	0.914	1.062	1.053	1.231	1.215
3.0	0.8160	0.815	0.8816	0.9077	0.904	0.9362	0.931	1.124	1.110	1.391	1.363
4.0	0.8163	0.815	0.8823	0.9088	0.905	0.9380	0.933	1.137	1.122	1.458	1.422
5.0	0.8163	0.815	0.8824	0.9089	0.905	0.9381	0.933	1.140	1.125	1.486	1.447
10.0	0.8163	---	0.8824	0.9089	---	0.9381	---	1.141	---	1.508	---
∞	0.8163	---	0.8824	0.9089	---	0.9381	---	1.141	---	1.508	---

According to Tab. 5, unless the forth digit, the present results adhere very well with those of Attia (1999) for all situations analyzed. As one can see from this table and from Figs. 4a and 4b, increasing the Hartmann number cause a deceleration on the flow, since the electromagnetic force, contrary to movement, is augmented. On the other hand, Tab. 6 and Figs. 5a and 5b show clearly that differently from the velocity, the temperature field is less influenced to Hartmann number. Some small discrepancies between the present results and those of Attia (1999) are again visualized. It is believed that as authors did not showed a grid convergence history and since the present results are fully converged in all digits showed, they may be considered as reference results.

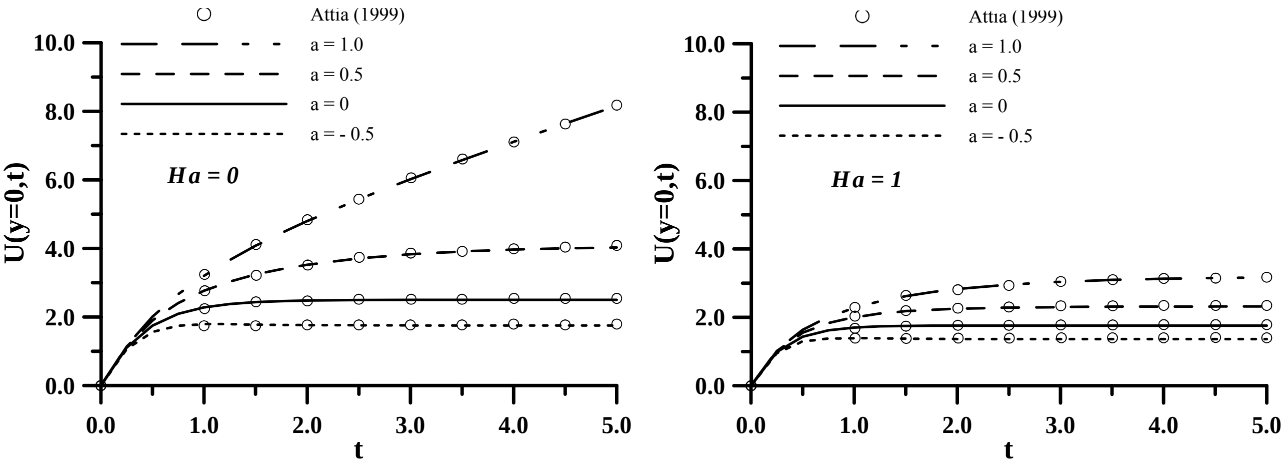

Figure 4. Influence of the viscosity parameter and Hartmann number, (a) *Ha=0* and (b) *Ha=1,* on the evolution of the centerline velocity

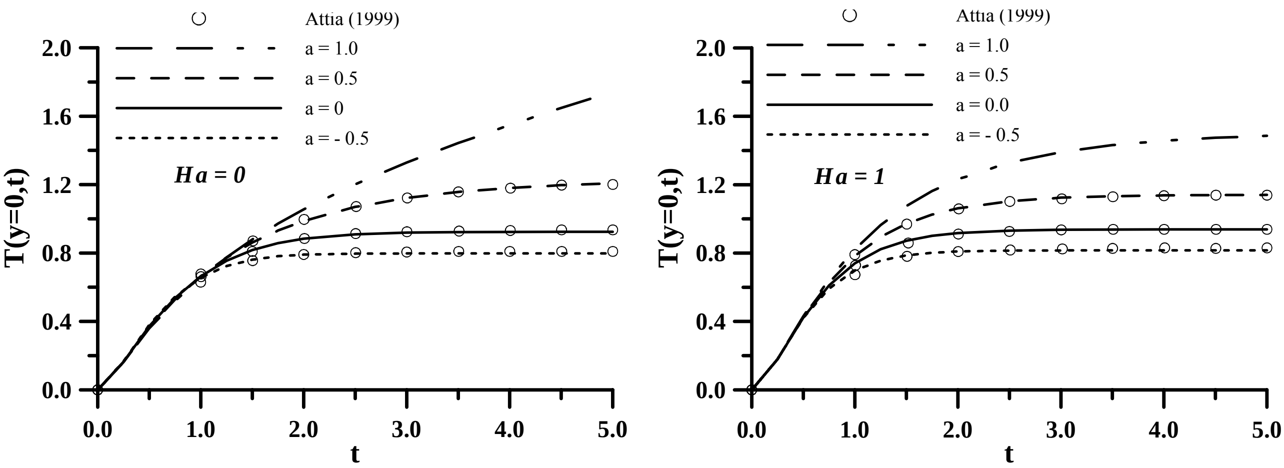

Figure 5. Influence of the viscosity parameter and Hartmann number, (a) *Ha=0* and (b) *Ha=1,* on the evolution of the centerline temperature

Another interesting feature illustrated by Tab. 6, but not visualized in Fig. 5b, is the slightly opposite behavior of the temperature with Hartmann number and viscosity parameter. For negative values of the viscosity parameter, an increase in Hartmann number (from 0 to 1) leads to an increase in the temperature field in all time instants, whereas for positive values of *a*, an increase in *Ha* leads to a decrease in the temperature field, notably for times reaching the steady-state regime.

5. CONCLUSION

The proposed integral transform approach provided reliable and cost-effective simulations of the unsteady and steady-state one-dimensional MHD flow with heat transfer in the laminar flow of

Newtonian fluid within parallel-plate channels. Benchmark results for the centerline velocity and centerline temperature were systematically tabulated and graphically presented for different values of Hartmann and viscosity parameter, illustrating the effectiveness of the present methodology. Comparisons with previous work in the literature were also performed, demonstrating excellent agreement and furnishing direct validations of the present results as well as showing that they were consistent.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

Alpher, R.A., 1961, “Heat Transfer in Magnetohydrodynamic Flow Between Parallel Plates”, Int. J. Heat Mass Transfer, Vol. 3, pp. 108-112.

Attia, H.A., 1999, “Transient MHD Flow and Heat Transfer Between Two Parallel Plates With Temperature Dependent Viscosity”, Mechanics Research Communications, Vol. 26, pp. 115-121.

Attia, H.A. and Kotb, N.A., 1996, “MHD Flow Between Two Parallel Plates With Heat Transfer”, Acta Mechanica, Vol. 117, pp. 215-220.

Cotta, R.M., 1993, “Integral Transforms in Computational Heat and Fluid Flow”, CRC Press, Boca Raton, FL, USA.

Cotta, R.M., 1998, “The Integral Transform Method in Thermal and Fluids Sciences and Engineering", Begell House Inc., NY, USA.

Nigam, S.D. and Singh, S.N., 1960, “Heat Transfer by Laminar Flow Between Parallel Plates Under the Action of Transverse Magnetic Field”, Quart. Journ. Mech. and Applied Math., Vol. 13, pp. 85-97.

Santos, C.A.C., Quaresma, J.N.N. and Lima, J.A., 2001, “Convective Heat Transfer in Ducts: The Integral Transform Approach”, E-Papers, Rio de Janeiro, Brasil.

Sutton, G.W. and Sherman, A., 1965, “Engineering Magnetohydrodynamics”, McGraw-Hill, NY, USA.

Rao, P.S. and Rao, J.A., 1988, “Numerical Solution of Non-Steady Magnetohydrodynamic Flow of Blood Through a Porous Channel”, J. Biomedical Eng., Vol. 10, pp. 293-295.