

# SENSITIVITY CURVES OF A LEAK DETECTION SYSTEM BASED ON VOLUME BALANCE: THEORY AND FIELD RESPONSES

**Alberto Herranz Lucas**

Laboratory of Theoretical and Applied Mechanics (LMTA), Mechanical Engineering Graduate Program (PGMEC), Universidade Federal Fluminense [herranz@mec.uff.br](mailto:herranz@mec.uff.br)

**José Henrique Carneiro de Araujo**

Laboratory of Theoretical and Applied Mechanics (LMTA), PGMEC – Computer Science Department, Universidade Federal Fluminense [jhca@ic.uff.br](mailto:jhca@ic.uff.br)

**Felipe Bastos de Freitas Rachid**

Laboratory of Theoretical and Applied Mechanics (LMTA), PGMEC – Mechanical Engineering Department, Universidade Federal Fluminense [rachid@mec.uff.br](mailto:rachid@mec.uff.br)

**Maria Laura Martins-Costa**

Laboratory of Theoretical and Applied Mechanics (LMTA), PGMEC – Mechanical Engineering Department, Universidade Federal Fluminense [laura@mec.uff.br](mailto:laura@mec.uff.br)

**Abstract.** *This work presents a theoretical development to derive a sensitivity curve for a leak detection system (LDS) in pipelines based on integrated mass balance. The uncertainties in the mass balance equation terms give rise to a leak detection criterion to predict the sensitivity curve. The algorithm also accounts for packing and unpacking along the line enabling its operation at unsteady regimes. Based on flowrate and pressure data measured by SCADA system at the pipeline ends, the pressure is continuously evaluated through the solution of mass and momentum conservation equations so that the linefill variation can be computed. The theoretical and field response sensitivity curves for a specified leak scenario are plotted together and compared. The obtained results indicates the efficiency of the leak detection system algorithm, and also shows the need of a validation modulus capable to identify the operational regime of the pipeline for enhancing its performance.*

**Keywords:** *Leak detection system, sensitivity curves, integrated mass balance, leak detection criterion, linefill correction.*

## 1. INTRODUCTION

Compensated volume balance is a type of leak detection system currently employed in pipelines around the world. Such kind of methodology infers the existence of a leak based on the balance of mass inside the pipeline segment. It can operate during steady state as well as transient regimes and is usually referred to as volume balance systems with linefill correction (Petherick and Pietsch, 1994).

The performance of a software-based leak detection system is evaluated according to the four following parameters (API 1155, 1995): reliability, accuracy, robustness and sensitivity. Reliability is related to the probability of detecting a leak, given that a leak does in fact exist, and the probability of incorrectly declaring a leak, given that no leak has

occurred. Accuracy is related to the precision with which the system estimates the leak parameters such as leak flow rate and leak location. Robustness is defined as a measure of the system to operate and provide useful information under degraded conditions. Finally, the sensitivity is a composite measure of the leak size that a system is capable of detecting and the time required for the system to detect it. The relation between leak size and response time is known as sensitivity curve and depends upon the nature of the leak detection system. Some types of leak detection systems have a peculiar sensitivity curve, such as the methods based on volume balance. On the other hand, other methodologies may even not possess a well-defined sensitivity curve such as those based on pattern recognition techniques (Liou and Tian, 1995).

The main objective of this paper is to explore the sensitivity curve of a Leak Detection System (LDS) in order to better identify among their features those susceptible to improvement. A theoretical expression aiming to express the sensitivity curve of a software-based compensated volume balance leak detection system, for both permanent and transient regimes, is presented, being subsequently compared with experimental data.

## 2. LEAK DETECTION SYSTEM BASED ON VOLUME BALANCE

The compensated volume balance equation is, essentially, the mass conservation principle for an instrumented pipeline segment, written in terms of a standard volume. Considering any time instant  $t \in (-\infty, +\infty)$ , it may be expressed as:

$$\frac{d}{dt}\hat{V}(t) + \hat{Q}_o(t) - \hat{Q}_i(t) = -\hat{Q}_l(t) \quad (1)$$

in which  $\hat{V}(t)$  represents the standard volumetric linefill,  $\hat{Q}_o(t)$  and  $\hat{Q}_i(t)$  the standard volumetric flowrates at the outlet and inlet of the pipeline segment and  $\hat{Q}_l(t)$  is the standard volumetric leak rate. All these variables have suffered pressure and temperature correction. The linefill and outlet and inlet flowrates corrected values are given by

$$\hat{V}(t) = \frac{1}{\mathbf{r}_0} \int_0^{\hat{L}(t)} \hat{\mathbf{r}}(x,t) \hat{A}(x,t) dx, \quad \hat{Q}_o(t) = \frac{1}{\mathbf{r}_0} \hat{\mathbf{r}}(x,t) \hat{q}_o(t) \text{ and } \hat{Q}_i(t) = \frac{1}{\mathbf{r}_0} \hat{\mathbf{r}}(x,t) \hat{q}_i(t) \quad (2)$$

where  $\hat{L}(t)$  and  $\hat{A}(x,t)$  denote the pipeline segment length and cross sectional area,  $\hat{\mathbf{r}}(x,t)$  the mass density of the fluid(s) inside the pipeline segment,  $\mathbf{r}_0$  a reference density, at 15°C and 1 atmosphere and, finally,  $\hat{q}_o(t)$  and  $\hat{q}_i(t)$  represent the volumetric flowrates at the outlet and inlet. The always nonnegative term  $\hat{Q}_l(t)$  is the standard volumetric leak rate, being zero only in non-leaking condition.

The estimated values of the quantities  $\hat{V}(t)$ ,  $\hat{Q}_o(t)$ ,  $\hat{Q}_i(t)$ ,  $\hat{L}(t)$ ,  $\hat{A}(x,t)$  and  $\hat{\mathbf{r}}(x,t)$  – available at selected successive time instants differing by a scan rate – are denoted by  $V(t)$ ,  $Q_o(t)$ ,  $Q_i(t)$ ,  $L(t)$ ,  $A(x,t)$  and  $\mathbf{r}(x,t)$ .

Integrating Eq. (1) between  $t - \mathbf{t}$  (with  $\mathbf{t} > 0$  denoting the time window) and the current time instant  $t$ , the following function may be defined:

$$F(t, \mathbf{t}) = \left| V(t) - V(t - \mathbf{t}) + \int_{t-\mathbf{t}}^t [Q_o(\mathbf{x}) - Q_i(\mathbf{x})] d\mathbf{x} \right| \quad (3)$$

Since the values  $V(t)$ ,  $Q_o(t)$  and  $Q_i(t)$  may present errors, the function defined by Eq. (3) is not expected to be identically zero, even in non-leaking conditions.

The values of  $L(t)$ ,  $A(x, t)$  and  $\mathbf{r}(x, t)$  are temperature and pressure-dependent, so, these two properties must be known along the line segment at any time instant in order that accurate values for  $V(t)$  may be obtained. As a consequence, the position of existing interface batches along the line and appropriate equations of state must also be known. Compensated volume balance leak detection systems make use of computational transient fluid flow models (based on the numerical solution of the continuity, momentum and energy equations along with suitable boundary conditions at the inlet and outlet of the pipeline segment) to estimate the linefill. Different models and algorithms have been proposed and analyzed in API 1149 (1993) and in Thompson and Skogman (1984).

A compensated volume balance leak detection criterion is automatically obtained whenever the uncertainty  $\mathbf{d}F(t, \mathbf{t})$  – associated with the computation of Eq. (3) – is known. If, for any time window  $\mathbf{t} > 0$  (in practice  $\mathbf{t} \geq \Delta t$  with  $\Delta t$  representing the employed scan rate) and any time instant  $t \in (-\infty, +\infty)$  the following inequality holds,

$$F(t, \mathbf{t}) < \mathbf{d}F(t, \mathbf{t}) \quad (4)$$

then it may be concluded, with the same confidence level used to obtain  $\mathbf{d}F(t, \mathbf{t})$ , that there is no leak in the instrumented pipeline segment. The term  $\mathbf{d}F(t, \mathbf{t})$  represents the total uncertainty – arising from inlet and outlet flowrate measurements and from the linefill evaluation at  $t - \mathbf{t}$  and  $t$ .

Assuming  $F$  as function of the independent quantities  $V(t - \mathbf{t})$ ,  $V(t)$ ,  $Q_o(t)$  and  $Q_i(t)$  for all  $t \in (-\infty, +\infty)$ , there is no mathematical error associated with the evaluation of the integral in Eq. (3); there is no uncertainty associated with the time instants  $t - \mathbf{t}$  and  $t$  and also the uncertainties associated with the flow measurements at the inlet and at the outlet are time independent, then the root-sum-square process (Moffat, 1988) may be used, leading to (Freitas Rachid et al, 2002)

$$\mathbf{d}F(t, \mathbf{t}) = \sqrt{[\mathbf{d}V(t)]^2 + [\mathbf{d}V(t - \mathbf{t})]^2 + [\mathbf{t} \mathbf{d}Q_o]^2 + [\mathbf{t} \mathbf{d}Q_i]^2} \quad (5)$$

with  $\mathbf{d}V(t)$  and  $\mathbf{d}V(t - \mathbf{t})$  representing the linefill uncertainties at time instants  $t$  and  $t - \mathbf{t}$ ,  $\mathbf{d}Q_o$  and  $\mathbf{d}Q_i$  denoting the uncertainties associated with the flow measurement equipment at the outlet and inlet, respectively.

The criterion presented in Eq. (4) states that an existing leak is reliably detected when the inequality (4) is no longer satisfied for any time instant  $t \geq T$  and for a particular value of the time window  $\mathbf{t}$ .

It is remarkable that, in general, uncertainties associated with flow measurements (as well as pressure and temperature) depend not only on the quality of the instrumentation at the pipeline segment but also on the time instant, since the instrumentation behavior changes due to its usage and to the fluid present in the pipeline. In the present work the time dependence of  $dF(t, \mathbf{t})$  has been assumed to be caused by the uncertainty associated with changes in the linefill only.

### 3. THEORETICAL SENSITIVITY CURVE OF A LEAK DETECTION SYSTEM

The connection between the actual standard volumetric balance of a leaking line - Eq. (1) with  $\hat{Q}_l(t) = 0$  - and the leak detection criterion of a typical compensated volume balance leak detection system, Eq. (4), is ultimately established when the time instant  $t = T$  spent by the software to announce a leak is identified. So, let us suppose that a leak has occurred at  $t = t^*$  and has been detected at  $t = T$ , with  $T > t^*$ . Thus,  $\mathbf{t} = T - t^*$  and the following relationship may be written (Freitas Rachid et al, 2002):

$$\bar{\hat{Q}}_l = \frac{dF(T, T - t^*)}{T - t^*} = \sqrt{(dQ_o)^2 + (dQ_i)^2 + \frac{[dV(T)]^2 + [dV(T - t^*)]^2}{(T - t^*)^2}} \quad (6)$$

where  $\bar{\hat{Q}}_l$  denotes a mean value of the leak flow rate from the time it has been initiated to the time it has been detected.

Equation (6) represents the sensitivity curve of compensated volume balance leak detection systems. It defines the behavior of the leak size  $\bar{\hat{Q}}_l$  as a function of the elapsed time  $T - t^*$  required by the software to detect it in both permanent and transient fluid flow regimes.

Transient fluid flow regimes are classified according to its severity  $I(t)$ , whose definition is given by

$$I(t) = \frac{|Q_o(t) - Q_i(t)|}{Q_r} \quad (7)$$

where  $Q_r$  is a flow rate reference value.

Following the idea presented by Liou in API 1149 (1993), two limiting cases are of interest here. The first one appears when  $Q_l$  is equal to the reference volumetric flow rate in the segment line, i. e.  $Q_l = Q_r$ . In such a case, the equality in (15) can be used to determine the minimum response time  $T = T_m$ . Since, according to the foregoing considerations, after a leak of this nature (a leak of magnitude equal to 100%) has occurred the fluid flow regime in the segment line is highly transient - with a transient severity probably equal to or greater than  $\ddot{e}_-$  for a sufficiently long period of time - it comes that  $\ddot{a}V(T_m) = [\ddot{a}V]_{max}$  and consequently,

$$T_m = \sqrt{\frac{(dV_{\max})^2 + [dV(0)]^2}{(Q_r)^2 - (dQ_o)^2 - (dQ_i)^2}} \quad (8)$$

Equation (17) reveals that the minimum response time depends on the linefill uncertainty at the time instant the leak has begun ( $t = 0$ ). Since, by hypothesis, the linefill uncertainty depends on the flow regime in the segment line, then it becomes evident that there will exist a lower and an upper bound for the minimum response time. The lower bound  $T_{Lm}$  will be obtained when at the time instant the leak has occurred ( $t = 0$ ), the line segment is at steady state (the transient severity is less than or equal to  $\ddot{a}\ddot{e}$ );

$$T_m^L = \sqrt{\frac{(dV_{\max})^2 + [dV_{\min}]^2}{(Q_r)^2 - (dQ_o)^2 - (dQ_i)^2}} \quad (9)$$

Accordingly, the upper bound value of the minimum response time  $T_{Um}$  will prevail when at the time instant the leak has occurred ( $t = 0$ ), the line segment is undergoing a severe transient (the transient severity is equal to or greater than  $\ddot{e}_-$ ). In such a situation,  $\ddot{a}V(0) = [\ddot{a}V]_{\max}$  and from (17) we obtain:

$$T_m^U = \frac{\sqrt{2} \, dV_{\max}}{\sqrt{(Q_r)^2 - (dQ_o)^2 - (dQ_i)^2}} \quad (10)$$

The second limiting case arises when the limit of (15) is taken as the elapsed time approaches infinity, i. e.  $T \rightarrow \infty$ . In this case, the equality in (15) can be solved for  $\bar{Q}_l$  in order to determine the size of the minimum detectable leak

$$\left(\bar{Q}_l\right)_{\min} = \sqrt{(dQ_o)^2 + (dQ_i)^2} \quad (11)$$

which is essentially the overall uncertainty of the flow measurement in the segment line. Contrary to the previous limiting case, the minimum detectable leak is indifferent to the fluid flow regime in the segment line at the time the leak has initiated. Such a feature has already been noticed in other works, such as in (Petherick and Pietsch, 1994).

Based upon the past considerations, the qualitative behavior of the sensitivity curve can be depicted in a plot  $\bar{Q}_l$  against  $T$ , as illustrated ahead in Figures 1 and 2. It should be emphasized that the analysis presented herein not only allows the determination of the sensitivity curve but also furnishes the bounds of its location, as given by Eq. (14).

#### 4. DESCRIPTION OF THE LEAK DETECTION SYSTEM

The leak detection algorithm requires the computation of the volumetric flowrates at the inlet and outlet of the instrumented pipeline segment as well as uncertainties at the time instants  $t$  and  $t - \tau$ .

The procedure employed to build the present leak detection system (Lucas, 2003) was based on the worst possible scenario – namely considering transient flow regime. Since, in this case, there are more parameters to account for than in steady-state flow, the uncertainty is considerably larger. This may be better understood by considering the following brief description of the computation of linefill uncertainty.

Basically, the linefill uncertainty at a given time instant  $t_j$  is given by the following expression

$$dV(t_j) = \left\{ \left[ \frac{\partial V}{\partial T}(t_j) d\bar{T} \right]^2 + \sum_{i=1}^{n+1} \left[ \frac{\partial V}{\partial p_{x_i}}(t_j) dp_{x_i} \right]^2 \right\}^{1/2} \quad (12)$$

in which  $dT$  denotes the uncertainty associated with the fluid temperature measuring device. In this expression  $p_{x_i}$  represents the pressure at a spatial point  $x_i$  and  $dp_{x_i}$  its uncertainty, both considered at the instant  $t_j$ , the latter being given by

$$dp(x_i, t_j) = \left\{ \left[ \frac{\partial p}{\partial T}(x_i, t_j) d\bar{T} \right]^2 + \left[ \frac{\partial p}{\partial H}(x_i, t_j) dH(x_i, t_j) \right]^2 + \left[ \frac{\partial p}{\partial h}(x_i, t_j) dh(x_i, t_j) \right]^2 \right\}^{1/2} \quad (13)$$

where  $d\bar{T}$  represents the fluid temperature uncertainty,  $dH(x_i, t_j)$  is the head loss uncertainty and  $dh(x_i, t_j)$  is the uncertainty associated to with the topographic height. All these quantities have been evaluated according to the position of the point  $(x_i, t_j)$  in the characteristics grid.

For short, once the pressure and flow rate at the pipeline ends are made available by the SCADA, these data are used as initial data for the mass and momentum conservation equations to estimate the pressure profile  $p(x_i, t_j)$  inside the pipeline, which is then employed to compute the linefill in the criterion given by Eq. (4). To perform this task, numerical simulations using the method of characteristics is used as described in detail in Lucas, 2003. In the specific case of the head loss, for any internal point, the uncertainties are given by (Lucas, 2003):

$$dH(x_i, t_j) = \frac{1}{2} \left\{ \left( \frac{\partial \mathbf{a}_A^{(k)}}{\partial H^*} dH_A^{(k)} \right)^2 + \left( \frac{\partial \mathbf{a}_B^{(k)}}{\partial H^*} dH_B^{(k)} \right)^2 + \left( \frac{H_r}{q_r} \frac{\partial \mathbf{a}_A^{(k)}}{\partial q^*} dq_A^{(k)} \right)^2 + \left( \frac{H_r}{q_r} \frac{\partial \mathbf{a}_B^{(k)}}{\partial q^*} dq_B^{(k)} \right)^2 + \left[ H_r \left( \frac{\partial \mathbf{a}_A^{(k)}}{\partial R} + \frac{\partial \mathbf{a}_R^{(k)}}{\partial R} \right) dR \right]^2 \right\}^{1/2} \quad (14)$$

It is remarkable that equation (14) requires the computation not only of the flow rate uncertainty but also of the uncertainties of the friction factor  $f$  and wave speed  $a$ , since the uncertainty of the parameter  $R$  is given by

$$dR = \left\{ \left( \frac{\partial R}{\partial f} df \right)^2 + \left( \frac{\partial R}{\partial a} da \right)^2 \right\}^{1/2} \quad (15)$$

where the uncertainties  $df$  and  $da$  are given by

$$df = \left\{ \left( \frac{\partial f}{\partial \bar{H}_i} d\bar{H}_i \right)^2 + \left( \frac{\partial f}{\partial \bar{H}_o} d\bar{H}_o \right)^2 + \left( \frac{\partial f}{\partial \bar{q}_i} d\bar{q}_i \right)^2 + \left( \frac{\partial f}{\partial \bar{q}_o} d\bar{q}_o \right)^2 \right\}^{1/2} \quad (16)$$

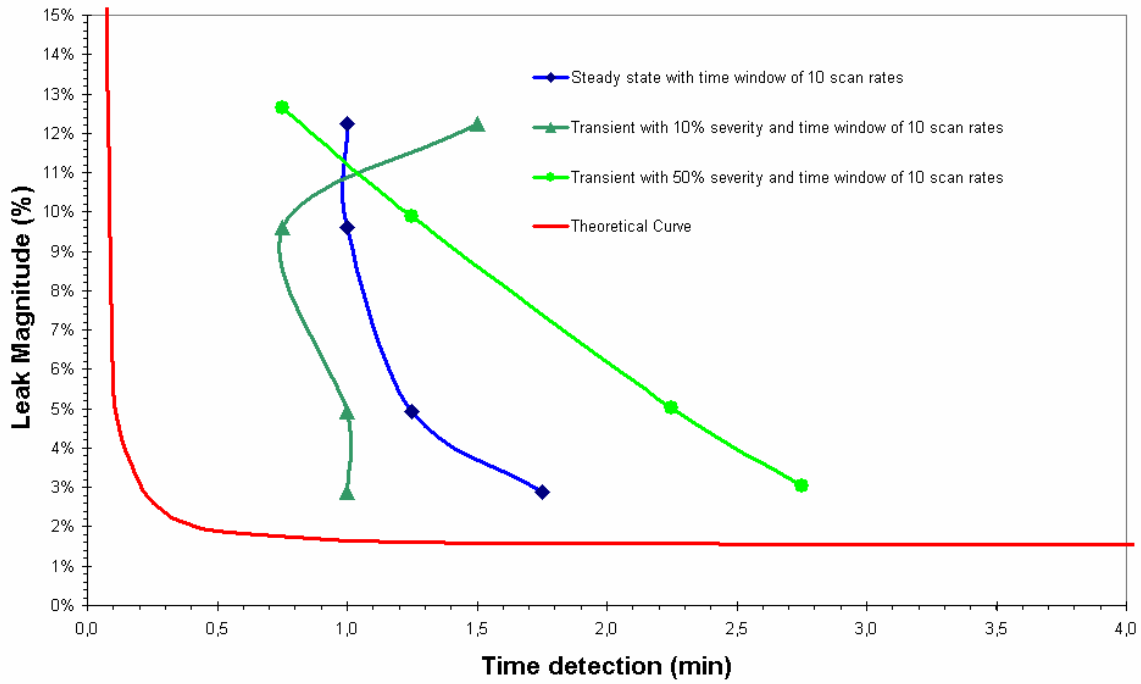
$$da = \left\{ \left( \frac{\partial a}{\partial T} (\bar{T}, \bar{p}, \mathbf{r}_o) d\bar{T} \right)^2 + \left( \frac{\partial a}{\partial p} (\bar{T}, \bar{p}, \mathbf{r}_o) d\bar{p} \right)^2 \right\}^{1/2}$$

The overall linefill uncertainty given by Eq.(12) along with equations (13-16) is computed at each SCADA time stamp, so that the leak detection criterion established through (4) can be evaluated. Although the process of computing this uncertainty be long, it does not take so much computation time and so can be implemented on real time. Due to the limited number of pages of this paper, the readers should be referred to the work of Lucas (2003) for more details.

## 5. PRACTICAL SENSITIVITY CURVE OF THE LEAK DETECTION SYSTEM

The sensitivity curve of a leak detection system (LDS) is a key parameter in the analysis of the LDS performance. Considering a previously described LDS based on volume balance, as well as given pipeline configuration, data acquisition system, flowing fluid, leak position and transient severity, the sensitivity curve of a LDS relates the leak magnitude to the time required for its detection.

Figures 1 and 2 present sensitivity curves for a leak detection system whose main features were presented in past section. To allow a direct comparison, the theoretical sensitivity curve given by expressions presented in section 3 is also plotted in these figures. Figures 1 and 2 refers to leaks placed at 15 km and 35 km, measured from the entrance of the pipeline segment. Both figures represent the sensitivity curves for three distinct pipeline operation regimens – steady state and transient with 10% and 50% severity. The instrumented segment line is 50km long and is used to transport crude oil with a nominal flow rate of 222m<sup>3</sup>/h. Leaks of magnitude ranging from 2.0% to 13% of the nominal flow rate were generated at the sites mentioned above within a scenario characterized by the closing and opening of two valves positioned 3km away of the pipeline segment extremities. The whole leak scenario was numerically simulated with the aid of a off-line software. Pressure and flow rate are made available at the inlet and outlet at every other 15 seconds. To ascribe the data a typical character of the SCADA, noise of random nature was artificially inserted on it. Linefill uncertainty estimated according to the expressions presented on the past section is approximately constant and of the order of 0.014 Sm<sup>3</sup>. Overall uncertainties of flow rate instruments are estimated as being 2.21 and 2.65 Sm<sup>3</sup>/h for inlet and outlet , respectively.

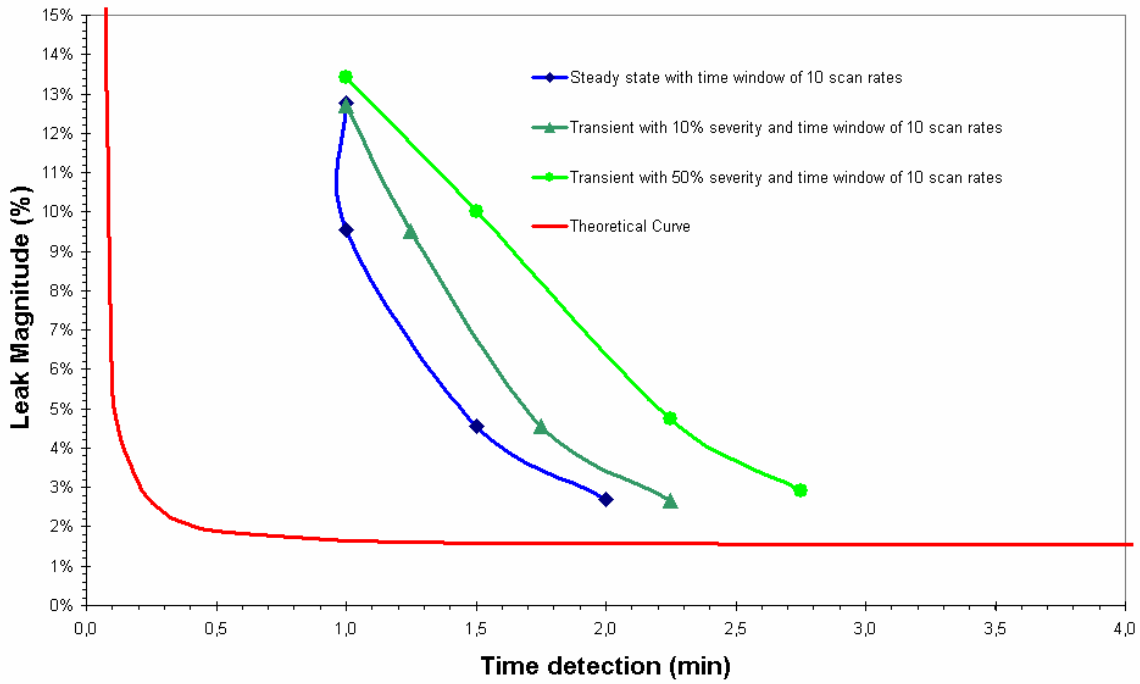


**Figure 1: Theoretical and field sensitivity curves at 15 km of the pipeline entrance.**

Regardless the leak position, it may be observed from figures 1 and 2 that the smaller the leak is, the greater the time required for its detection. This behavior is apparent not only in the theoretical curve but also in the field response. As it can be observed, the minimum detectable leak in filed response (2.67%) is greater than the one associated to the theoretical curve (1.55%). Spurious oscillations (noise) introduced intentionally in the flow and pressure data do not allow the SDV to detect leaks whose magnitude is inferior to 2.67%. It explains the reason for not detecting the leaks of magnitude between 2.0 and 2.6% that were generated.

Another feature to be noted between the theoretical and filed responses is that first curve is right-shifted in relation to the second one. This behavior is intrinsically related to the necessity of the SDV to require a number of scan rates ahead of the current time instant in order to reconstruct the pressure field at this instant, and so estimate the linefill variation. In this particular case, it corresponds to approximately six scan rates what is equivalent to 1.5 minute.





**Figure 2: Theoretical and field sensitivity curves at 35 km of the pipeline entrance.**

An important feature of the sensitivity curves of a leak detection system is their relative position, considering distinct operational regimes. In the present work, both the linefill and its uncertainty are computed considering a transient flow regime – or, in other words,  $dV(t) = dV_{\max} \sim \text{cte}, \forall t$ . As a consequence, the sensitivity curves referring to the considered LDS do not present a significant variation as the operational regime is altered (either steady-state or moderate transient or severe transient). This may be noticed by observing figures 1 and 2 in which no pattern related to the relative position of the curves representing these three flow regimes – namely steady-state and transient with 10% and 50% severity – is identified. It should be emphasized, however, that in general the greater the transient severity, the longer the time required detecting a leak. Such a behavior is associated with the inaccuracy with which the linefill variation is computed during transient events. To reduce the number of false alarms, the SDV is set to announce a leak after a certain elapsed time interval. The analysis carried out clearly demonstrates that if the linefill uncertainty were computed separately for steady and unsteady regimes, leaks taking place at the former regime could be detected in a shorter time. However, such a task would require a validation modulus to identify the regime the line is being subjected before a leak detection algorithm is launched. This strategy is being implemented so that the leak detection performance can be enhanced.

## 6. CONCLUDING REMARKS

It has been presented in this paper a theoretical development which allows the determination of the sensitivity curve of a general compensated volume balance leak

detection system. Besides of its determination, the analysis gives the bounds of its location as a function of the flow rate measurement and linefill uncertainties. Field response to a leak scenario involving leaks of 2 to 13%, under steady and transient regimes, are used to build the sensitivity curve of a SDV recently constructed. The comparison carried out between the theoretical and field sensitivity curves clearly demonstrates the need to compute the linefill uncertainty not only during transient events but also at steady state, assigning the SDV a distinguished behavior during these regimes.

## 7. ACKNOWLEDGEMENTS

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