BUBBLE FREE OSCILLATIONS IN LIQUIDS

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Abstract: *A brief discussion about the modified Rayleigh-Plesset equations is first presented here with the purpose to choose the better form of the equation to study the oscillations of a bubble containing a mixture of water vapour and air in a viscous compressible liquid. Once decided by a Keller form equation, it is applied to compressible water, taking into account the physical properties of water and air. The equations are derived and then solved using the finite difference method to model the oscillations of the bubble wall, considering that the process is adiabatic. Radius of the bubble versus time curves are plotted where oscillations of diminishing period are observed. Such oscillations occur until the bubble reaches an equilibrium radius. Such attenuation of the oscillations are caused by dissipation mechanisms such as viscosity and liquid compression.*

Keywords: *bubble, oscillations, cavitation.*

1. INTRODUCTION

Since Rayleigh (1917), many equations have been proposed to explain the collapse and growth of a spherical bubble in a liquid. They are, in fact, modifications of the classical Rayleigh-Plesset equation (Brennen, 1995). An analysis of these further equations (or modified Rayleigh-Plesset equations) was made by Prosperetti; Lezzi (1986), searching for the best equation or the best set of equations.

In the classical Rayleigh-Plesset equation (Hammitt, 1980), it is described the behaviour of a spherical bubble of radius *R* (as a function of time *t*) in an infinite domain of liquid where the pressure far from the bubble (and made as a constant) is P_∞. The surrounding liquid is considered as incompressible, the bubble contents is assumed to be homogeneous, and the temperature and the pressure within the bubble are always uniform. A good approximation is to disregard mass and heat transfer across the bubble because the collapse process is very fast. According to Brennen (1995), with such considerations, the bubble will oscillate indefinitely. A simple deduction of the Rayleigh-Plesset equation can be found in Hammitt (1980), and in Brennen (1995).

Fujikawa; Akamatsu (1980) made analytical and numerical analysis of the behaviour of the bubble, taking into account condensation of the water vapour, heat conduction, and temperature discontinuity at the phase interface. The bubble contents was considered to obey the perfect gas law. Although this is a very complex model, no significant differences was found when disregarding mass and heat transfer across the bubble wall, as can be seen in Bazanini (2003). Besides, to solve a full set of radial equations for the conservation of mass, momentum and energy in the bubble and in the surrounding liquid would be a huge computation. Recently, a work that has the purpose to investigate efficient methods of incorporating heat and mass transfer effects for spherical bubbles with the aim to reduce computation time was made by Preston et al. (2001). Even though, the authors assumed that the perfect gas law holds for the mixture of air and water vapour, and that the liquid is incompressible. It seems such an unnecessary complication of the problem, since maintains assumptions as incompressible liquid and perfect gas law, what will not be done in the present work, because of the importance of the liquid compressibility and real gas assumption in the process, as can be seen in Brennen (1995), and Bazanini (2001). Let us now focus on the modified Rayleigh-Plesset equations, which take into account the liquid compressibility and the physical properties of the fluids involved in the phenomenum.

Perhaps the most important work about the bubble behaviour since Rayleigh is the one by Gilmore (1952). Working with the liquid enthalpy, it takes into account the compressibility of the surrounding liquid through the use of the sonic velocity in the Rayleigh-Plesset equation. Every important work since then is somehow based on it, like the ones by Trilling (1952), Keller; Kolodner (1956), Prosperetti; Lezzi (1986), and Löfstedt et al. (1993), among others.

The bubble contents (vapour and air) compression and expansion during the oscillations can be treated as isothermal or adiabatic, although adiabatic is a more realistic assumption because of the rapidity of each collapse and growth (Young, 1989).

2. EQUATIONS OF BUBBLE BEHAVIOUR

A comparative analysis of many modified Rayleigh-Plesset equations such as those by Herring (1941), Trilling (1952), and Keller; Kolodner (1956), among others, was made by Prosperetti; Lezzi (1986). Most of them were clearly influenced by Gilmore's report (1952). The conclusion was that these equations are entirely equivalent and form a family of equations having the same degree of accuracy, being the Keller form equation (Keller; Kolodner, 1956) slightly more accurate. Another conclusion is that the liquid compressibility is important, especially when thermal effects are unimportant (that is, in our case), and because in the final stages of collapse there are high velocities and pressures (Brennen, 1995 and Bazanini, 2003a).

Based on the above analysis, a Keller form equation as below Eq. (1) shall be used here to model the bubble oscillations. But now viscous and surface tension effects will be considered in the present work, in Eq. (2).

$$\left(1-\frac{R'}{C}\right)RR''+\frac{3}{2}\left(1-\frac{R'}{3C}\right)R'^2=\frac{1}{\rho_L}\left[\left(1+\frac{R'}{C}\right)P_L+\frac{R}{C}\frac{dP_L}{dt}-\left(1+\frac{R'}{C}\right)P_\infty\right] \tag{1}$$

where R is the bubble radius, C is the sonic velocity, ρ_L is the liquid density, P_∞ is the pressure far from the bubble, and P_L is the liquid pressure at the bubble wall as below. The prime denotes time derivative.

$$P_L=\frac{P_{g_0}\,R_0^{\,3K_g}}{\left(R^3-a_g^3\right)^{K_g}}+\frac{P_{v_0}\,R_0^{\,3K_v}}{\left(R^3-a_v^3\right)^{K_v}}-\frac{2S}{R}-\frac{4\left(\mu_g+\mu_L\right)}{R}R' \tag{2}$$

where P_{g0} is the initial gas pressure, P_{v0} is the initial vapour pressure, R_0 is the initial bubble radius, S is the surface tension, and μ_g and μ_L are the dynamic viscosity of gas and liquid, respectively.

As already discussed in the Introduction, it will be considered here the adiabatic process for the bubble contents, including the effect of the van der Waals hard core a_g and a_v for for gas and vapour, respectively (see Barber; Putterman, 1991). In fact, gas and vapour are being considered to obey the van der Waals equation of state for real gases. This is important because of the raising pressures within the bubble during the collapse.

To use Eq. (1), it is first necessary to derive Eq. (2) respect to time. Then results for the term dP_L/dt the following Eq. (3):

$$\frac{dP_L}{dt} = -3R^2R'\left[\frac{K_g P_{g_0} R_0^{3K_g}}{\left(R^3 - a_g^3\right)^{1+K_g}} + \frac{K_v P_{v_0} R_0^{3K_v}}{\left(R^3 - a_v^3\right)^{1+K_v}}\right] + \frac{2SR'}{R^2} - 4(\mu_g + \mu_L)\left(\frac{RR'' - R'^2}{R^2}\right) \tag{3}$$

3. RESULTS

Equations (2) and (3) are used in Eq. (1), which is solved using the finite difference method in an explicit time integration scheme. The liquid considered is the water, and the gases within the bubble are the air (that is always present on the formation of the bubble, since the bubble nucleate from microbubbles of air, as described in Hammitt, 1980), and the water vapour. The heat and mass transfer through the bubble wall are disregarded, and the process is considered as adiabatic, that is a better assumption than the isothermal one (Bazanini et al., 1998). For R_0/a_g, and R_0/a_v, are used the values 8.54 and 10.79, respectively (Barber et al., 1997). For the sonic velocity is used the value 1481 m/s (Löfstedt et al., 1993).

Oscillations of the bubble wall in water using Eqs. (1) to (3) are shown in Figs. (1) to (4). Figures (1) and (2) are for a initial bubble radius of 3.56 mm, the same used by Knapp; Hollander (1948); initial and boundary conditions are also the same, that is: P_{g0}= 40 Pa; P_{v0}= 2,340 Pa; P_∞= 27,579 Pa. From Fig. (1) it is possible to see the attenuation of the oscillations with time. Figure (2) has the time scale enlarged to see the convergence to the equilibrium radius. It can be seen that the bubble will reach an equilibrium radius of approximately 1.9 mm in a period of time of about 300 ms.

Figures (3) and (4) are for another initial and boundary conditions, for comparison purposes. It was now used a smaller bubble radius, very common in more recent works. The initial and boundary conditions are the same used by Fujikawa; Akamatsu (1980), that is: R_0=1.0 mm; P_{g0}= 702.5 Pa; P_{v0}= 2,305 Pa; P_∞= 70,250 Pa. Unfortunately, in their model, that takes into account the condensation of the vapour and heat conduction at the phase interface, the authors did not let the bubble oscillate, modelling the two first collapses only. Once again, Fig. (3) shows the beginning of the attenuation process, and Fig. (4) is for enlarged time scale.

One can see in Fig. (4) that the bubble reaches its equilibrium radius of approximately 0.54 mm in a period of time of about 80 ms.

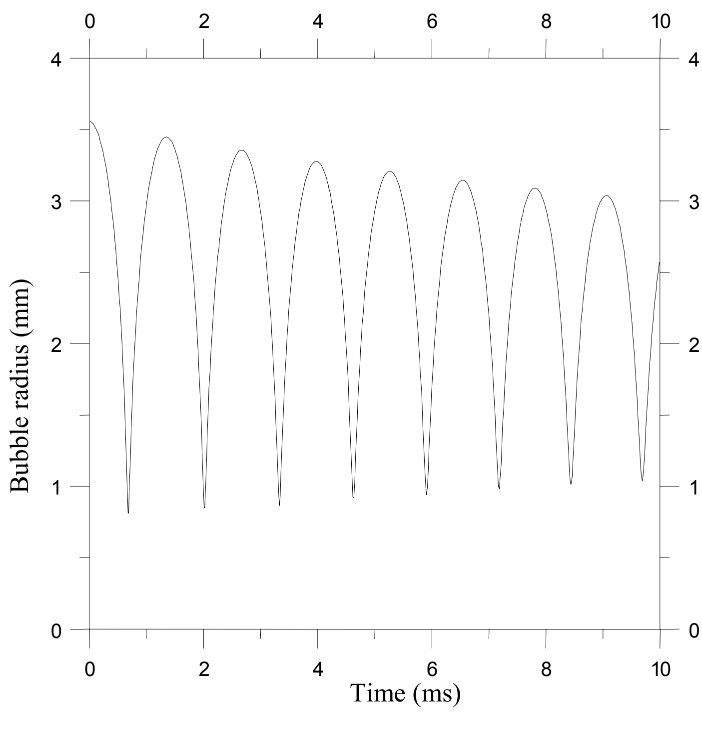

Figure 1. Bubble radius versus time for initial bubble radius of 3.56 mm.

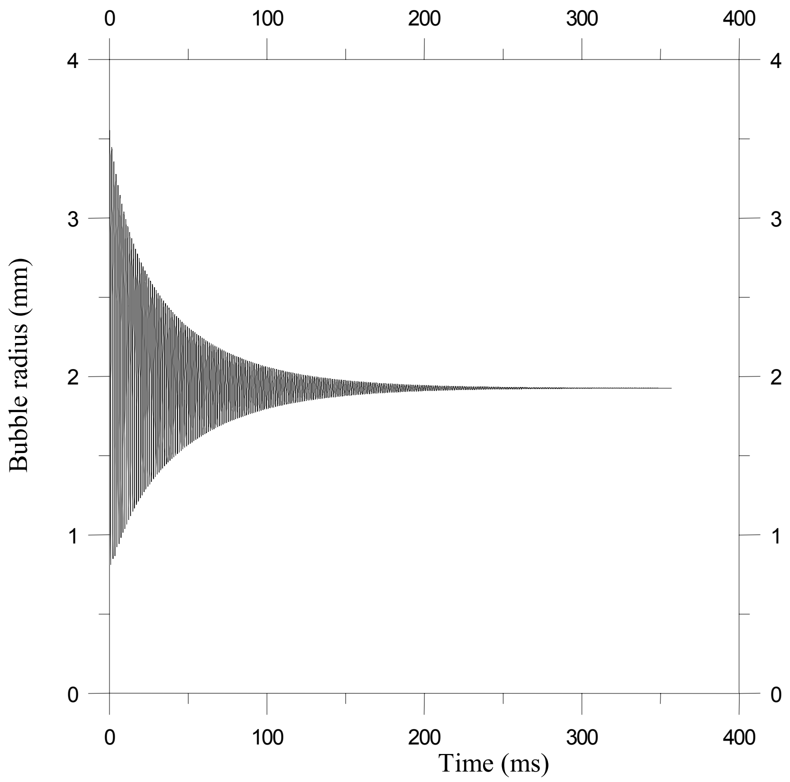

Figure 2. Bubble radius versus time for initial bubble radius of 3.56 mm.

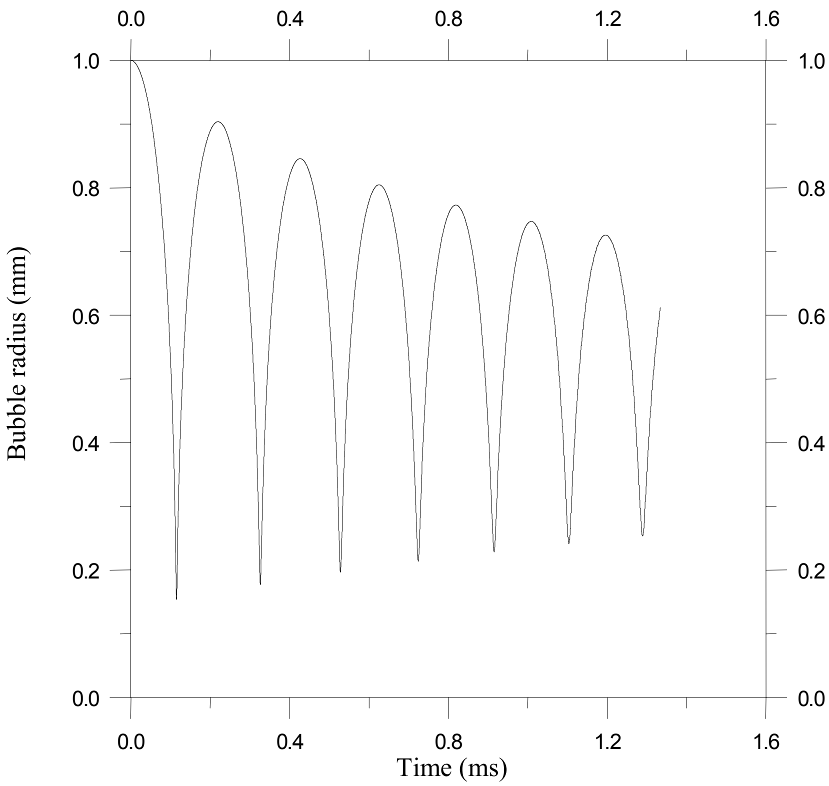

Figure 3. Bubble radius versus time for initial bubble radius of 1.0 mm.

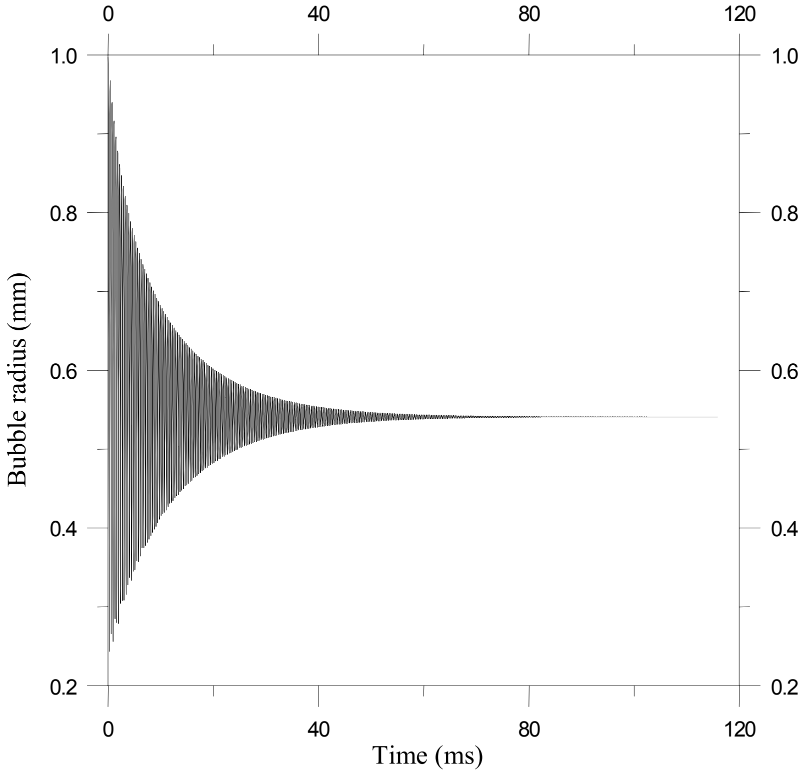

Figure 4. Bubble radius versus time for initial bubble radius of 1.0 mm.

4. CONCLUSIONS

The attenuation of the bubble oscillations is observed in Figs. (1) to (4). The Figs. (1) and (3) are useful to see the beginning of them, for different initial bubble radius. This attenuation is due to dissipation mechanisms such as liquid viscosity and compressibility, resulting in oscillations of diminishing period. The viscosity acts at the bubble surface as a brake, whether the bubble is expanding or contracting, and some energy is expended to compress the surrounding liquid. This energy expenses leads the bubble to an equilibrium radius. Without any dissipation mechanisms the bubble would oscillate indefinitely, as demonstrated by Brennen (1995) for the Rayleigh-Plesset equation.

The observed equilibrium radii in Figs. (2) and (4) are very close to half of the respective initial bubble radii. It was obtained an equilibrium radius of 1.9 mm for an initial bubble radius of 3.56 mm, and an equilibrium radius of 0.54 mm for an initial radius of 1.0 mm.

As expected, a bubble of a greater radius (and besides, submitted to a smaller external pressure P_∞) should take more time to reach its equilibrium radius. That could be seen in Figs. (2) and (4).

In the present work, only the free oscillations of the bubble were treated. But it is also possible to study forced oscillations by using a term of driving pressure in a modified Rayleigh-Plesset equation, in an attempt to control bubble oscillations, as made by Löfstedt et al. (1993) and by Gumerov (2000).

5. REFERENCES

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OSCILAÇÕES LIVRES DE BOLHAS EM LÍQUIDOS

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Resumo: *Inicialmente é apresentada uma breve discussão a respeito das equações modificadas de Rayleigh-Plesset com o objetivo de escolher a melhor forma da equação para estudar as oscilações* *de uma bolha contendo uma mistura de ar e vapor d'água em um líquido viscoso compressível. Uma vez decidido por uma equação na forma da de Keller, a mesma é aplicada à água, levando em conta as propriedades físicas dos fluidos envolvidos no processo. As equações são derivadas e depois resolvidas utilizando o método das diferenças finitas para modelar as oscilações da parede da bolha, considerando o processo como adiabático. Figuras na forma raio da bolha versus tempo são obtidas, onde são observadas oscilações de período decrescente, o que ocorre até que a bolha atinja um raio de equilíbrio. Estas atenuações das oscilações são devido a mecanismos de dissipação, tais como viscosidade e compressão do líquido.*

Palavras-chave: *bolha, oscilações, cavitação.*