A FINITE ELEMENT FORMULATION FOR INVISCID COMPRESSIBLE FLOWS

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Abstract. In this paper, we describe a space-time finite element numerical formulation for the solution of the compressible Euler equations written in their conservation form. Two supersonic flow applications are analysed and the numerical outputs are compared to analytical solutions and to results found in the literature, and good agreement is found. The algorithm is based on a space-time approach and on a simplified SUPG stabilizing matrix. The methodology used has formed a basis for future works in three dimensions and other sets of variables.

Keywords. Euler equations, Compressible flows, SUPG.

1. INTRODUCTION

It is well know that the Euler equations possess some peculiar aspects, not easily dealt with, in the context of Finite Element Methods. First, a Petrov-Galerkin method must be used in order to stabilise the numeric negative diffusion that appears with pure Galerkin weighting. Also, at very high speed flows, the presence of shocks is inevitable, rendering the solution strong discontinuities and, therefore, requiring additional localised numeric dissipation. Last, the resulting system of linearised equations which must be solved through several interactions gets ill conditioned as the flow speed lowers down. These three aspects just mentioned have motivated a lot of research work through the last decades. In this paper, we choose to use the SUPG[1] method for solving the set of compressible Euler equations written in its conservative form, with a simplified version of the stabilising matrix. A computer code written in FORTRAN 90 has been developed and some results are shown.

2. SPACE-TIME FORMULATION

Consider the Euler equations:

$$\frac{\partial \boldsymbol{U}}{\partial t}+\frac{\partial \boldsymbol{F_k}}{\partial x_k}=\mathbf{0}\ ;\ k=1\ldots nd \qquad (1)$$

1 SUPG : Strealine-Upwind Petrov- Galerkin

In equation (1), $\boldsymbol{U}$ represents the set of conservation variables, $\boldsymbol{F_k}=\boldsymbol{F_k}(\boldsymbol{U})$ is the k[th] convective flux vector and "*nd"* is the number of spatial dimensions. A solution $\boldsymbol{U}=\boldsymbol{U}(t,\boldsymbol{x})$ is sought in the space-time domain $Q=\Omega\times[0,T]$. Expressions for $\boldsymbol{U}$ and $\boldsymbol{F_k}$ can be found in the literature (see, for example, Hirsch,1988)

Let $\hat{Q}$ be an approximate partition of the Q, such that $\hat{Q}$ is, actually, a union of $N-1$ *space-time slabs*, as shown in Fig.(1). With this choice of partitioning, the spacial domain Ω_n is approximately discretized into $\hat{\Omega}_n$ and the time domain $[0,T]$ is equally divided into $N-1$ open intervals: $I_n=]t_{n.},t_{n+1}[$. Notice that a triangular discretization is used to exemplify. It follows that we can define the n[th] *space-time slab* as (Shakib, 1988):

$$\hat{Q}_n=\hat{\Omega}_n\times I_n \tag{2}$$

With $\hat{Q}_n$ given by Eq. (2), the discretized dominion, $\hat{Q}$, is given by:

$$\hat{Q}=\left(\bigcup_{n=0}^{N-1}\hat{Q}_n\right)\cup\{t_{0,}t_1\ldots t_N\} \tag{3}$$

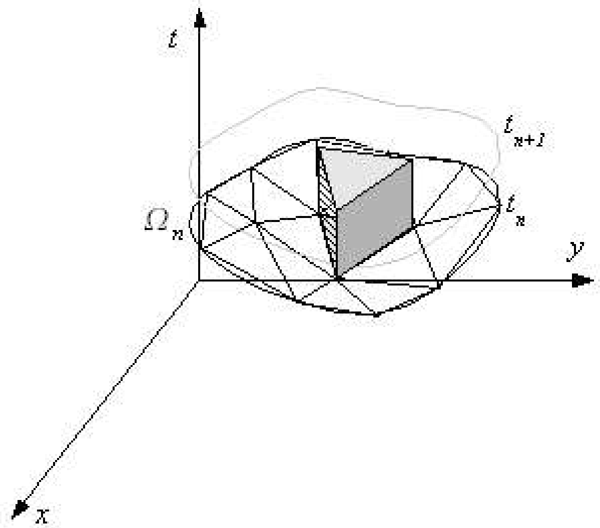

Figure 1. A space-time slab.

Figure (1) also shows a prismatic element $\hat{Q}^e_n=\hat{\Omega}^e_n\times I_n$. Let us denote the border of the spatial domain $\hat{\Omega}_n$ by $\hat{\Gamma}_n$ and the corresponding element's border by $\hat{\Gamma}^e_n$; the space-time borders at level *n* will likewise be labeled $\hat{P}_n$ and $\hat{P}^e_n$. One should also notice that the spatial domain partitions need not have the same pattern at each time step *n*. This feature makes space-time methods specially well suited for meshes which deform with time (Aliabadi and Tezduyar, 1995).

2.1 A constant-in-time approach

Let $\hat{\boldsymbol{U}}\in S^h_n$ and $\boldsymbol{W}\in V^h_n$ where S^h_n is the set of trial functions and V^h_n is the space of weighting functions given by:

$$S_n^h = \{\hat{\boldsymbol{U}} \mid \hat{\boldsymbol{U}} \in \left[C^0\left(\hat{Q}_n\right)\right]^m, \hat{\boldsymbol{U}}|_{\hat{Q}_n^e} \in \left[p_k\left(\hat{\Omega}_n^e\right)\right]^m, \boldsymbol{q}(\hat{\boldsymbol{U}}) = \boldsymbol{g}(t), \forall \hat{\boldsymbol{U}} \in \left(\Gamma_n \times I_n\right)\} \tag{4}$$

$$V_n^h = \{\boldsymbol{W} \mid \boldsymbol{W} \in \left[C^0\left(\hat{Q}_n\right)\right]^m, \boldsymbol{W}|_{\hat{Q}_n^e} \in \left[p_k\left(\hat{\Omega}_n^e\right)\right]^m, \boldsymbol{q}(\boldsymbol{W}) = \boldsymbol{0}, \forall \hat{\boldsymbol{U}} \in \left(\Gamma_n \times I_n\right)\} \tag{5}$$

In equations (4) and (5), m is the number of degrees of freedom per node (four, in the two-dimensional case) and $p_k = p_k(\boldsymbol{x}, t)$ is a polinomial of order k. In the *constant-in-time* approach, we restrict ourselves to $p_k = p_k(\boldsymbol{x})$. The variational formulation of equation (1) can then be stated as: within each $\hat{Q}_n$ ($n = 0, \ldots N-1$), find $\hat{\boldsymbol{U}} \in S_n^h$ such that for all $\boldsymbol{W} \in V_n^h$ the following equation is satisfied:

$$\begin{aligned}\int_{\hat{Q}_n} \left[-\boldsymbol{W}_{,t} \cdot \hat{\boldsymbol{U}} - \boldsymbol{W}_{,k} \cdot \boldsymbol{F_k}(\hat{\boldsymbol{U}})\right] dQ + \int_{\hat{\Omega}_n} \{\left[\boldsymbol{W}\left(t_{n+1}^-\right)\right] \cdot \left[\hat{\boldsymbol{U}}\left(t_{n+1}^-\right)\right] - \left[\boldsymbol{W}\left(t_n^+\right)\right] \cdot \left[\hat{\boldsymbol{U}}\left(t_n^+\right)\right]\} d\Omega + \\ \sum_e \int_{\hat{Q}_n^e} \left(\boldsymbol{A_k} \boldsymbol{W}_{,k}\right) \cdot \tau \left(\boldsymbol{A_k} \boldsymbol{U}_{,k}\right) dQ + \sum_e \int_{\hat{Q}_n^e} \delta \boldsymbol{W}_{,k} \cdot \hat{\boldsymbol{U}}_{,k} dQ = -\int_{\hat{P}_n} \boldsymbol{W} \cdot \left[\boldsymbol{F_k}(\hat{\boldsymbol{U}}) n_k\right] dP\end{aligned} \tag{6}$$

with $\boldsymbol{A_k} = \dfrac{\partial \boldsymbol{F_k}}{\partial \boldsymbol{U}}$. Values for $\boldsymbol{A_k}$ can be found in Hirsh, 1990.

In equation (6), the first integral and the integral on the boundary are the Galerkin contributions, the second integral comes from the integration by parts of the time-flux term ($\boldsymbol{W} \cdot \hat{\boldsymbol{U}}_{,t}$) plus the so called *jump condition*; the sums are the SUPG and discontinuity capturing terms, respectively. The role of the jump condition plus considerations on stability can be found in the literature (see, for instance: Bauer, 1995)

In a constant-in-time approach, equation (6) reduces to:

$$\begin{aligned}-\Delta t \int_{\hat{\Omega}_n} \boldsymbol{W}_{,k} \cdot \boldsymbol{F_k}(\hat{\boldsymbol{U}}) d\Omega + \int_{\hat{\Omega}_n} \boldsymbol{W} \cdot \left[\hat{\boldsymbol{U}}\left(t_{n+1}^-\right) - \hat{\boldsymbol{U}}\left(t_n^+\right)\right] d\Omega + \\ \Delta t \sum_e \int_{\hat{\Omega}_n^e} \left(\boldsymbol{A_k} \boldsymbol{W}_{,k}\right) \cdot \tau \left(\boldsymbol{A_k} \boldsymbol{U}_{,k}\right) d\Omega + \Delta t \sum_e \int_{\hat{\Omega}_n^e} \delta \boldsymbol{W}_{,k} \cdot \hat{\boldsymbol{U}}_{,k} d\Omega = \\ -\Delta t \int_{\hat{\Gamma}_n} \boldsymbol{W} \cdot \left[\boldsymbol{F_k}(\hat{\boldsymbol{U}}) n_k\right] d\Gamma\end{aligned} \tag{7}$$

where $\Delta t = t_{n+1} - t_n$.

3. FINITE ELEMENT DISCRETIZATION

The solution $\hat{\boldsymbol{U}}$ in equation (7) can be piecewisely approximated at the time levels *n* and *n+1* as:

$$\hat{\boldsymbol{U}}^{\boldsymbol{n}} = \sum_{j=1}^{Nnodes} \phi_j \hat{\boldsymbol{u}}_{\boldsymbol{j}}^{\boldsymbol{n}}, \quad \hat{\boldsymbol{U}}^{\boldsymbol{n}+1} = \sum_{j=1}^{Nnodes} \phi_j \hat{\boldsymbol{u}}_{\boldsymbol{j}}^{\boldsymbol{n}+1} \tag{8}$$

where $\phi = \phi(\boldsymbol{x})$ represents the usual linear *shape function. Nnodes* stand for the number of nodes in the spatial discretized domain.

The weighting functions, likewise, are written as:

$$\boldsymbol{W}^{\boldsymbol{n}}=\sum_{j=1}^{Nnodes}\phi_j\boldsymbol{w}_{\boldsymbol{j}}^{\boldsymbol{n}} \tag{9}$$

Substitution of equations (8) and (9) into (7) ultimately gives:

$$\begin{aligned}-\Delta t\int_{\hat{\Omega}_n}\phi_{i,k}\boldsymbol{F}_{\boldsymbol{k}}\left(\hat{\boldsymbol{U}}^{\boldsymbol{n}}\right)d\,\Omega+\int_{\hat{\Omega}_n}\phi_i\phi_j\left(\hat{\boldsymbol{u}}_{\boldsymbol{j}}^{\boldsymbol{n+1}}-\hat{\boldsymbol{u}}_{\boldsymbol{j}}^{\boldsymbol{n}}\right)d\,\Omega\\ \Delta t\sum_e\int_{\hat{\Omega}_n^e}\tau^{\boldsymbol{T}}\left(\phi_{i,k}\boldsymbol{A}_{\boldsymbol{k}}\right)\left(\boldsymbol{A}_{\boldsymbol{k}}\phi_{j,k}\hat{\boldsymbol{u}}_{\boldsymbol{j}}^{\boldsymbol{n}}\right)d\,\Omega+\Delta t\sum_e\int_{\hat{\Omega}_n^e}\delta\,\phi_{i,k}\phi_{j,k}\hat{\boldsymbol{u}}_{\boldsymbol{j}}^{\boldsymbol{n}}d\,\Omega=\\ -\Delta t\int_{\hat{\Gamma}_n}\phi_i\left[\boldsymbol{F}_{\boldsymbol{k}}(\hat{\boldsymbol{U}}^{\boldsymbol{n}})n_k\right]d\,\Gamma\\ i,j=1\ldots Nnodes\,;\;\;k=1\ldots nd\end{aligned} \tag{10}$$

Of course, the matrices built through Eq. (10) are to be assembled element by element, following the usual finite element procedure. One can then reformulate Eq.(10) in an element-wise manner. Using a matricial notation we get:

$$\boldsymbol{M}_e\left(\hat{\boldsymbol{u}}_{\boldsymbol{e}}^{\boldsymbol{n+1}}-\hat{\boldsymbol{u}}_{\boldsymbol{e}}^{\boldsymbol{n}}\right)+\Delta t\left(-\boldsymbol{V}_{\boldsymbol{e}}^{\boldsymbol{f}}+\boldsymbol{V}_{\boldsymbol{e}}^{\boldsymbol{fpg}}+\boldsymbol{V}_{\boldsymbol{e}}^{\boldsymbol{fdc}}\right)-\Delta t\,\boldsymbol{F}_{\boldsymbol{eb}}^{\boldsymbol{ab}}=\boldsymbol{0} \tag{11}$$

Note that we dropped the superindex *n* for the sake of clarity. We should keep in mind, therefore, that $\boldsymbol{V}_{\boldsymbol{e}}^{\boldsymbol{f}},\boldsymbol{V}_{\boldsymbol{e}}^{\boldsymbol{fpg}},\boldsymbol{V}_{\boldsymbol{e}}^{\boldsymbol{fdc}}$ and $\boldsymbol{F}_{\boldsymbol{eb}}^{\boldsymbol{ab}}$ are calculated at time level *n*. Matrix $\boldsymbol{M}_{\boldsymbol{e}}$ is the consistent mass matrix and the terms $\boldsymbol{V}_{\boldsymbol{e}}^{\boldsymbol{f}}$, $\boldsymbol{V}_{\boldsymbol{e}}^{\boldsymbol{fpg}}$ and $\boldsymbol{V}_{\boldsymbol{e}}^{\boldsymbol{fdc}}$ correspond, respectively, to the convective flux vector, the SUPG correction to the convection flux vector and the discontinuity capturing operator vector. The last term $\boldsymbol{F}_{\boldsymbol{eb}}^{\boldsymbol{ab}}$ is related to the convective flux through the boundaries.

Equation (11) gets somewhat simplified under the assumption that the element fluxes and matrices are constant and functions of the arithmetic average of $\hat{\boldsymbol{u}}_{\boldsymbol{e}}$ over the element's connectivities (Catabriga, 2000). This line of thought is adopted herein. If follows that in the corresponding integrals:

$$\boldsymbol{A}_{\boldsymbol{k}}^{\boldsymbol{e}},\boldsymbol{F}_{\boldsymbol{k}}^{\boldsymbol{e}},\tau,\delta^e=f\left(\frac{1}{nc}\sum_{i=1}^{nc}\hat{\boldsymbol{u}}_{\boldsymbol{e}}^{\boldsymbol{i}}\right) \tag{12}$$

where *nc* is the number of connectivities of the element.

With the assumption made in Eq. (12) and considering linear shape functions we obtain:

$$\boldsymbol{M}_e\left(\hat{\boldsymbol{u}}_{\boldsymbol{e}}^{\boldsymbol{n+1}}-\hat{\boldsymbol{u}}_{\boldsymbol{e}}^{\boldsymbol{n}}\right)=\left[\int_{\Omega_e}\phi_i\phi_j\,d\,\Omega\right]\left(\hat{\boldsymbol{u}}_{\boldsymbol{e}}^{\boldsymbol{n+1}}-\hat{\boldsymbol{u}}_{\boldsymbol{e}}^{\boldsymbol{n}}\right);\;\;i,j=1\ldots nc \tag{13}$$

$$\boldsymbol{V}_{\boldsymbol{e}}^{\boldsymbol{f}}=-\left[\Delta t\,\phi_{i,k}\int_{\Omega_e}d\,\Omega\right]\boldsymbol{F}_{\boldsymbol{k}}^{\boldsymbol{e}};\;\;k=1\ldots2\,;\;\;i=1\ldots nc \tag{14}$$

$$V_e^{fpg}=\left[\Delta t \tau^T(\phi_{i,k} A_k)(A_k \phi_{j,k})\int_{\Omega_e} d\Omega\right]\hat{u}_e^n\ ;\ k=1\ldots nd\ ;\ i,j=1\ldots nc \tag{15}$$

$$V_e^{fdc}=\left[\Delta t\, \delta\, \phi_{i,k}\phi_{j,k}\int_{\Omega_e} d\Omega\right]\hat{u}_e^n\ ;\ k=1\ldots nd\ ;\ i,j=1\ldots nc \tag{16}$$

$$F_{eb}^{ab}=-\left[\Delta t\int_{\Gamma_e}\phi_i\, d\Gamma\right](F_k^e n_k)\ ;\ k=1\ldots nd\ ;\ i=1\ldots nc \tag{17}$$

Equations (13) to (17) can be easily evaluated analytically. There's still the SUPG matrix τ and the discontinuity capturing parameter δ which are the subject of the following section.

4. STABILIZATION PARAMETERS

The stabilization matrix can be derived from the SUPG method applied to a scalar advection-diffusion equation. Hughes et.al. (b) (1986), following the work of Hughes and Brooks (1982), proposed an expression for the scalar τ considering a non-transient pure advection problem :

$$\tau=\left[\left(\frac{\partial \epsilon_i}{\partial x_k}\right)a_k^2\right]^{-\frac{1}{2}}\ ;\ i,k=1\ldots nd \tag{18}$$

where ϵ_i are natural coordinates, supposing there is a mapping $(x_i)\rightarrow(\epsilon_i)$.

The extension to advective systems is not straightforward and can be found in Hughes and Mallet (1986). In this case, the stabilizing matrix is given by a similar expression:

$$\tau=\left(\frac{\partial U}{\partial V}\right)\left[\left(\frac{\partial \epsilon_i}{\partial x_k}\right)A_k^2\right]^{-\frac{1}{2}} \tag{19}$$

where V is a set of symmetrizing variables (see Hughes et al. (a), 1986).

Equation (19) can only be evaluated numerically, though, for rectangular elements, one can deduce analytical expressions. Even so, they are much clumsy and complicated. Several simpler alternatives have been proposed which produce equally good results. However, the drawback seems to be an over addition of numerical diffusion to the original system of equations. In this paper, we choose to use a very simple stabilization matrix introduced by Aliabadi and Tezduyar (1995) and also found in Catabriga (2000), where τ is given by:

$$\tau=\tau I \tag{20}$$

where, for the space-time implicit formulation described before, the parameter τ is given by:

$$\tau=\mathbf{max}\left[0,\tau_t+\zeta(\tau_a-\tau_\delta)\right] \tag{21}$$

and:

$$\tau_a=\frac{h}{2(c+|u\cdot\beta|)}\ ;\ \tau_\delta=\frac{\delta}{(c+|u\cdot\beta|)^2}\ ;\ \tau_t=\frac{2\tau_a}{3(1+2Cr)}\ ;\ \zeta=\frac{2}{3}Cr \tag{22}$$

The Courant number (Cr) is given by:

$$Cr=\frac{\left(c+|\boldsymbol{u}\cdot\beta|\right)\Delta t}{h} \tag{23}$$

where c is the sound speed and h is a characteristic dimension of the element. For the case of two-dimensional problems ,we take $h=\sqrt{A_e}$, where A_e is the element's area.

Before writing expressions for β and δ in equations (22), some definitions are convenient. Let $\boldsymbol{Y}$ be a $(nd.m\times 1)$ vector and $\boldsymbol{M}$ a $m\times m$ matrix, let us define the following norms:

$$\|\boldsymbol{Y}\|_{\boldsymbol{M}}=\left\{\boldsymbol{Y}\cdot\left(\begin{bmatrix}\boldsymbol{M} & \cdots & \boldsymbol{0}\\ \vdots & \ddots & \vdots\\ \boldsymbol{0} & \cdots & \boldsymbol{M}\end{bmatrix}\boldsymbol{Y}\right)\right\}^{\frac{1}{2}}\ ;\ \|\boldsymbol{Y}\|=\{\boldsymbol{Y}\cdot\boldsymbol{Y}\}^{\frac{1}{2}} \tag{24}$$

We also define the gradient of the vector $\boldsymbol{Y}$ with respect to the cartesian coordinates x_k :

$$\nabla\boldsymbol{Y}=\begin{bmatrix}\dfrac{\partial\boldsymbol{Y}}{\partial x_1}\\ \vdots\\ \dfrac{\partial\boldsymbol{Y}}{\partial x_{nd}}\end{bmatrix} \tag{25}$$

and with respect to the natural coordinates ϵ_k :

$$\nabla_\epsilon\boldsymbol{Y}=\boldsymbol{J}^{-1}\nabla\boldsymbol{Y} \tag{26}$$

where $\boldsymbol{J}^{-1}$ is given by:

$$\boldsymbol{J}^{-1}=\begin{bmatrix}\dfrac{\partial\epsilon_1}{\partial x_1} & \cdots & \dfrac{\partial\epsilon_1}{\partial x_{nd}}\\ \vdots & \ddots & \vdots\\ \dfrac{\partial\epsilon_{nd}}{\partial x_1} & \cdots & \dfrac{\partial\epsilon_{nd}}{\partial x_{nd}}\end{bmatrix} \tag{27}$$

We chose to use the discontinuity capturing parameter found in Aliabadi and Tezduyar, 1986. Then, δ in equation (22) is given by:

$$\delta=\frac{\left\|\boldsymbol{A_k}\dfrac{\partial\boldsymbol{U}}{\partial x_k}\right\|^2_{\tilde{\boldsymbol{A}}_0^{-1}}}{\|\nabla_\epsilon\boldsymbol{U}\|^2_{\tilde{\boldsymbol{A}}_0^{-1}}}\ ;\ k=1\ldots nd \tag{28}$$

The parameter β is given by:

$$\beta = \frac{\nabla \|\boldsymbol{U}\|^2}{\left\| \nabla \|\boldsymbol{U}\|^2 \right\|} \tag{29}$$

It is worth noticing that the gradient in equation (29) is given by:

$$\nabla \|\boldsymbol{U}\|^2 = \begin{bmatrix} \dfrac{\partial \|\boldsymbol{U}\|^2}{\partial x_1} \\ \vdots \\ \dfrac{\partial \|\boldsymbol{U}\|^2}{\partial x_{nd}} \end{bmatrix}_{nd \times 1} \tag{30}$$

4.1 Numerical implementation

For the solution of the non-linear system of equations given by equation (11), a predictor-multicorrector algorithm has been used. Details of such implementation can be found in Shakib (1986). To solve the resultant system of linear algebraic equations we have used a block-diagonal preconditioned GMRES with reverse communication. The dimension of the Krylov Space was 50 and we assumed a stopping tolerance for the GMRES interactions of 10^{-4}. The code for the GMRES algorithm was obtained from the CERFACS[2] home page.

5. NUMERICAL RESULTS

5.1 Oblique shock wave

A supersonic air flow comes into the unit sided square domain at Mach number M (see Fig. (2)), and is reflected at the bottom wall. At the steady state, a shock is formed at an angle of 29.3º with the horizontal. Density, speed and mach number are prescribed at boundaries 3 and 4.

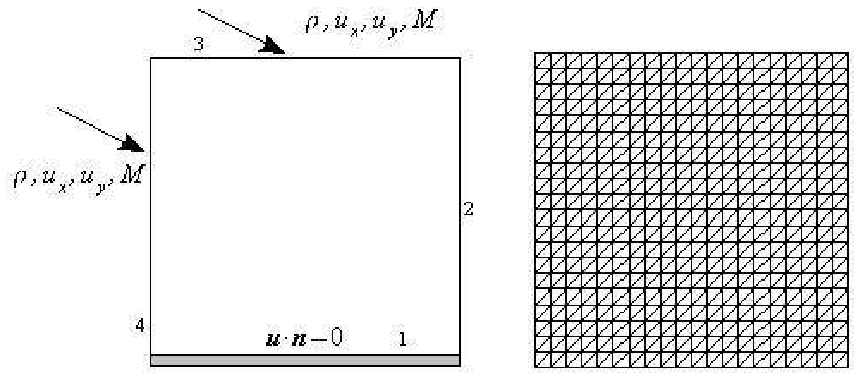

Figure 2 Boundary conditions and spatial domain discretization for the oblique shock problem.

A solid-wall condition ($\boldsymbol{u} \cdot \boldsymbol{n} = 0$) is imposed at boundary 1 and, at boundary 2, the flow is left free. The problem marches from a *free stream initial condition*[3] to a steady state and the boundary conditions at 3 and 4 are:

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2 "European Centre for Research and Advanced Training in Scientific Computation" (http://www.cerfacs.fr)

3 That is, the boundary conditions at walls 3 and 4 are extended to all spatial domain.

$$\rho=1.0\ ;\ u_x=\cos(10^o)\ ;\ u_y=-\sin(10^o)\ ;\ M=2.0 \qquad (31)$$

A structured mesh with 400 elements was used to discretize the spacial domain (Fig. (2)). Figure (3) shows the density and pressure profiles at steady state, using a Courant number of 1.0 for the marching in time (see equation (23)). All graphs show a plane cut, parallel to the vertical boundaries, at x = 0.9. We see that the shock was captured at its correct location though a diffusive behaviour is observed, when comparing the numerical with the analytical solution. Nevertheless, it agrees quite well with other published results (see Catabriga, 2000).

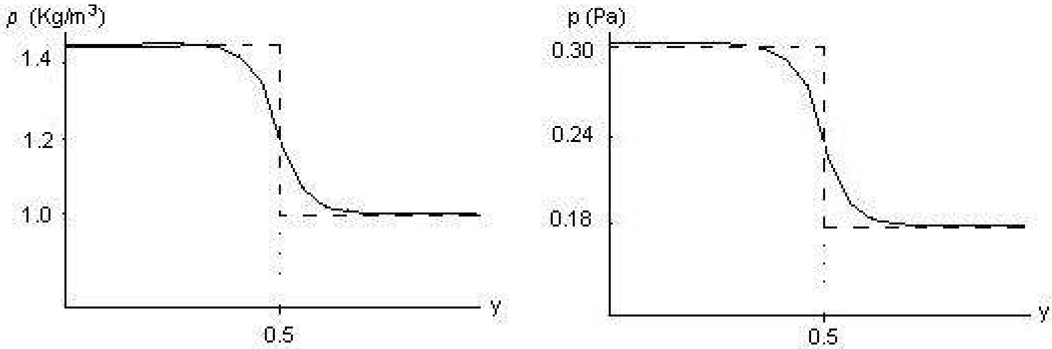

Figure 3. The density and pressure distribution for the oblique shock problem: numerical and analytical solutions.

5.2 Flow around a cylinder

The second example consists on the study of a supersonic flow around a cylinder. We used an unstructured mesh consisting of 2642 elements, somewhat refined in front of the cylinder and in the rarefaction zone. Fig. (4), shows the mesh alongside with the boundary conditions. We assumed a free flow at all the boundaries but the leftmost one, where values are prescribed. Also, a solid-wall condition is imposed on the cylinder's surface.

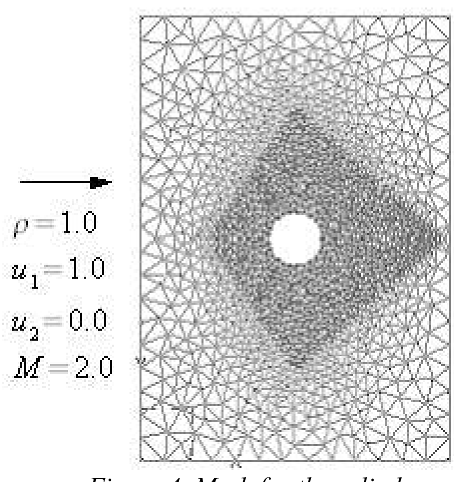

Figure 4. Mesh for the cylinder problem.

A marching in time procedure is again used with the free stream initial conditions. The Courant number adopted was 1.0 and the solutions were plotted after 350 steps. Figure (5) shows the isolines of Mach number, temperature and density at the steady-state. The capturing of the shock is quite evident and there is a good qualitative agreement with results published in the literature for similar problems (see Lyra, 1994).

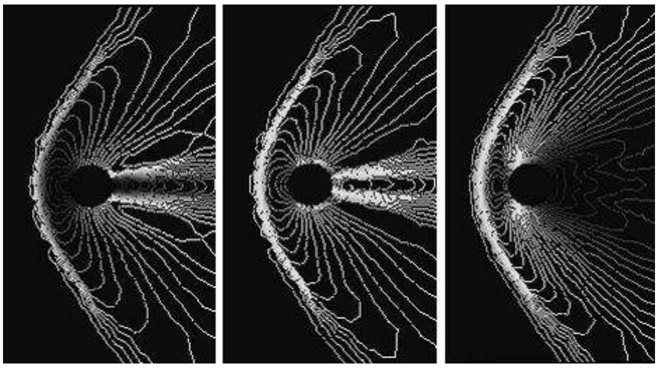

Figure 5. Mach number, temperature and density (Mach 2.0 flow around a cylinder)

6. CONCLUSIONS

A numerical solution for the compressible two-dimensional Euler equations has been implemented and the results have been validated by comparison with analytical and numerical results published in the literature. The implementation is the basis for a three dimensional Navier-Stokes solver which is under conclusion. Three dimensional results for different sets o variables will be the goal of a near future work.

7. ACKNOWLEDGEMENTS

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8. REFERENCES

Aliabadi, S. K., Tezduyar, T. E., 1995, " Parallel Fluid Dynamics Computations in Aerospace Applications", Comp. Meth. Appl. Mech. And Engng., Vol 21, pp. 783-805

Bauer, R. E., 1995, "Discontinous Galerkin Methods for Ordinary Diferential Equations", MSc Thesis, University of Colorado, USA

Catabriga, L., 2000, "Soluções Implícitas das Equações de Euler Empregando Estrutura de Dados por Arestas", Doctoral Thesis, COPPE/ UFRJ, Brazil. (in portuguese)

Hirsch, C., 1988, " Numerical Computations of Internal and External Flows, Vol. 1", John Wiley & Sons, UK, 515p.

Hirsch, C., 1990, " Numerical Computations of Internal and External Flows, Vol. 2", John Wiley & Sons.

Hughes, T. J. R., Brooks, A. N., 1982, " Streamline Upwind/Petrov-Galerkin Formulations for Convection Dominated Flows with Emphasis on the Incompressible Navier-Stokes Equations", Comp. Meth. Appl. Mech. And Engng., Vol 32, pp. 199-259

Hughes, T. J. R., Franca, L. P., Mallet, M., 1986, " A New Finite Element Formulation for Computational Fluid Dynamics:I. Symmetric Forms of the Compressible Euler and Navier-Stokes Equations and the Second Law of Thermodynamics", Comp. Meth. Appl. Mech. And Engng., Vol 54, pp. 223-234

Hughes, T. J. R., Mallet, M., 1986, " A New Finite Element Formulation for Computational Fluid Dynamics:III. The Generalized Streamline Operator for Multidimensional Advective-Diffusive Systems", Comp. Meth. Appl. Mech. And Engng., Vol 58, pp. 305-328

Hughes, T. J. R., Mallet, M., Mizukami, A., 1986, " A New Finite Element Formulation for Computational Fluid Dynamics:II. Beyond SUPG", Comp. Meth. Appl. Mech. And Engng., Vol 54, pp. 341-355

Lyra, P. R. M., 1994, "Unstructured Grid Adaptive Algorithms for Fluid Dynamics and Heat

Conduction", PhD Thesis, University of Wales, Swansea-UK
Shakib, F., 1988, "Finite Element Analysis of the Compressible Euler and Navier-Stokes Equations", PhD Thesis, Stanford University, USA