DIMENSIONAL ANALYSIS OF THE ANAEROBIC BIODIGESTION

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Abstract. *This paper aims at simulating the methane yield from the anaerobic digesting process by dimensionless equations. Through mathematical models based on mass balance and empirical relations, one builds a numerical simulating platform comprising of four differential equations, the anaerobic digesting of organic matter, biogas yielding, methane molar fraction and methane exergy flux, which thus set up a framework for deriving the dimensionless equations. A dimensionless phase-space plot is created for monitoring system performance and a specific exergy equation is derived by combining the dimensionless equations developed.*

Keywords: *anaerobic digestion, dimensional analysis*

1. INTRODUCTION

The global strategy for sustainable development needs high scale solutions for effectively protecting Earth from human pollution. Anaerobic digesting processes can potentially satisfy the two goals of sustainable development: generation of renewable energy and pretreatment of organic sewage to be disposed of in water bodies. This paper ends up doing an exergetic analysis of the methane yield from the anaerobic digestion, thus lined up with Wall's (1977) extented concept of exergy as a resource accounting tool which can be applied to the whole planet.

According to Capra (1996), the anaerobic digesting process is a natural one dating back about 3.5 billions years when the first bacteria evolved in a planet Earth with little oxygen. More 1.5 billions years had to pass for increasing the oxygen content in order to enable aerobic digesting.

Actually the anaerobic digesting process is a complex one consisting of several bacterial groups competing with one another for cracking the organic matter down to their nutrients and releasing a gas mixture containing from 50% up to 70% CH_4 and the remaining ones being CO_2 and H_2S.

Nowadays there have been evolving high performance anaerobic digesting reactors for waste waters which optimize the bacterial work, which are generically called UASB-Upflow Ascendant Sludge Bed, according to Van Haandel and Lettinga (1994).

2. PARAMETER DIAGRAM

Singh at al (1983) proposed a first-order model for substract utilization and biogas formation from cattle waste, which was then modified by Converti et al (1998) and Azeiteiro et al (2001) to comply with non-linearities of the anaerobic digesting. Nevertheless, those papers concentrate on the organic substract utilization and leave little room for the methane yield.

The equations to be developed are based on high performance anaerobic digesting reactors for waste waters (UASB-type ones) with the following assumptions: a) isothermal processes; b) steady-state biodigester operation; c) disturbance factors as per Fig. (1) kept controlled at suitable ranges; d) gradients of biomass density and temperature disregarded in the biodigester. For the anaerobic digesting processes being sensitive to temperature and for complying with assumption (a), the time derivatives of all variables are partial ones, so making implicit they do vary with temperature as well.

Figure (1) depicts a parameter diagram for the anaerobic digesting process. Their inputs and outputs to be considered in this work are shown in the left and in the right boxes respectively. The variables in the bottom box are classified as *parameters*, which means they do not vary beyond process inherent fluctuations. The variables in the upper box are classified as *disturbance* factors, which means they do interfere with process outputs but positively, so their values ought to be kept at controllable ranges.

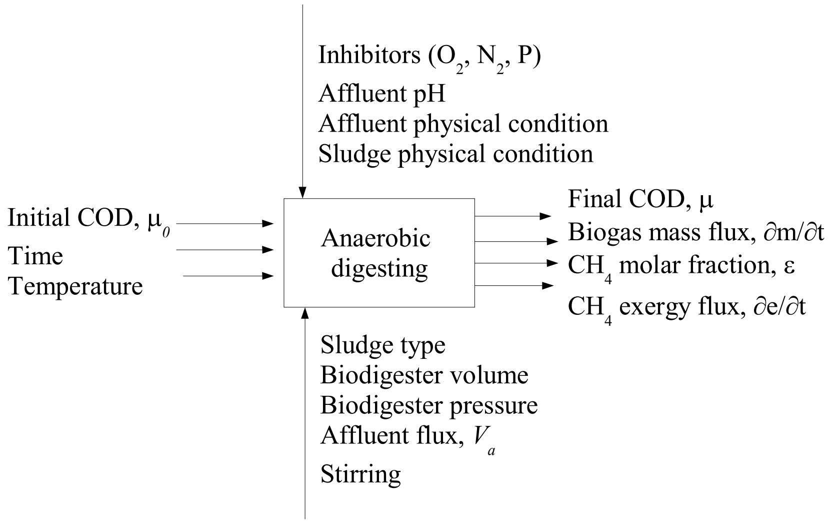

Figure 1. Parameter diagram for the anaerobic digesting process

Equation (1) models the anaerobic digesting of organic matter, μ [ML^{-3}], also known as COD – Chemical Oxygen Demand, by an exponential decrease as per Converti et al (1998). The *a* [$M^{1-z}L^{3(z-1)}T^{-1}$] empirical coefficient depends highly on temperature and the types of organic matter, digesting process. The *z* [1] empirical exponent accounts for the best experimental data fitting.

$$\frac{\partial \mu}{\partial t} = -a\mu^{z} \tag{1}$$

Actually Eq. (1) is not a model in terms of stemming from an isothermal mass balance over the chemical reactions taking place in the biodigester chamber, which would be quite a complex task for the non-linearities involved. So Converti et al (1998) set forth in fitting an exponential decaying differential equation and having added a needed non-linearity represented by a parameter *z* distinct from unity.

Equation (2) models the biogas mass flux, ∂m/∂t [MT^{-1}], based on an isothermal mass balance in the biodigester control volume by Deo (2004), where *m* [M] is the biogas mass; *y* [1] is an empirical diffusion coefficient of biogas into the effluent; V_a [L^3T^{-1}] is the affluent volume flux; and t_p [T] is the retention time, which is the ratio between the biodigester volume and V_a. The α [T^{-1}] empirical coefficient is a digesting time constant.

$$\frac{\partial^2 m}{\partial t^2} = -(1+y)^{-1} V_a \left(\frac{\partial \mu}{\partial t}\right)(1+\beta) \tag{2}$$

Where $\beta = -e^{a(t-tp)}$

Equation (2) was checked out against experimental data from a sewage treatment station of a beer plant in Brazil by Sept. 2003. The measured biogas average daily yield was about 3,000m^3 from the following inputs: m_0=2.88kg/m^3; V_a=450m^3/h; t_p=8h. By tweaking the constants to be y=0.3; a=0.5$kg^{-2}m^6h^{-1}$; a=0.05h^{-1}; z=3; w=1 and taking into account the biogas average specific volume at room temperature and multiplying it to *m* from integrating Eq. (2) twice over an eight-hour retention time, one obtains about 990 m^3; as the beer plant operates in three shifts, one should multiply that value by three, thus finding a theoretical value of 2,970m^3 a day, which is about 1% different from the measured biogas yield.

Equation (3) models the methane molar fraction in biogas, ε [1], as per Deo (2004), by setting up a time-dependent relation proportional to *m*, with *b* [$M^{-1}L^3T^{-1}$] *, w* [1] being empirical coefficients.

$$\frac{\partial \varepsilon}{\partial t} = b\mu\varepsilon^{w}(1+\beta) \tag{3}$$

Similar to Eq. (2), Eq. (3) was checked out against experimental data by making b=0.6m^3/kg.h, resulting about 70% CH_4 molar fraction at eight-hour average retention time, which agrees very closely with the average experimental value.

Equation (4) models the methane exergy flux as per Deo (2004), where Q_b is the methane specific lower combustion energy, about 50 MJ/kg or 14 kWh/kg by Van Wylen and Sonntag (1973).

$$\frac{\partial e}{\partial t} = \varepsilon \frac{\partial m}{\partial t} Q_b \tag{4}$$

Considering the same data from the sewage treatment station of a beer plant in Brazil, one finds an average value of 1,550kW over an eight-hour retention time, which means about 37MWh a day. Such an exergy has been burned up in flares to date.

3. DIMENSIONAL ANALYSIS

Equation (1) sets the anaerobic digesting of organic matter by modeling the reduction of its organic load or COD, μ, the boundary condition at zero time being μ_0, the initial COD. A performance metric comes up naturally by comparing the two COD extreme values, the initial one and the final one after a determined retention time, as to Eq. (5). Thus the smaller the final COD the better.

$$\eta_\mu = 1 - \frac{\mu}{\mu_0} \tag{5}$$

By multiplying $\partial m/\partial t$ from Eq. (2) and ε from Eq. (3) one obtains the methane mass flux, which can be divided by the product between V_a and μ_0, which means the affluent mass flux, in order to develop another performance metric relating both mass fluxes as to Eq. (6). Thus the larger such a value the more methane is yielded from a determined affluent mass flux.

$$\eta_\varepsilon = \varepsilon \frac{\partial m / \partial t}{V_a \mu_0} \tag{6}$$

Figure (2) exemplifies a phase-space plotting of the above performance metrics. The upper limit for η_ε, 0.25, refers to the stechiometric one according to Van Haandel and Lettinga (1994). The upper limit for η_μ, 1.00, is an unreachable one for anaerobic digesting only, the current processes getting about 0.85 average without external stirring.

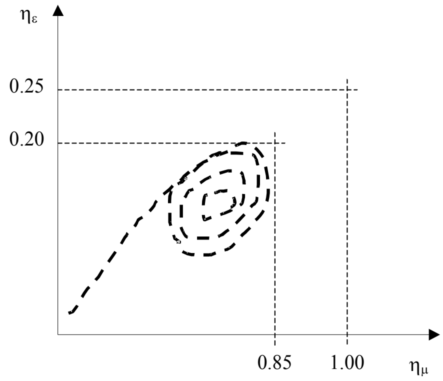

Figure 2. Performance metrics in a phase-space plotting

Such a plotting can be used as a process monitoring visual chart: after process startup phase represented by the ascending portion of the curve, it should go on a pattern which shows the expected process fluctuations, otherwise the process shall not have reached a stable configuration.

By combining Eqs. (4) and (6) one obtains another equation for the exergy flux as to Eq. (7). By dividing Eq. (7) by the affluent mass flux given by the product between V_a and μ_0 one obtains Eq. (8), which means a specific exergy, or how much of the affluent mass flux turns out to be converted into exergy per mass unit. Equation (8) is useful for estimating a specific exergy output for an anaerobic digesting process. With η_ε being 15%, such a specific exergy output would be about 2 kWh/kg biomass.

$$\frac{\partial e}{\partial t} = (\eta_\varepsilon V_a \mu_0) Q_b \tag{7}$$

$$\hat{e} = \eta_\varepsilon Q_b \tag{8}$$

Once obtained Eq. (2), whose integration yields the biogas mass flux, it can turn out to become dimensionless by manipulating Eqs. (5), (6) into that one, thus resulting in Eq. (9) as per Deo (2004), where C is an integration constant. Such an equation shows clearly the performance metrics from Eqs. (5), (6) are close related to each other as well as to the methane molar fraction ε, the diffusion coefficient y, and the anaerobic digesting factor β. Therefore, the methane yield efficiency increases proportionally to the methane molar fraction and the organic load and inversely to the diffusion coefficient.

$$\eta_\varepsilon = \frac{\varepsilon}{1+y} \int \frac{\partial \eta_\mu}{\partial t} (1 + \beta) dt + C \tag{9}$$

4. NUMERICAL SIMULATION

By utilizing a specific software for solving differential equations (like SIMNON™ from SSPA, www.sspa.se), one can plot the outputs from each Eqs. (1) to (4) thus far developed and their combinations easily. Nevertheless, Eqs. (5) and (6) comprise all the variables in dimensionless outputs, which is very convenient to analyze. Figure (3) shows the plotting of Eqs. (5) and (6) with the following parameters: μ_0=1.5kg/m^3; V_a=1.5m^3/h; t_p=20h; y=0.3; a=0.5kg^{-2}m^{6}h^{-1}; b=0.6 m^3/kg.h; α=0.05 h^{-1}; z=3; w=1.

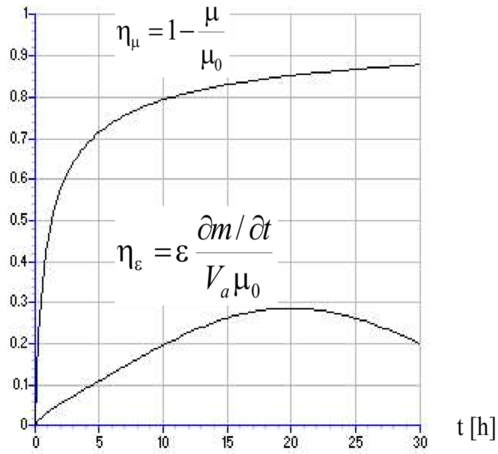

Figure 3. Dimensionless performance metrics for the anaerobic digestion

5. CONCLUSIONS

The methane yield from the anaerobic digesting process was analyzed by solving a four-differential equation set comprised of the COD reduction, biogas yield, methane molar fraction and methane exergy flux, Eqs. (1) to (4). These four equations brought about dimensionless equations for the COD reduction and methane yield, Eqs. (5), (6), which were then plotted together in a phase-space format, Fig. (3). A specific methane exergy relationship, Eq. (8), was derived for quick evaluating the energy potential of a given biomass yield, thus allowing decision-making processes for biomass energy investments to be made.

As mentioned in the mathematical assumptions, the models developed in this paper are isothermal ones, meaning they are valid for specific or short range temperatures. Further research should be done to include the temperature factor in the equations, thus making possible more precise predictions of methane yielding from anaerobic digesting processes in real environments subject to wide temperature gradients.

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