NONLINEAR DYNAMIC BEHAVIOR OF FIXED OFFSHORE STRUCTURES

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***Abstract**: In this paper, a simplified model is used to investigate the nonlinear dynamic behavior of fixed offshore platforms under the action of ocean waves. The simplified model is capable of reproducing the levels of fundamental frequencies observed experimentally for reduced size fixed offshore structures. The wave forces are determined by Morison's equation, with the velocity and acceleration obtained from the Airy's first order wave theory. Design waves with typical parameters of the Brazilian coast are used in the analysis. The nonlinear equation of motion is derived using Hamilton's principle and expanded up to the cubic term. A comparison between the nonlinear analysis and the linear dynamic analysis is presented. The possibility of this kind of structures, under the action of ocean waves, exhibiting chaotic motion is investigated. The results presented highlight the fact that fixed offshore structures may respond in a chaotic way, depending on the wave and the structure's characteristics.*

***Keyword:** nonlinear dynamics, fixed offshore platforms, chaotic motion.*

1. INTRODUCTION

Offshore technology is growing rapidly. Platforms have been used in the oil industry for drilling, producing, storage, materials handling, living quarters, etc. In general, there are two types of offshore structures. They are fixed and compliant structures. Usually, fixed structures are designed to withstand environmental forces without any substantial displacement. Therefore, one could conclude that a linear dynamic analysis should be sufficient. But in fact, for these structures, the dynamic responses may have nonlinear characteristics, which need to be explored fully (Han and Benaroya, 2000).

The nonlinear dynamic analysis of a fixed offshore structure is, undoubtedly, a complex subject due to the variety of topics that play a significant part in the overall response, such as: the three-dimensional characteristic of the structure and its size; the action of ocean waves, currents and tides; the fluid-structure interaction, to name just a few. In order to get an insight into the nonlinear dynamic behavior of fixed offshore structures, a simplified one-degree of freedom model is used and its characteristics are discussed in the next session of this paper. The study here presented takes into account the action of ocean waves on the model. The adequacy of such model for the understanding of the response of fixed offshore structures is illustrated by the comparison between the results obtained with it and those obtained experimentally by Sotelino and Roehl (1982), and Teixeira and Roehl (1986), who tested reduced size steel and acrylics fixed platforms under the action of ocean waves typical of the Brazilian coast. Although in both experimental works the authors mention that no effort was made to simulate any particular prototype, the nature and quality of the tests justify the use of the results as means of comparison.

In the present paper the possibility of chaotic motion occurring is investigated using different ocean wave characteristics. The results obtained show that fixed offshore structures under the action of ocean waves may exhibit some typical nonlinear system behaviors, such as chaotic motion.

2. THE SIMPLIFIED MODEL

The vibration of the simplified model used in this paper comes from experimental results carried out by Sotelino and Roehl (1982), and Teixeira and Roehl (1986), who tested reduced size fixed platform under the action of an ocean wave typical of the Brazilian coast. In their experiment the similitude criteria was used for the design of the structure with a geometrical scale 1:100. The tested reduced size offshore structures are shown, respectively, in Fig. 1.

The experimental results were compared to those obtained with an equivalent clamped-free column (same height, mass and stiffness) with an equivalent mass M_{eq} concentrated at the top, and expressed as (Pinto, 1993):

$$M_{eq} = M_d + \frac{ml}{4.12} \qquad (1)$$

where M_d is the mass of the platform's deck, m the distributed mass of the platform per unit length (not including the deck) and l represents the height of the platform. The factor 4.12 dividing the total mass of the platform ml (minus the deck) in Eq. (1), comes from the analogy between a column with distributed mass m along the length l and no mass at the top ($M_d = 0$) and the column with $m = 0$ and a mass M_d at the top. For the first case ($m > 0$ and $M_d = 0$) the fundamental frequency of vibration, ω_1, is (Clough and Penzien, 1989):

$$\omega_1 = 1.875^2 \sqrt{\frac{EI}{ml^4}} \qquad (2)$$

where E is the Young's Modulus and I the moment of inertia of the cross section of the column. For the second case ($m = 0$ and $M_d > 0$) the corresponding fundamental frequency is expressed as (Clough and Penzien, 1989):

$$\omega_1 = \sqrt{\frac{3EI}{M_d l^3}} \qquad (3)$$

Note that when in the presence of water one has to include the distributed mass along the cylindrical rigid-bar element, m_o, and the distributed added mass, m_a, resulting the following expression (Mc Cormick., 1973)

$$m = m_o + m_a \left(\frac{d}{l}\right)^2 \qquad (4)$$

where l is the length of the rigid-bar and d the water depth.

The tested reduced platforms had different values for M_d. Once the fundamental frequency corresponding to the smallest value of M_d is known, f_{m0}, the values of the natural frequency corresponding to greater values of M_d, f_m, can be determined from Eq. (3) as follows:

$$f_m = f_{m0} \sqrt{\frac{M_{eq0}}{M_{eq}}} \qquad (5)$$

where M_{eq0} and M_{eq} represent the value of the equivalent mass of the smallest value of M_d and for a greater value of M_d, respectively. The natural frequencies in Eq. (5) are expressed in Hertz.

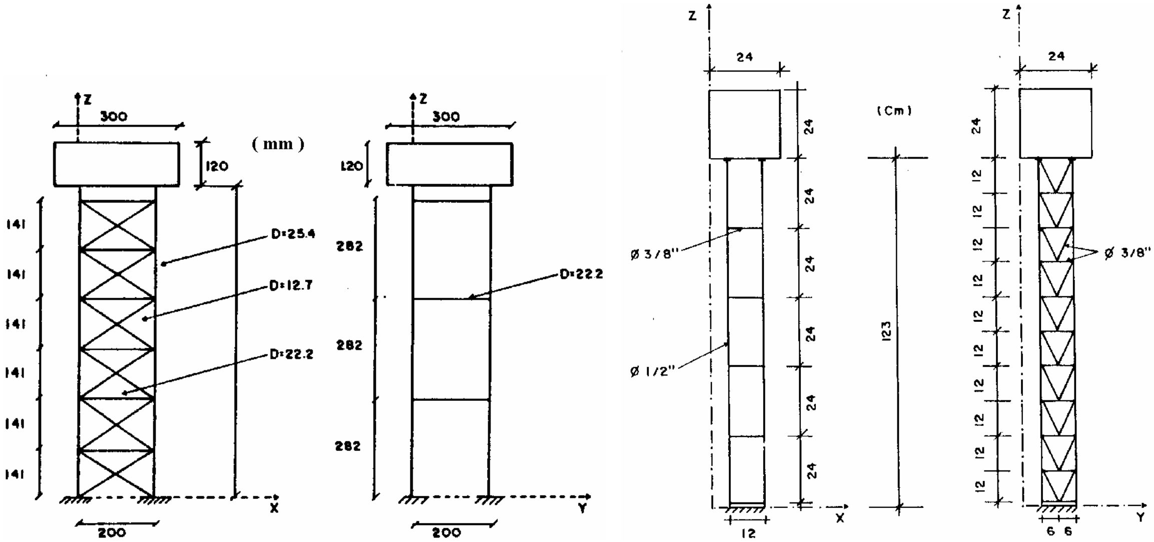

Figure 1. Reduced size models (Sotelino and Roehl 1982, Teixeira and Roehl 1986).

Table 1 shows a comparison between the values of the frequency obtained experimentally (Exp) for Sotelino and Roehl (1982) and those obtained using Eq. (4) for m_l = 11.33 Kg and the following cases of M_d: m_1 = 4.8 Kg, m_2 = 24.87 Kg, m_3 = 44.8 Kg and m_4 = 64.8 Kg. The tests were carried out in the presence of water and without water.

Table 1 - Comparison of Frequencies (Hz)

	m_1		m_2		m_3		m_4	
	Exp	Eq4	Exp	Eq4	Exp	Eq4	Exp	Eq4
Air	12.8	12.8	6.7	6.7	4.9	5.1	4.0	4.3
Water	12.0	12.0	6.4	6.4	4.7	4.9	4.0	4.1

The worst result in the presence of water corresponds to an error of 4.26% (for m_3) and in the absence of water 7.50% (for m_4). Results of the same order were found for the model of Teixeira and Roehl (1986). Based on these comparisons, it is possible to conclude that the behavior of the equivalent column may be a good approximation to the behavior of the fixed platform.

In this paper the equivalent clamped-free column is modeled by the rigid-bar element shown in Fig. 2, for which the nonlinear dynamic analysis is carried out.

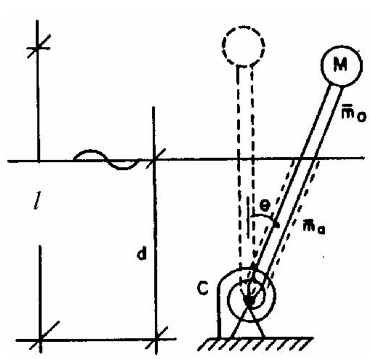

Figure 2. The simplified model.

It consists of a rigid-bar element with a concentrated mass at the free end and a rotational spring of stiffness C at the support.

One can see in those works (Sotelino and Roehl 1982, Teixeira and Roehl 1986) that, regarding to the displacements of the deck, the answer is basically the same of a single degree of freedom system with the fundamental frequency of the structure. It is also observed that a simpler system, as a clamped-free column, can serve as a base for an analysis of the behavior of a system as complex as a fixed platform (Pinto, 1993).

The idea of this work is that given a certain structure there is an equivalent column, with the same mass and the same rigidity that the structure in subject, which has a very close behavior. This equivalent column is used in order to specify the characteristics of the simplified model.

As mentioned above, the model shown in Fig. 2 consists of a rigid-bar element free at the top and fixed at the bottom. It has at the support a rotational spring of constant stiffness C which is determined in such a way that the lateral displacement at the top of both the equivalent column and the model is the same for a concentrated load applied at the top. Therefore,

$$C = \frac{3EI}{l} \tag{6}$$

where EI and l have already been defined previously. The value of EI for the equivalent column is obtained from the fundamental frequency of vibration of the platform, f_0, obtained experimentally:

$$EI = \frac{4}{3}\pi^2 f_0^{\,2} M_{eq} l^3 \tag{7}$$

The displacement coordinate is chosen to be the total angular rotation θ, Fig. 2.

The length of the rigid-bar element is the same as the length of the equivalent column and, therefore, corresponds to the height of the platform. The cross section of the model is an annulus for which the external and the internal diameter, D_e and D_i, respectively. D_e is determined in such a way that using the same added mass coefficient C_a, obtained experimentally, the volume of water displaced is the same. Therefore,

$$D_e = 2\sqrt{\frac{S_a}{\pi}} \tag{8}$$

where S_a is the apparent area defined as

$$S_a = \frac{M_a}{\rho d C_a} \tag{9}$$

M_a being the added mass, ρ the fluid density and d the water depth. S_a can also be expressed as

$$S_a = \frac{M_0}{\rho d} \tag{10}$$

where M_0 is the mass displaced by the model. D_i is determined in such a way that the total mass of the model is equal to the total mass of the platform not including the deck. Therefore,

$$D_i = \sqrt{D_e^{\,2} - \frac{4V_p}{\pi l}} \tag{11}$$

where V_p is the volume of the platform excluding the deck.

Table 2 shows the values of the frequency obtained with the simplified model (Mod) for the same cases shown in Tab. 1. For the comparison, the characteristics of the model are: D_e = 5.11cm, D_i =3.35cm and C = 73.88 KN/m.

Table 2- Frequencies (Hz)

	m_o		m_1		m_2		m_3	
	Exp	Mod	Exp	Mod	Exp	Mod	Exp	Mod
Air	12.8	12.0	6.7	6.6	4.9	5.0	4.0	4.2
Water	12.0	11.7	6.4	6.5	4.7	*5.0*	4.0	4.2

The results shown in Tab. 2 highlight the capability of the simplified model to reproduce the measured frequencies of the fixed platform obtained in the experiments reported by Sotelino and Roehl (1982).

The model adopted in the present study and shown in Fig. 2 was previously used for post-buckling non-linear dynamic analysis (Souza and Mook, 1991) and for non-linear dynamic analysis of fixed offshore structures (Souza and Pinto, 1993, Pinto, 1993). Its main stability and dynamical characteristics were discussed in detail in those references.

3. EQUATION OF MOTION

The equation of motion is obtained by means of a perturbation around a static equilibrium configuration, θ corresponding to a given load level. In the pre-buckling state the static equilibrium configuration corresponds to θ = 0 and, therefore, the motion takes place around such a configuration. In the post-buckling state the motion will take place around the static equilibrium configuration θ, depending on whether the model is perfect or initially imperfect, respectively (Souza and Pinto, 1993).

The procedure adopted leads to the following nonlinear equation of motion:

$$I^*\ddot{\phi} + \mu^*\dot{\phi} + C[\theta + \phi - p\sin(\theta + \phi)] = C\theta_0 + M_t(t) \tag{12}$$

where θ_o is an initial imperfection (for a perfect model $\theta_o = 0$), ϕ, $\dot{\phi}$ and $\ddot{\phi}$ represent the perturbed displacement, velocity and acceleration, respectively, I^* is the generalized inertia given by

$$I^* = \frac{l^2}{3}\left(3M_d + m_0 l + m_a\frac{d^3}{l^2}\right) \tag{13}$$

μ^* is the damping coefficient and M_t (t) is the external excitation coming from the action of the ocean waves which will be discussed in the next session, and p is the load parameter defined as

$$p = \frac{gl}{2C}\left(2M_d + m_0 l + m_a\frac{d^2}{l}\right) \tag{14}$$

where g is the acceleration of gravity.

Assuming the perturbation to be small, the exact Eq. of motion (12) can be expressed as

$$I^*\ddot{\phi} + \mu^*\dot{\phi} + \omega_0{}^2\phi + \alpha\phi^2 + \beta\phi^3 = M_t(t) \tag{15}$$

where terms of order O(4) or higher are neglected and the coefficients of ϕ, ϕ^2 and ϕ^3 are defined as

$$\omega_0^2 = C(1 - p\cos\theta), \quad \alpha = \frac{1}{2}Cp\sin\theta\,, \quad \beta = \frac{1}{6}Cp\cos\theta \tag{16}$$

It is worth mentioning that in the pre-buckling state (for a perfect model) the parameter α is identically zero and the nonlinear equation of motion becomes a Duffing type equation.

4. WAVE FORCES

In the dynamic analysis of a marine structure, the wave loading is usually the most important of all environmental loadings for which the structure must be designed. The horizontal force exerted by waves on a cylindrical object consists of two parts: a drag force, which is related to the kinetic energy of the fluid, and an inertial or mass force, that is related to the inertia of the fluid. For representing the wave forces acting on fixed offshore platforms, usually it is used the well-known Morison's equation (Morison et. al., 1950):

$$F = \frac{1}{2}\rho C_d D_e u|u| + \rho C_m \frac{\pi D_e{}^2}{4}\frac{\partial u}{\partial t} \tag{17}$$

In Eq. (17), F is the transverse wave forces per unit length, ρ is the fluid density, C_d the drag coefficient, D_e the external diameter of the cylindrical member, u and $\partial u/\partial t$ the velocity and acceleration of the fluid perpendicular to the cylinder, respectively, C_m is the inertia coefficient and t is time. Note that Morison's equation is applicable when the drag force is predominant, which occurs when the structural diameter is small compared to the water wavelength (Dean, and Dalrymple, 1984)

The proper values of C_d and C_m depend in part on the wave theory being used. Typical values for cylindrical members are $0.6 \le C_d \le 1.0$ and $1.5 \le C_m \le 2.0$, and the values selected should not be smaller then the lower limits of these ranges (API, 1989). Some available experimental results for fixed offshore structures of reduced size (Sotelino and Roehl, 1982, Teixeira and Roehl, 1986) point larger values for C_m. In this work the values C_d = 0.8 and C_m = 3.0 obtained experimentally by Sotelino and Roehl (1986) are used. The higher value of C_m obtained reflects the fact that it corresponds to the actual structure, where besides the additional mass of each element, a portion of water that is confined in the interior exists and it vibrates together with the structure, increasing the total additional mass, and not to just one cylinder (Pinto, 1993).

Water particle velocity and acceleration are functions of wave height (H), wave period (T), water depth (d), distance above bottom and time (t). These parameters may be determined by any defensible method, e.g., Stokes fifth order wave theory, Airy's linear theory, modified solitary wave theory, cnoidal wave theory, etc. Here, the velocity of the fluid is determined from the Airy's first order wave theory because in the cases studied in this paper the wave height is small compared to the wave length or water depth and, therefore, in this case it gives accurate results (Dean, and Dalrymple, 1984).

The wave's parameters to be used in Airy's theory are shown in Fig. 3, where we have, besides the basic characteristics of the wave, c = celerity, H = water surface elevation measured from the mean or still water level (SWL).

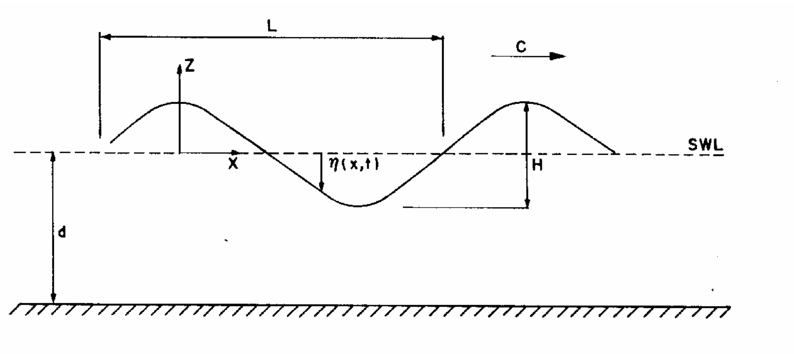

Figure 3. Wave profile.

The velocity and acceleration components, u and $\partial u/\partial t$, respectively, are expressed as

$$u = \frac{\pi H}{T} \frac{\cosh[K(d+Z)]}{\sinh(Kd)} \cos\theta \tag{18}$$

and

$$\frac{\partial u}{\partial t} = \frac{2\pi^2 H}{T^2} \frac{\cosh[K(d+Z)]}{\sinh(Kd)} \sin\theta \tag{19}$$

where

$$\theta = Kx - \omega t\,, \quad K = \frac{2\pi}{L}, \quad \omega = \frac{2\pi}{T} \tag{20}$$

The wave-length L can be obtained from the transcendental equation

$$L = \frac{gT^2 H}{2\pi} \tanh(Kd) \tag{21}$$

using Newton's numerical method.

In the nonlinear Eq. of motion (12) or (15), $M_t\,(t)$ represents the moment about the fixed end of the rigid-bar element. The moment is obtained by a numerical integration procedure using Morison's equation with Airy's wave theory.

The moment at the support function $M_t\,(t)$ was found to be approximately of the form

$$M_t(t) = S_m + A_m \cos(\omega t - \varphi) \tag{22}$$

where

$$S_m = \frac{1}{2}(M_{\max} - M_{\min}) \tag{23}$$

$$A_m = \frac{1}{2}(M_{\max} + M_{\min}) \tag{24}$$

$$\varphi = \pi\left(\frac{t_{\max} + t_{\min}}{T} - \frac{1}{2}\right) \tag{25}$$

M_{max} and M_{min} representing the maximum and minimum values of the moment at the support, respectively. t_{max} and t_{min} represent the instants of time when M_{max} and M_{min} occur, respectively. The approximate function, Eq. (22), is obtained in such a way that the amplitude A_m is the same as the one from Morison's equation and the phase angle φ is such that the difference between the exact moment and the approximate function be a minimum regarding the occurrence of the peaks, M_{max} and M_{min}. Comparisons between $M_t\,(t)$ obtained directly from Morison's equation and from Eq. (22) for the wave used and the model show that the approximation is extremely close to the exact solution (Souza and Pinto, 1993, Pinto, 1993). In this paper, the moment function $M_t\,(t)$ is used in its approximate form given by Eq. (22).

5. RESULTS

The possibility of chaotic motion occurring was investigated using a 5th-order Runge-Kutta procedure for the integration of the non-linear Eq. of motion (12). Fast Fourier transformation and Poincaré sections were also used in the analysis.

The search for chaos was initially carried out for waves of heights between H = 120mm to 180mm and periods between T = 1.0s to 1.2s, data which correspond to actual waves of H = 120m and 180m and periods T = 10s to 12s, respectively. For such typical waves of the Brazilian coast no situation of chaotic motion was found.

Chaotic motion was obtained for the post-buckling load level p = 1.195. The following are the values of the different parameters which led to the chaotic motion: $I^* = C = 1.0$, $\mu^* = 0.15$, M_{ot} = 1.8, $\Omega = 0.43$ and $\varphi = 0$.

Figures 4 to 6 illustrate the results obtained and represent, respectively, phase portrait, Poincaré section and the frequency spectrum.

The phase portrait of Fig. 4 corresponds to the response from time $t = 10{,}000T_\Omega$ to $t = 10{,}020\ T_\Omega$ ($T_\Omega = 2\pi/\Omega = 14.612$s).

Figure 5 represents Poincaré sections corresponding to $12{,}000T_{\Omega,}$ typical of chaotic motion.

In the power spectrum shown in Fig. 6 it can be seen the different peaks corresponding to the frequency of the excitation Ω, and its multiples. It can also be observed considerable noise, also a characteristic of the chaotic motion.

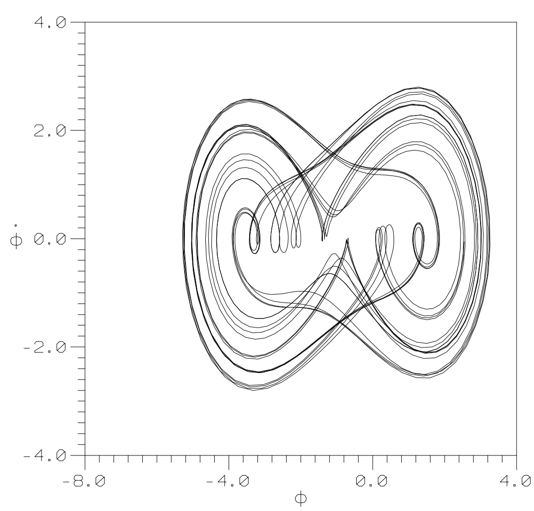

Figure 4. Phase Portrait.

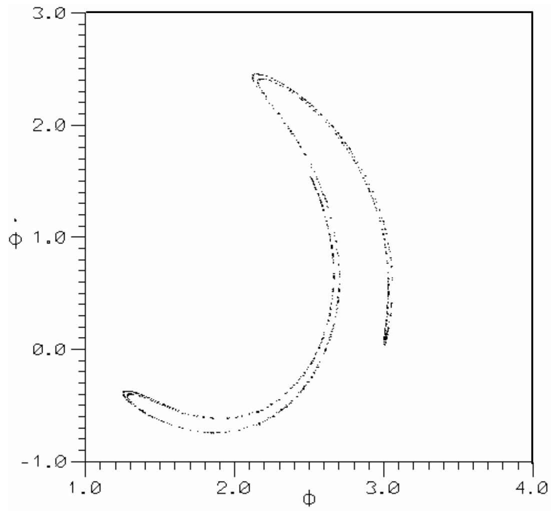

Figure 5. Poincaré Section.

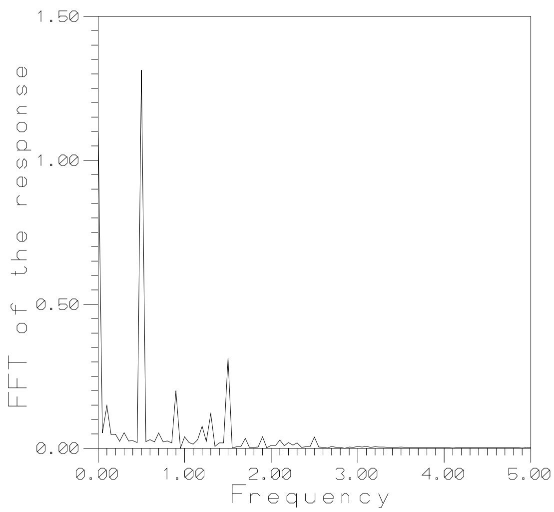

Figure 6. Frequency Spectrum.

6. CONCLUSIONS

The comparison between the experimental results and those obtained with the model in terms of the fundamental frequency of vibration emphasizes the adequacy of the simplified model for the study of the nonlinear dynamic response of fixed offshore structures.

For the typical wave of the Brazilian coast used in the analysis the results obtained show that a linear dynamic analysis provides a good approximation for the response. It was also shown that due to the nature of the nonlinear equation of motion, for different wave characteristics or different excitation the linear response is not satisfactory, with the possibility that chaotic behavior can occur.

The non-linear dynamic response of fixed offshore structures was discussed in terms of the possibility of the occurrence of chaotic motion. The equation of motion was integrated and a situation of chaotic motion was identified. The study here reported illustrates the fact that fixed offshore structures can respond in a chaotic way depending on the wave and the structure's characteristics. Although the validation of the model was done using experimental results corresponding to tests carried out on reduced size fixed structures, the results of the present work highlight the importance of taking into account the possibility of the chaotic response in the analysis and design of fixed offshore structures.

In summary, the simplified model used in the analysis is a useful tool for the understanding of the nonlinear dynamic behavior of complex structures such as the fixed platforms.

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