AN INSIGHT ON FREE SURFACE FLOWS DUE TO SUBMERGED SINGULARITIES

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Abstract. *The study of free surface flows represents one of the most interesting and challenging problems in fluid mechanics. Indeed it has attracted the attention of researchers for centuries, motivated by natural observations and several applications. For high Reynolds number flows, wave phenomena on water surfaces appear as the most cited natural example. In this context singularities have been employed as a common tool to model underlying currents. In this paper a review of analytical and numerical works that have been developed aiming to model free surface flows induced by vortices, sinks, sources and dipoles is presented. The effects of nonlinearity at the free surface of a body of water when it interacts with an underlying flow are discussed. A fully nonlinear, unsteady, boundary-integral method is employed to two free surface flow problems. The first is related to the interaction between water waves and currents (Moreira & Peregrine 2001, Moreira 2003a), which has been the subject of discussion of many theoretical and experimental works. The second concerns the disturbances generated at a free surface by a horizontal cylinder in a uniform stream flow (Moreira 2003b).*

Keywords. *Potential flows, water waves, nonlinear effects, boundary integral method.*

1. SINGULARITIES IN MATHEMATICAL MODELS OF FREE SURFACE FLOWS

The use of singularities in the modelling of fluid flows, especially when studying their effects at a free surface, has been a common tool for researchers for many years, aiming to understand the basics and complexities of several natural phenomena. Several text books have shown the value of singularities when modelling fluid flows and thus helped to disseminate this concept. Singularities become widely employed in fluid mechanics, whether in the investigation of nonlinear effects at the free surface due to an underlying flow, whether in the understanding of free surface disturbances induced by, for instance, vortex flows.

1.1. Free surface flows due to a dipole in a uniform current

Perhaps one of the most cited and successful attempts to model a free surface flow with an underneath singularity under the light of linear theory was that due to Lamb (1913). Kelvin was the first to suggest that problem in 1905, followed by Lamb who analysed it formally in the light of linear water wave theory in 1913 (for a more accessible version of this paper see Lamb 1932, §247). Lamb's method consisted in replacing the cylinder by the equivalent dipole at its centre and then finding the fluid motion due to this doublet. Supposing a steady irrotational potential flow with a linearised free surface boundary condition, he found the appearance of a local disturbance immediately above the obstacle followed by a train of stationary sinusoidal waves on the downstream side (see figure 1). This solution is applicable when the cylinder is of small radius a compared with the depth of submergence d; then the disturbance to the free stream U due to the presence of the cylinder causes small amplitude waves that can be approximated by linear theory.

Havelock (1926) proceeded to further approximations by the method of successive images, applying it to horizontal and vertical doublets. The contribution of each part of the image system to the surface disturbance was indicated and the corresponding surface elevations were determined numerically. In these higher-order approximations, only the flow in the vicinity of the body has been corrected, while the free surface condition has been kept in its first-order version.

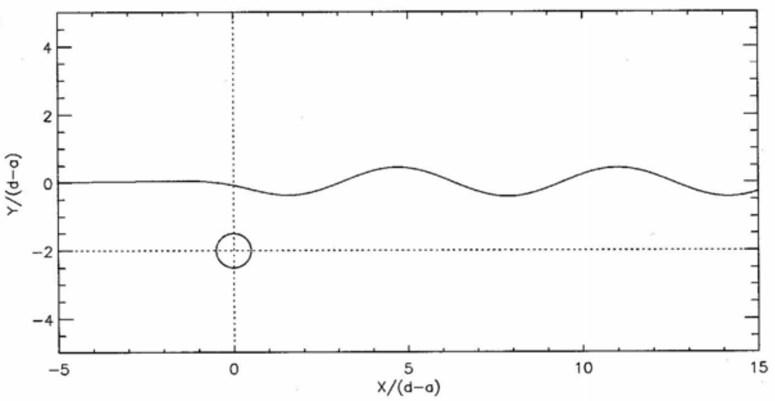


Figure 1. Stationary waves due to a cylinder on a uniform stream flow. *a/d=1/4*, *U=1*.

Later Tuck (1965) investigated the limitations of the linear solution and the importance of the nonlinear term in the free surface boundary condition. Using Lamb's formulation, he replaced the submerged cylinder by a doublet in a uniform stream flow; numerical values for the potential and stream function were then obtained via a modified Wehausen scheme. By plotting the streamlines and the first-order surface elevation for a cylinder close to the free surface, he found that some of the streamlines break up at wave crests into something resembling splashes. He suggested that in these cases the exact nonlinear solution would involve highly non-sinusoidal or even breaking waves and that a simple linearisation would be inadequate for cases in which the cylinder is close to the free surface, whereas it does not satisfy the assumption $a/d \ll 1$. Tuck showed that second-order nonlinear effects become important when the cylinder is not too far from the free surface. Moreover, he found that the second-order correction associated with the nonlinearity of the free surface condition is more important than the one related to the body condition.

Following Tuck's work, Dagan (1971) investigated the flow past a circular cylinder close to a free surface at high Froude number by the method of matched asymptotic expansions. The inner flow model is that of a non-separated, nonlinear, gravity free surface flow past a doublet, while the linear outer solution is that of a singularity at the free surface. A good agreement with linear theory was found for deep submerged bodies. At moderate immersion depths the linearised solution is still valid, provided that the depth is replaced by an effective depth, larger than the actual one. For a body close to the free surface, Dagan found that the nonlinear calculations differed significantly from the linearised solution, suggesting that in this case the free surface would break or create a detached jet.

As it is discussed later, these results are restricted to a region where the amplitude of the disturbances are linear, thus satisfying the inequality $a/d \ll 1$. As the ratio *a/d* increases, the roughness of the free surface is augmented and waves become “steeper”, with nonlinear effects taking over. The implications of the nonlinear results for experimental observations are not completely clear. In section 3.2 the effects of nonlinearity at the free surface when a steady stream flow interacts with a submerged cylinder are discussed. A version of the fully nonlinear boundary-integral method developed by Dold & Peregrine (1986) is adapted to this situation. For details of the boundary value problem and the numerical scheme see Moreira (2003b).

1.2. Free surface-vortex flow interactions

At an initially undisturbed free surface, considerable disturbances due to vortex interactions have been reported by several authors, based on linearised approaches and numerical simulations for various Froude numbers. Experiments were also carried out showing this evidence (Willmarth *et al.* 1989). In the theoretical models here reviewed the fluid flow is assumed to be in deep water and irrotational, except at the location of the discrete vortices. To characterise the fluid motion two different Froude numbers are defined based on different length scales, $Fr = k/\sqrt{gd^3}$ and $Fr_s = k/\sqrt{gl^3}$, where *k* is the strength of the vortices, *g* is the acceleration due to gravity, d is the initial depth of the point vortices below the free surface and l is the initial separation between two vortices; Fr_S is called the spacing Froude number. For clarity, in the present work a vortex pair (or eddy pair) is assumed to be two vortices with the same sign, while a vortex dipole (or eddy couple) has opposite signs.

Telste (1989) observed two different free surface features when a pair of counter-rotating point vortices approaches a free surface. For a weak circulation ($Fr=0.04$, $Fr_S=0.5$) no wave breaking occurs and a small depression on the free surface is formed. If the reference frame is considered moving with one of the point vortices then this solution approaches the linear steady state profile predicted by Novikov (1981). For larger circulations ($Fr=0.2$ and 0.6, $Fr_S=2.2$ and 7.1) a central hump is formed followed by free surface breaking. To solve the unsteady nonlinear two-dimensional free surface potential flow problem Telste used a boundary-integral method. Marcus & Berger (1989) solved the same unsteady nonlinear problem via a finite-difference method. In their work strong vortices ($Fr=0.5$, $Fr_S=2.8$) also displace a mound of fluid before the free surface breaks. For weaker vortices ($Fr=0.06$, $Fr_S=0.3$) disturbances are gentler, with a shallower scar. Nevertheless their numerical method suffered from instabilities which excluded longer simulations.

The effect of a single vortex near a free surface was investigated by Tyvand (1991), who observed that all supercritical vortices ($Fr \geq 6.3$) lead to surface breaking while subcritical vortices ($Fr<6.3$) tend to accumulate a surface mound until surface breaking eventually occurs. Tong (1991) introduced a point vortex into the boundary-integral method developed by Dold & Peregrine (1986), showing that small waves could be generated by a single weak vortex ($Fr<0.2$), with the steepness of the waves depending on the direction of the vortex circulation, while a stronger vortex ($Fr \geq 0.2$) leads to free surface breaking.

Barnes *et al.* (1996) used Tong's algorithm as an approach to model the region of strong vorticity generated by a plunging breaker. In their model vorticity is represented by two-dimensional discrete vortices interacting with a nonlinear free surface initially at rest. Each point vortex moves under the influence of the free surface, the other vortices and its images, while the free surface moves due to the vortices and gravity. Figure (2) shows the free surface displacement due to a single vortex (continuous lines) and an equivalent system of *10* vortices distributed initially around the same point (dashed lines). The two cases have $Fr=0.5$ and the results are remarkably similar, showing that the point approximation can model patches of vorticity quite well under certain conditions.

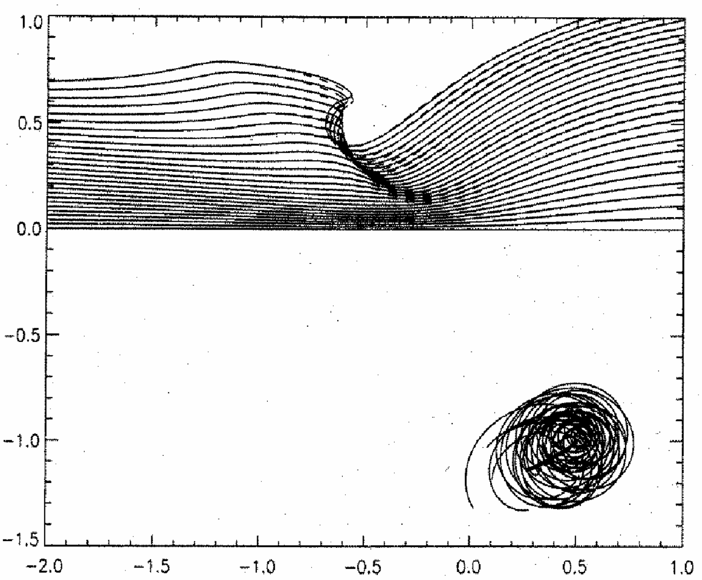


Figure 2. Stacked free surface displacement due to a single vortex (solid lines) and *10* vortices (dashed lines) and the corresponding vortex paths. $Fr=0.5$. (Barnes *et al.* 1996)

1.3. Free surface flows due to sinks and sources

The problem of a free surface flow of an ideal fluid induced by a submerged source or sink with gravity as the restoring force has been the subject of many papers. Solutions with a stagnation point immediately above the source or sink have been given by Havelock (1926) and Vanden-Broeck *et al.* (1978). Peregrine (1972) suggested that a limiting flow would eventually be achieved for sufficiently high withdrawal rate at which the interface would form a secondary stagnation point enclosing a 120^0 corner. For a sink at the vertex of a sloping bottom, Craya (1949) found an exact solution when the bottom sloped downward at an angle $\pi/3$ from the vertical, and Hocking (1985) obtained numerical solutions for a sequence of angles ranging from 0 to $\pi/2$. For a sink in fluid at infinite depth, a numerical solution for the problem was found by Tuck & Vanden-Broeck (1984). They appear to have been the first to encounter numerically the formation of a cusp at the free surface directly above the sink when $Fr=1.776$. (Note that in this case the Froude number is defined as in §1.2 but with k defined as the volume flux per length unit of the sink.)

Cusped flows due to a sink within a fluid of finite depth, with gravity acting, have been considered by Vanden-Broeck & Keller (1987), who confirmed numerically the solutions of Craya, Hocking and Tuck & Vanden-Broeck and showed that cusped flows exist for Froude numbers larger than some particular value. Collings (1986) considered the flow due to a line source or sink within a fluid of finite depth and above a horizontal bottom, but with no restoring force, and found cusped solutions when the source/sink was on the flat bottom and when the source depth was *0.56742* of the far fluid depth. King & Bloor (1988) used a conformal transformation and integral equation technique to construct solutions to the steady flow induced by a submerged source beneath a cusped free surface and above a flat horizontal bottom when there is no restoring force. They found explicit closed-form results for the equation of the free surface and the cusp height, confirming the numerical and asymptotic results of Collings and Hocking. In a following paper, King & Bloor (1989) investigated the free surface flow of a uniform stream of ideal fluid around a Rankine body formed by a source and a sink at finite depth. Linear and nonlinear numerical solutions are presented in their work for a variety of body shapes, for both supercritical and subcritical flows.

Solutions having a stagnation point on the free surface directly above the sink also were found in finite-depth flows in two dimensions. Results for supercritical (where the depth-based Froude number is greater than 1) and subcritical flows ($Fr<1$) were computed, respectively, by Mekias & Vanden-Broeck (1989), via a numerical scheme based on a series expansion, and Hocking & Forbes (1992), who used an integral equation approach. Both works confirm the formation of a stagnation point on the free surface directly above the sink plus a uniform stream flow at infinity. In an independent study, Mekias & Vanden-Broeck (1991) also solved the subcritical flow problem in finite depth, but unlike Hocking & Forbes who concluded that waves far from the line sink would not occur, these authors obtained a regular wave train downstream.

All the works mentioned in the last three paragraphs assume that the free surface flow is steady. Recently several other authors have obtained numerical solutions for the unsteady motion of a free surface flow due to a line source and a point sink. They include Tyvand (1992), Miloh & Tyvand (1993), Xue & Yue (1998) and Stokes *et al.* (2002).

2. MATHEMATICAL MODELS OF SINGULARITY DISTRIBUTIONS

As reviewed in the last section the inclusion of a distribution of singularities has been a common tool for researchers aiming to model underlying flows. In this section the mathematical modelling of three of these flows is presented. In the first two models a pair of vortices and a distribution of sinks and sources are selected aiming to represent “rapidly” and “slowly” varying surface currents in a periodic domain. In order to apply Cauchy's integral theorem to the periodic problem, a conformal mapping of the form $\varsigma(\xi,t)=e^{-iZ(\xi,t)}$ is applied to the singular potential velocity ϕ_S (for more details see Moreira 2003a). Finally, in the last model, a single doublet is used to replace a submerged cylinder in a uniform stream flow (Moreira 2003b). This model is in fact an approximation of the flow about a horizontal circular cylinder and it was first employed by Lamb (1913). In this case the fluid domain is assumed to be unbounded and then no conformal mapping is required.

2.1. Vortex model

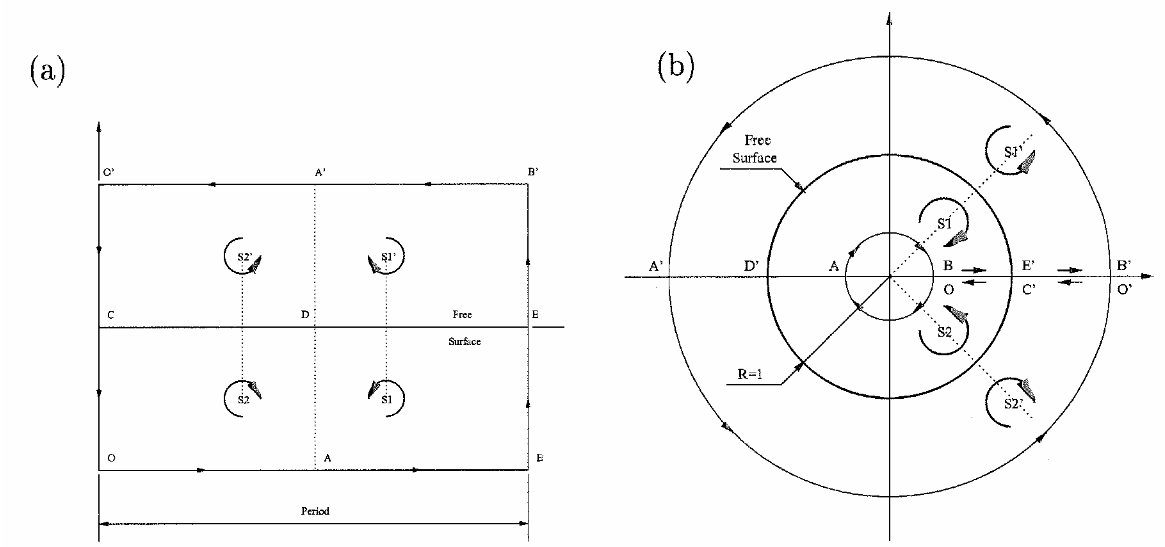


Figure 3. (a) A sketch of the z-plane with two singularities and their images reflected onto the free surface. (b) The corresponding ς-plane obtained via conformal mapping $\varsigma(\xi,t)=e^{-iZ(\xi,t)}$.

Assuming that the singular points S_1 and S_2 shown in figure (3a) are a vortex couple with strength k, occupying respectively the positions z_1 ($=x_1+i\,y_1$) and z_2 ($=x_2+i\,y_2$), then the complex potential induced by those vortices in the transformed ς-plane is given by (Batchelor 1967, p.410),

$$\omega(\varsigma)=-ik\,\log\left(\frac{\varsigma-\varsigma_1}{\varsigma-\varsigma_2}\right), \tag{1}$$

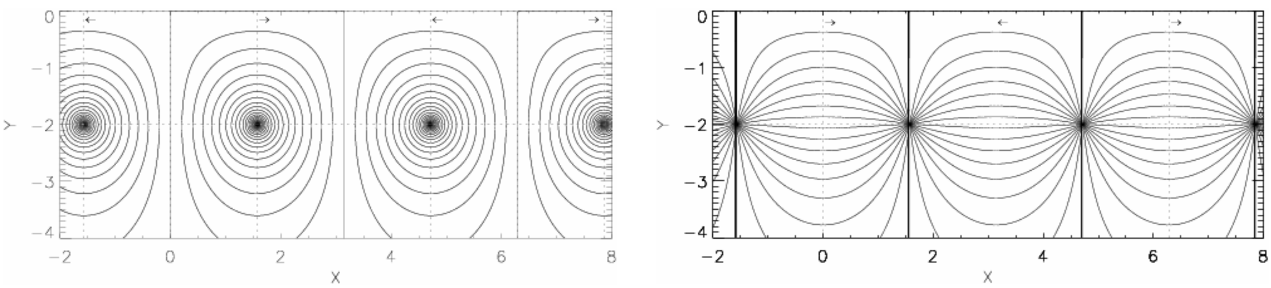
where $\varsigma_1(=e^{y_1+ix_1})$ and $\varsigma_2(=e^{y_2+ix_2})$ represent the corresponding positions of the vortex couple in the ς-plane (see figure 3b). From the circle theorem (Milne-Thomson 1962, p. 154) the complex potential of the flow induced by the pair of point vortices and their reflected images in the ς-plane is given by,

$$\omega_S(\varsigma)=-ik\log\left[\left(\frac{\varsigma-\varsigma_1}{1/\varsigma-\overline{\varsigma_1}}\right)\left(\frac{1/\varsigma-\overline{\varsigma_2}}{\varsigma-\varsigma_2}\right)\right], \tag{2}$$

where $\overline{\varsigma_1}$ and $\overline{\varsigma_2}$ are the complex conjugate of ς_1 and ς_2. Note once again that the reflection of the vortices onto the free surface represents a convenient choice for deep water only. They are placed outside the body of the fluid and used to approximate the complex potential within the fluid. For an unbounded domain with a bed, a vertically periodic set of vortices reflected onto the bed is more convenient. Then the velocity potential ϕ_S of a pair of counter-rotating vortices and its corresponding image can be expressed in the transformed plane by,

$$\phi_s(\varsigma)=-k\Re\left\{i\log\left[\left(\frac{\varsigma-\varsigma_1}{\varsigma-\varsigma_2}\right)\left(\frac{1/\varsigma-\bar{\varsigma}_2}{1/\varsigma-\bar{\varsigma}_1}\right)\right]\right\}. \tag{3}$$

The first term inside the logarithm represents the contribution of the two vortices to the system, while the second term refers to their reflected images. In our examples the point vortices are prescribed to be at fixed positions in time. The free surface moves under the influence of the eddy couple and gravity since our first aim is to analyse the contribution of not advected vortices beneath a free surface. The total circulation Γ (positive and equal to $2\pi k$, in the case of a single counter-clockwise point vortex) vanishes around the pair of counter-rotating vortices. The contribution of the eddy couple to the "total" velocity $\boldsymbol{u}$ is then given by $\nabla\phi_S$. The stream function ψ_S induced by the singularities is obtained by simply taking the imaginary part of the complex potential ω_S. Figure (4a) shows the streamlines plotted in the *z*-plane for a periodic line of counter-rotating vortices in deep water. Two counter-rotating vortices per period, equally spaced in the fluid domain, are showed in this case.



(a) (b)

Figure 4. Streamlines obtained for a periodic distribution of singularities in deep water. (a) Counter-rotating vortices. (b) Sources and sinks. In both cases $Fr=1.0x10^{-2}$, $Fr_s=1.8x10^{-3}$. Depressions at the free surface cannot be seen due to its small slope.

2.2. Sink-source model

If the singularities shown in figure (3a) are chosen as a single pair of source and sink, located at ς_1 and ς_2 respectively, then their complex potential turns to (Batchelor 1967, p.410),

$$\omega(\varsigma) = k\log\left(\frac{\varsigma-\varsigma_1}{\varsigma-\varsigma_2}\right). \tag{4}$$

Here *k* is defined as the volume flux per length unit of each of the sinks and sources. From the circle theorem, the complex potential of the flow induced by the source and sink and their reflected images in the ς-plane can be similarly determined and, taking its real part, a new velocity potential ϕ_S can be defined,

$$\phi_s(\varsigma) = k\Re\left\{\log\left[\left(\frac{\varsigma-\varsigma_1}{\varsigma-\varsigma_2}\right)\left(\frac{1/\varsigma-\bar{\varsigma}_1}{1/\varsigma-\bar{\varsigma}_2}\right)\right]\right\}. \tag{5}$$

Figure (4b) shows the corresponding streamlines. Depressions at the free surface cannot be seen due to its small slope.

2.3. Dipole model

The complex potential for an irrotational flow due to a circular cylinder of radius *a* held in a stream with uniform velocity *U* far from the cylinder is given by (Batchelor 1967, p.424),

$$\omega(z) = U\left(z + \frac{a^2}{z}\right), \tag{6}$$

where $z=x+iy$. To apply Cauchy's integral theorem to the non-periodic free surface flow problem in deep water, the complex potential ω includes for convenience the reflection of the cylinder in the free surface,

$$\omega(z) = U\left(z - z_0 + \frac{a^2}{z - z_0} + \frac{a^2}{z - \overline{z_0}}\right), \tag{7}$$

where z_0 ($=x_0+iy_0$) is the position of the centre of the cylinder and $\overline{z_0}$ its complex conjugate. The velocity potential ϕ_S is then given by the real part of (7),

$$\phi_S(x,y) = U(x - x_0)\left[1 + \frac{a^2}{(x-x_0)^2 + (y-y_0)^2} + \frac{a^2}{(x-x_0)^2 + (y+y_0)^2}\right]. \tag{8}$$

The corresponding streamlines and linear steady free surface are shown in figure (5). The contour of the cylinder approaches a perfect circle in this case and corresponds to a closed streamline in which $\psi_S =$ constant. Thus the stream advances towards the cylinder axis (*y=-2*) until the first stagnation point *A* is reached, then divides and proceeds in opposite directions round the cylinder, joins up again at the second stagnation point *B* and moves off along *y=-2*. The disturbances generated at the free surface, though steady, may deform the closed circular streamline and therefore a perfect circle is no longer obtained. However, for the purpose of our study, we assume that this is sufficiently close to a circle. Note also that surface waves cannot be seen in figure (5) due to their small wave steepness.

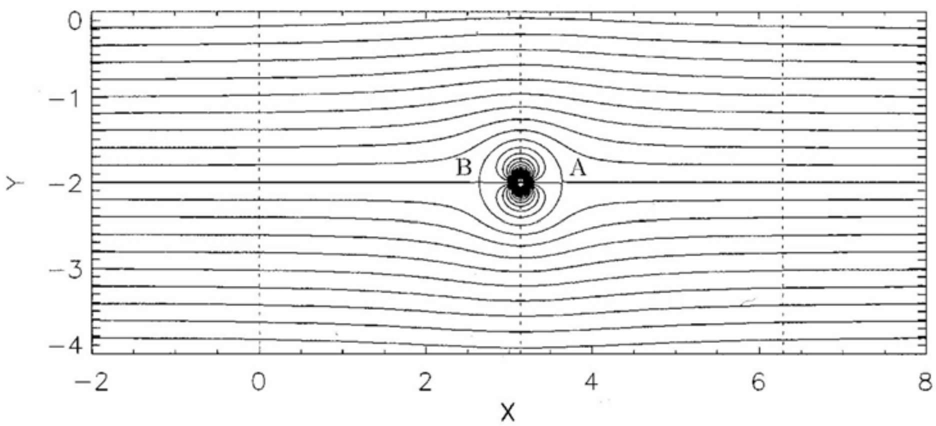


Figure 5. Streamlines obtained for a horizontal doublet in a uniform stream flow in deep water. $Fr=8.2x10^{-2}$.

3. FREE SURFACE FLOWS DUE TO SINGULARITIES: CASE STUDIES

To study the effects of nonlinearity at the free surface, the singularity models presented in the last section are introduced in the fully nonlinear boundary-integral solver developed by Dold & Peregrine (1986). Our main interest is to study the effects on a free surface of a stationary underlying flow, particularly when a train of linear water waves interacts with an underlying current, or when a submerged cylinder meets a uniform stream flow. To model the required underlying flow, a combination of vortices, sinks, sources and dipoles is employed. Depending on whether the domain is periodic or unbounded, a conformal mapping may or not be needed. Details of the boundary value problem and its numerical solution can be found in Moreira (2003a,b).

3.1. Nonlinear interactions between water waves, sinks, sources and vortices

A sink/source distribution is employed to generate a near-linear surface current. 16 sinks and 16 sources are distributed symmetrically in the period domain at the same depth $d=0.25$. For a scheme of the fluid domain see figure (6). The effect of the singularity distribution on the waves then depends on the strength k of the sinks and sources. The desired maximum and minimum velocities are then obtained choosing suitable values for k. The steady sinks and sources are "turned on" at time $t=0$ and impose a volume flux perpendicular to the plane of motion. Figure (6) shows the evolution of short surface waves ($A_0 K_0=0.04$) interacting with a near-linear surface current ($U_{min}=-0.25\ c_0$). The nonlinear results are vertically exaggerated 40 times. It is clear the wave transformation that occurs due to the underlying current. Roughly speaking, a steep and a smooth region can be identified, respectively, downstream and upstream of the U_{min} region after a certain period of time. A strong increase in wave steepness is observed close to the U_{min} region, leading to wave breaking, while wave amplitudes decrease beyond this region. Some of the waves are steep enough to be noticeably affected by nonlinearity. Partial wave blocking is predicted by linear ray theory and thus confirmed by the nonlinear computations.

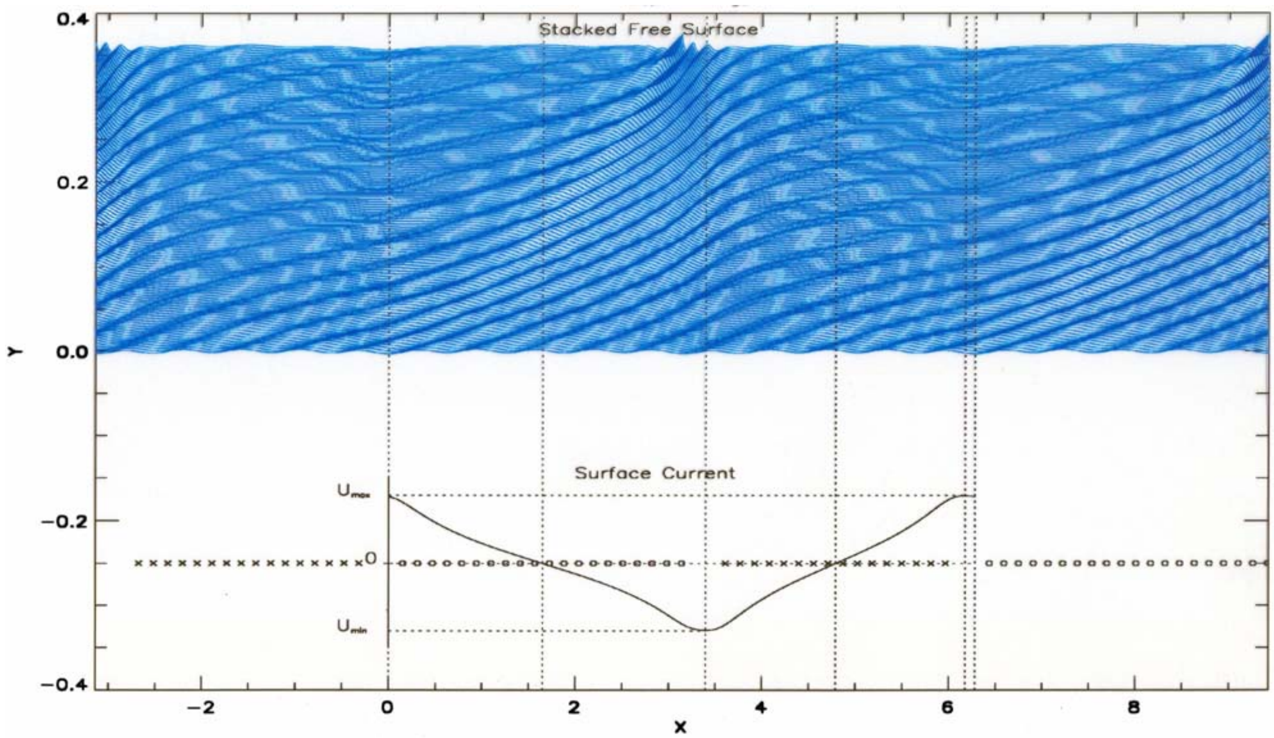


Figure 6. Stacked free surface and surface current profile due to a distribution of sources (x) and sinks (o). $Fr=0.232$, $U_{min}=-0.25\ c_0$, $A_0\ K_0=0.04$. $t_{breaking}=25.4$. Vertical exaggeration 40:1.

For "sharper" current gradients a vortex couple is positioned underneath the free surface. For a scheme of the fluid domain see figure (7). The maximum and minimum velocities are defined by choosing suitable values for the depth of submergence d. Figure (7) shows the stacked free surface deformation of a wave train due to a stationary vortex dipole flow, with $U_{min}=-0.250\ c_0$. This "rapidly" varying surface current is switched on at time $t=0$ and starts to interact with the wave train. The short waves have initial steepness of $A_0K_0=0.04$. The fully nonlinear results show that surface currents induced by vortices are sufficient enough to cause wave steepening and breaking. The wave transformation due to the underlying current is visible in figure (7). The incident waves are clearly deformed near the maximum and minimum velocity regions U_{max} and U_{min}, while their group velocity remains unchanged near the regions where $U=0$. The positive current accelerates the surface waves nearby the U_{max} region, increasing locally their kinetic energy and group velocity, while in the U_{min} neighbourhood waves start to be partially blocked. Since a strong surface current gradient is applied over one wavelength nearby the U_{min} region, ray theory assumptions are not fully satisfied there, with nonlinear effects taking over. Furthermore, since we are ignoring dissipation, in the light of the linear approximation wave action is conserved in the system as a whole. This implies that wave energy increases for rays moving into regions of greater frequencies and is lost when frequencies decrease.

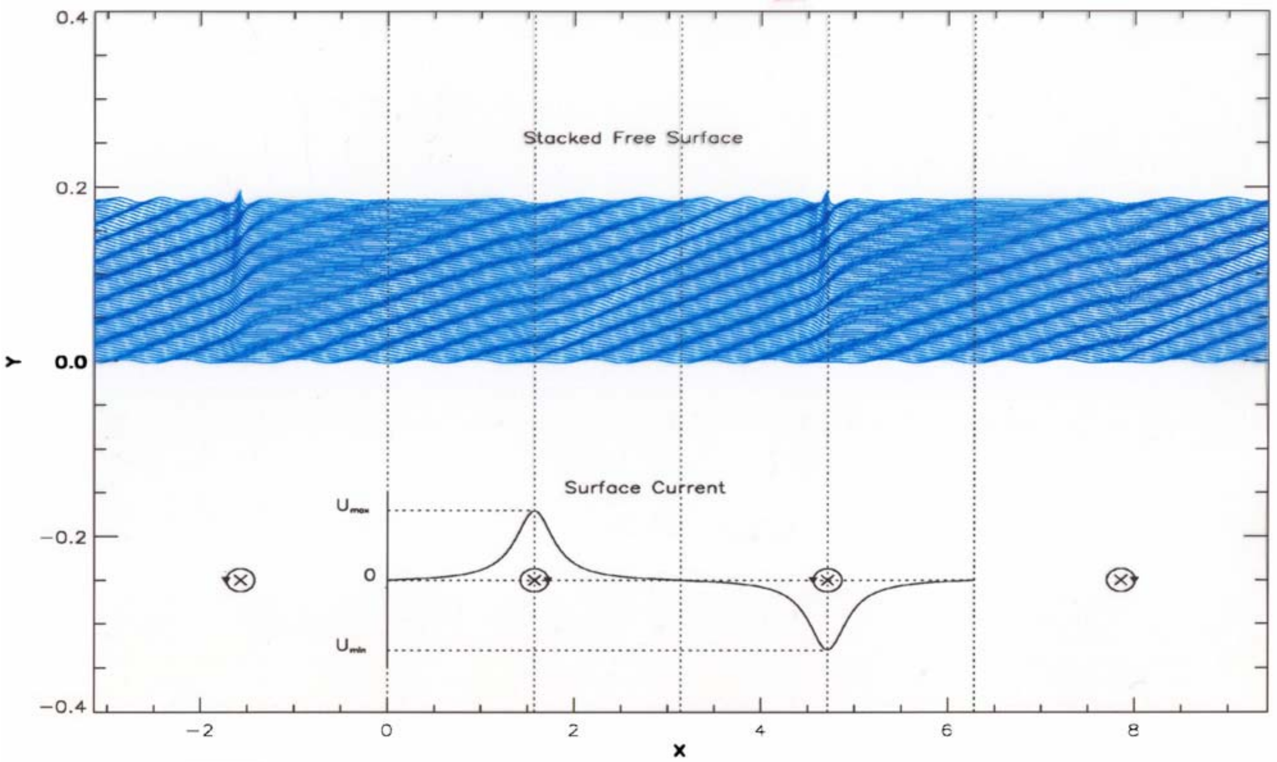
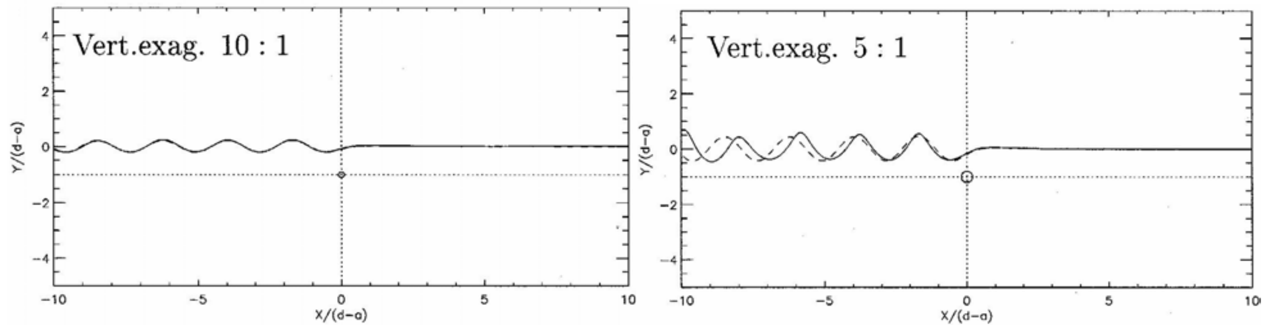


Figure 7. Stacked free surface and surface current profile induced by a pair of counter-rotating vortices. $Fr=0.08$, $U_{min}=-0.250\ c_0$, $A_0\ K_0=0.04$. $t_{breaking}=14.8$. Vertical exaggeration 40:1.

3.2. Nonlinear interactions between a free surface flow and a dipole

Figure (8) compares the linear stationary free surface profiles with quasi-steady nonlinear results for two different values of *a/d*. For clarity the cylinder is also represented in scale based on the values attributed for *a* and *d*. As expected a good agreement between linear and nonlinear results is found in figure (8a). In this case the cylinder is sufficiently far from the free surface for no nonlinear interactions to take place. On the other hand discrepancies between the profiles can be clearly noticed in case (b). In this case the cylinder becomes closer to the free surface as *a* increases for a fixed *d*. In particular figure (8b) shows that the nonlinear result becomes "steeper" than the linear one, with sharper crests and shorter wavelengths. For larger values of *a/d* e.g. $a/d=1/3$, disturbances induced by the underlying flow are unsteady and strong enough to cause wave breaking. For these cases the nonlinear computations do not reach a stationary profile after a certain time. Breaking occurs isolated downstream the cylinder, in a region very close to it. This result confirms the investigations of Tuck (1965) and Dagan (1971) who suggested that wave breaking occurs when the inequality $a/d<<1$ is not satisfied. In this case linear theory fails and Lamb's classical solution of a sinusoidal wave train downstream the cylinder does not apply.



(a) (b)

Figure 8. Comparison between nonlinear (------) and linear steady results (-- -- --).
(a) *a/d=1/10*; (b) *a/d=1/5*. For all cases *U=-0.6* and *Fr=0.44*.

4. SUMMARY

This work was motivated by previous natural observations and experiments which reported interesting features at the surface waves when meeting an underlying flow. In this paper a summary of the analytical and numerical works that have been developed aiming to model free surface flows induced by vortices, sinks, sources and dipoles is presented. The mathematical modelling of three singularity distributions is presented and used to model stationary surface flows, which is shown to affect significantly the behaviour of the free surface motion. The effects of nonlinearity at the free surface of a body of water when it interacts with these singularities are investigated through a fully nonlinear, unsteady, boundary-integral method. The nonlinear results show the importance of singularities in the modelling of free surface flow problems.

5. ACKNOWLEDGEMENT

I am grateful for the guidance and useful discussions with Prof. D.H.Peregrine of the School of Mathematics, University of Bristol. This research is supported by CNPq under contract no. 62.0018/2003-8-PADCT III / FAPERJ.

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