ON THE CHAOTIC DYNAMICS OF NON-IDEAL FRICTION-DRIVEN VIBRATING SYSTEM

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Abstract. *In engineering practical systems the excitation source is generally dependent on the system dynamic structure. In this paper we analyze a self-excited oscillating system due to dry friction which interacts with an energy source of limited power supply (non ideal problem). The mechanical system consists of an oscillating system sliding on a moving belt driven by a limited power supply. In the oscillating system considered here, dry friction acts as an excitation mechanism for stick-slip oscillations. The stick-slip chaotic oscillations are investigated because the knowledge of their dynamic characteristics is an important step in system design and control. Many engineering systems present stick-slip chaotic oscillations such as machine tools, oil well drillstrings, car brakes and others.*

Keywords: *non-ideal excitation, friction-induced vibrations, chaotic vibration,* bifurcational-like behavior

1. INTRODUCTION

A non-linear phenomenon present in mechanical systems is the dry friction between contact surfaces (Andronov et al., 1966). The characteristics of the friction produce in mechanical systems two effects: the energy dissipation and the self-excitation effects (Hagedorn, 1988). The self-excitation occurs in mechanical systems where the friction forces have significant influence on the system operation which generates stick-slip oscillations. Because of that, the dry friction has been the object of many investigations, see for instance, the references below, for some examples, with deserve others (Liang and Feeny, 1998; Oancea and Laursen, 1998) and so on. Friction between two surfaces in contact depends on many parameters, including, surface materials, lubrication, normal load, sliding velocity, and sliding acceleration (Berger, 2002; Wasfy, 2003).

In engineering systems, in general, the possible influence of the oscillating system motion on its power supply or external excitation is disregarded. However, in many engineering systems were observed that the excitation or its energy source is influenced by the system response (the Sommerfeld effect). The traditional dynamic system model is denominated system with an ideal energy source, where the existence of the interaction phenomenon between the dynamical system and the energy source is not considered. In this way, the adoption of another model based on the concept of non-ideal dynamic system, that is, a dynamic system with an energy source of limited power supply (non-ideal source) is required (Kononenko, 1969; Balthazar et al., 2002; Pontes et al, 2003-a).

In the present paper we analyze a self-excited oscillating system due to dry friction which interacts with an energy source of limited power supply (non ideal problem). In engineering practical systems the excitation source is generally dependent on the system dynamic structure. The mechanical system consists of an oscillating system sliding on a moving belt driven by a limited power supply. In the oscillating system considered here, the dry friction and the energy source act as an excitation mechanism for stick-slip chaotic oscillations. Many engineering systems present stick-slip chaotic oscillations such as machine tools, oil well drillstrings, car brakes and others. Here, the stick-slip chaotic oscillations are investigated for system design and control purpose.

2. THE ANALYZED PROBLEM

In the present work an oscillating block-belt-motor system as in (Pontes et al, 2000-a; Pontes et al, 2000-b; Pontes et al, 2002 and Pontes et al., 2003-b) is analyzed. The analyzed non-ideal self-excited system is described by the oscillating block-belt dynamical system and the rotational motion equations which can be written as

$$\begin{aligned} &m\ddot{x} + kx = F_f(x, \dot{x}, v_{\mathrm{Re}l}) \\ &I\ddot{\varphi} = T_{Motor}(\dot{\varphi}) - rF_f(x, \dot{x}, v_{\mathrm{Re}l}) \end{aligned} \tag{1}$$

where x, $\dot{x}$ and $\ddot{x}$ are the oscillating block displacement, velocity and acceleration, respectively; m the block mass; k the elastic constant; $v_{\mathrm{Re}l} = v_B - \dot{x} = r\dot{\varphi} - \dot{x}$ the relative velocity between block and belt; v_B the belt velocity; r the radius of the belt pulley or transmission rate; φ, $\dot{\varphi}$ and $\ddot{\varphi}$ are the angular displacement, velocity and acceleration of the DC motor shaft (power source), respectively; I the inertia moment of the system rotate part; $T_{Motor}(.)$ the mechanical torque, described by the DC motor characteristic; $F_f(.,.,.)$ the friction force interaction function which represents the static and dynamic friction (the Coulomb friction) effects and is as in (Karnopp, 1985). Also are defined F_D the dynamic friction force amplitude; F_S the static friction force amplitude; $\omega_N = \sqrt{k/m}$ the natural frequency of the mass-spring system.

In the ideal system, the system of equations is obtained considering the equilibrium between the motor torque and the required torque for the oscillating block-belt system. However, the angular acceleration is null and the angular velocity is constant. Then, the ideal system model may be represented by the oscillating system equation only, that is, the first equation of system (1).

3. NUMERICAL SIMULATION RESULTS

In this section we address the motor influences on the non-ideal self-excited system represented by equations (1). The numerical simulation results presented were obtained using the Matlab-Simulink™ from Mathworks®.

3.1. Motor Characteristic Influence On Chaotic Vibrations Appearance

To analyze the dependence of the non-ideal system on the motor characteristic, two set of results were obtained, as showed in Figures 1 and 2. The torque constant values assumed are K_T=0,5Nm/A and

K_T=0.05Nm/A. In both cases, the same applied voltage values are used. Each set of results show the block and motor angular velocity, the phase portrait velocity-displacement, the Poincaré map and the frequency spectrum. The results for the torque constant K_T=0.5 and K_T=0.05 are presented in Figures 1 and 2, respectively. A torque constant K_T=0.5 represents an energy source with a high power supply. The results showed that for a torque constant K_T=0.05, the oscillating system response suffers more influence from the energy source. The interaction between the motor and the oscillating system is evidenced in the different phase portraits and Poincaré maps also showed in Figures 1 and 2.

The sequence of results presented in Figures 1 and 2 shows the occurrence of quasi-periodic (regular) dynamics and non-periodic (irregular) dynamics. For instance, an irregular motion with chaotic characteristic is detected in Figure 2 for a torque constant K_T=0.05.

3.2. Dynamic Friction Influence on Bifurcational Behavior of the Ideal System

A bifurcation diagram is obtained as a set of Poincaré maps when varying chosen control parameters (Parker and Chua, 1989). The system control parameter considered here is the dynamic friction force amplitude F_D (the Coulomb friction) and the Poincaré map points are obtained when the oscillating system trajectory crosses the zero relative velocity, that is, the motion transition points of stick-slip oscillations. By this bifurcation diagram, as shown in Figure 3, the influence of the friction force amplitudes on the slip-stick, slip-slip and slip-stick transition points of the ideal system is investigated. The motion transition points as a function of the dynamic friction force amplitude characterized a bifurcational-like behavior. For a large difference between the friction force values during the stick-slip oscillations, the motion presented three transition points.

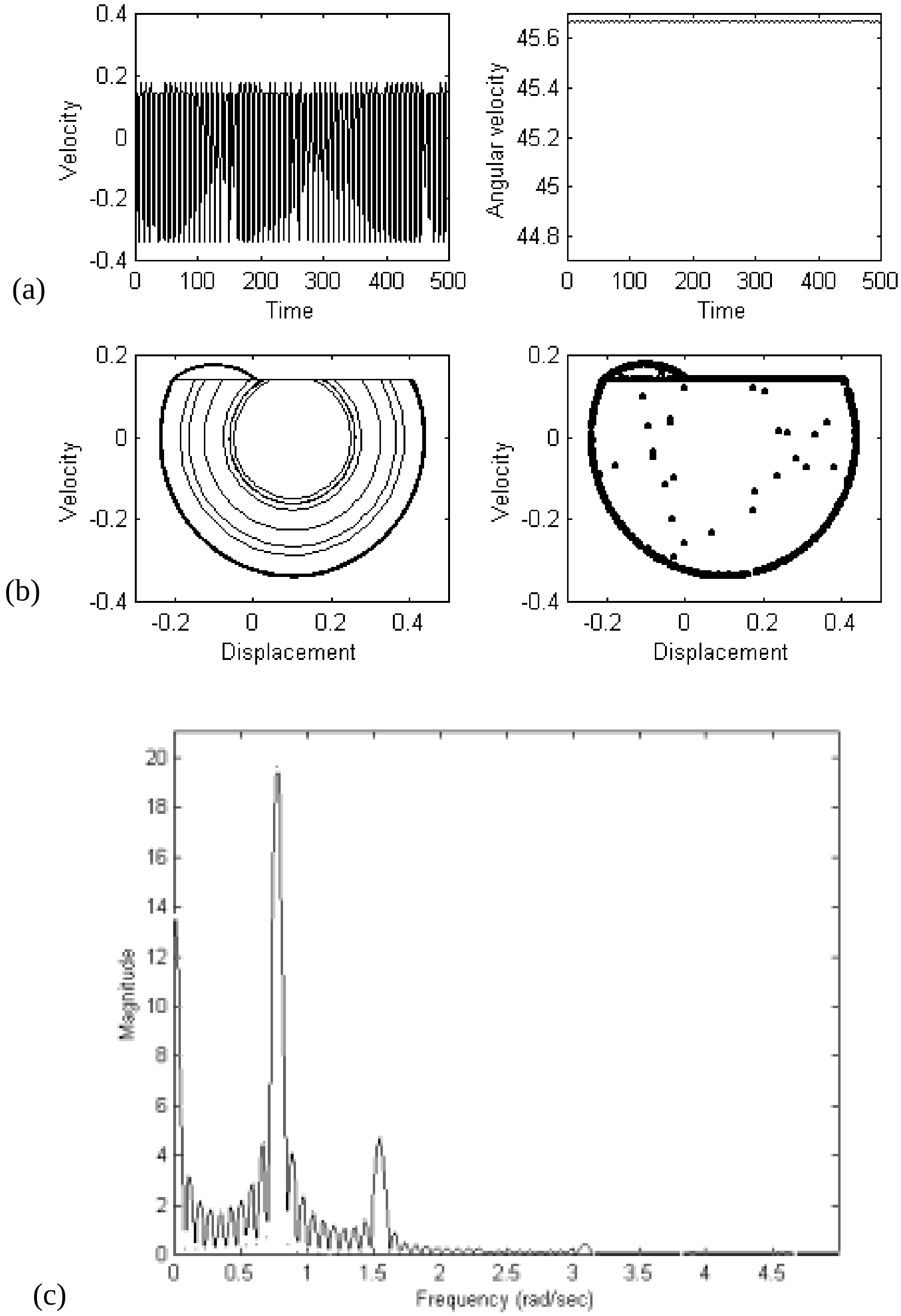

Figure 1. Non-ideal system behavior for K_T=0.5Nm/A, v_B≈0.14m/s, F_D=0.1N/kg and F_S=0.4N/kg: (a) velocity and angular velocity time responses; (b) phase portrait velocity-displacement and Poincaré section (6400 points); (c) frequency spectrum.

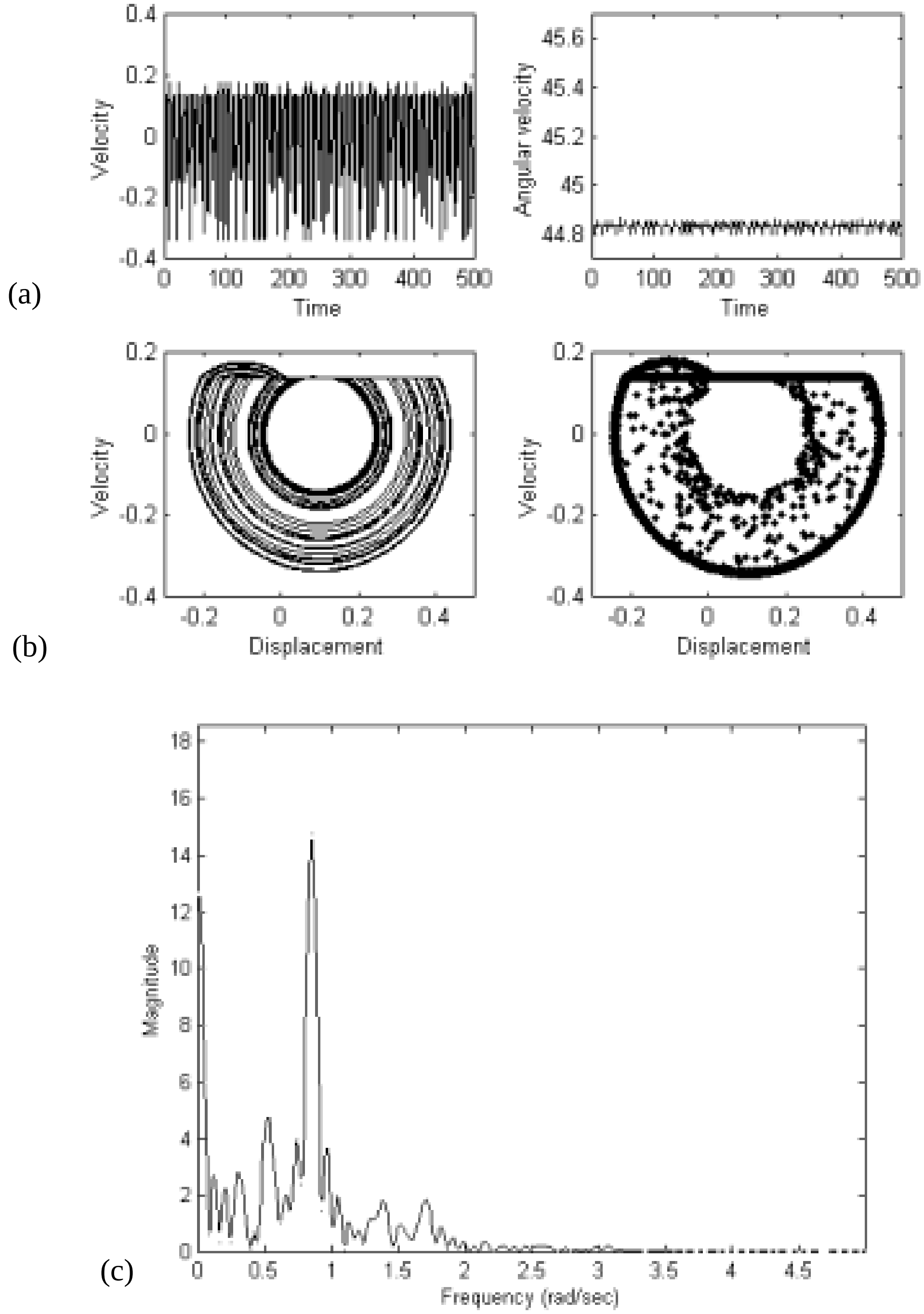

Figure 2. Non-ideal system behavior for K_T=0.05Nm/A, v_B≈0.14m/s, F_D=0.1N/kg and F_S=0.4N/kg: (a) velocity and angular velocity time responses; (b) phase portrait velocity-displacement and Poincaré section (6400 points); (c) frequency spectrum.

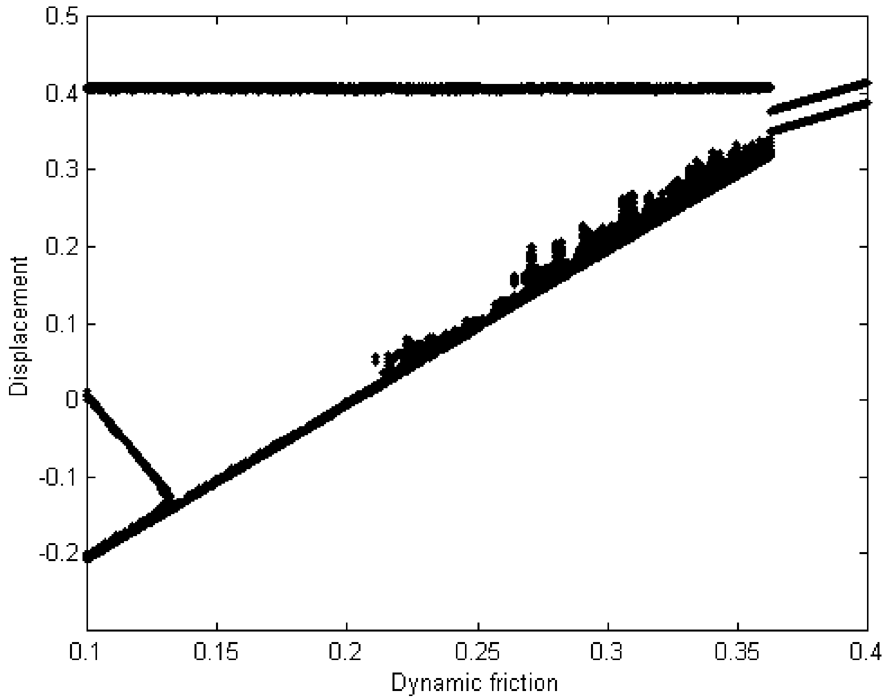

Figure 3. Ideal system: bifurcation diagram depending on the dynamic friction force amplitude F_D for v_B=0.14m/s, F_S=0.4N/kg.

4. CONCLUSION

This work analyses the behavior of a non-ideal self-excited system. The main results obtained are on the interaction between the system power supply and the self-excited system with a dry friction model. The numerical results showed the occurrence of different types of dynamic interaction. For a torque constant K_T=0.05, the oscillating system response suffers more influence from the energy source evidencing the motor influence on the oscillating system. The influence of the motor interaction on the self-excited system responses are observed along with non-periodic motions with chaotic characteristics when the motor characteristic represents a source of limited power. Then, on the chaotic vibrations appearance was detected the influence of the motor characteristic and the bifurcational behavior.

5. ACKNOWLEDGMENTS

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