SELF-SYNCHRONIZATION IN THE PARAMETRICALLY AND SELF-EXCITED SYSTEM WITH TWO NON-IDEAL SOURCES

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Abstract. *In this work, the phenomenon of self-synchronization and synchronization in parametrically and self-excited system subjected to two non-ideal energy source are examined by numerical simulations. Considered model consists of a nonlinear spring with periodically changing stiffness, a nonlinear damper described by Rayleigh's term and two unbalanced identical direct current motor with limited power. A non-ideal source depends on the response of the nonlinear system and acts it on. The non-ideal systems have two additional degrees of freedom in the original nonlinear system.*

Keywords: Self-synchronization, non-ideal system, parametrical self-excited system.

1. INTRODUCTION

In engineering practice, we can distinguish systems which oscillations are caused by different reasons. The well-known oscillations are: self-excited systems in which, roughly speaking, a constant input produces periodic output; parametrically excited systems characterized by periodically changing in time parameters and systems excited by an external force. All these systems were comprehensively analyzed in literature separately. However, we can find some papers devoted interaction between different kinds of vibrations, for example: self and external excited vibrations, self and parametric excited vibrations, as well as between self-parametric and external excited vibrations (Warminski et al., 2001; 2002).

When a forcing function is independent of the system it acts on, then the function is called ideal. In such case, the excitation may be formally expressed as a pure function of time. If in a certain model its ideal source is replaced by a non-ideal source, the excitation can be put in the form, where it is a function, which depends on the response of the system. Therefore, non-ideal source cannot be expressed as a pure function of time but rather as an equation that relates the source to the system of equations that describes the model. Hence, non-ideal models always have one additional degree of freedom as compared with similar ideal (Kononenko, 1969; Balthazar, et al., 2001, 2002).

The goal of this paper is to analyze the behavior of the phenomenon of self-synchronization (Blekhman, 1998, 2000; Dimentberg, 2001, Palacios, 2002, 2003), which may be arisen, if we taken into account two DC motors, with a limited power supply and with masses attached eccentrically to their rotating shafts. Here, we are interested, in analyze: the influenced of the response of the parametrically and self-excited system on the DC motors. Furthermore, we consider the problem of synchronization for type systems.

2. DYNAMICAL MODEL OF THE SYSTEM

Let consider the parametric and self-excited model, which includes two direct current (DC) motors with limited power, operating on a structure (Fig. 1). The excitation of the system is limited by the characteristic of the energy source. Vibration of the system depends on the motion of the motors, and the energy sources motion depends on vibration of the system, as well. Then, coupling of the vibrating oscillator and the two DC motors takes place. Hence, it is important to analyses what will happen to the motors, as the response of the system changes.

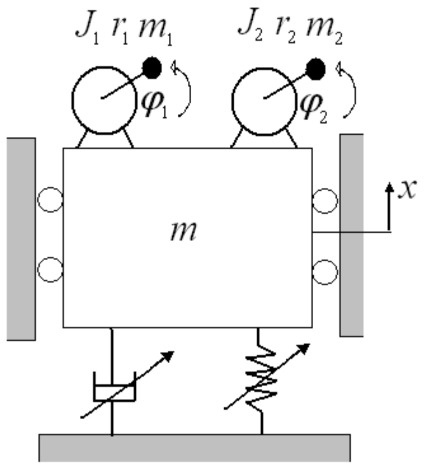

Figure 1. Non-ideal parametrically and self excited system model

Taking into account the extension of the differential equation of the complete electro-mechanical system presented in (Warminski and Balthazar, 2001) and Fig. 1 we can write:

$$J_s\ddot{\varphi}_s = L_s(\dot{\varphi}_s) - H_s(\dot{\varphi}_s) + m_s r_s \ddot{x}\cos\varphi_s - m_s g r_s \cos\varphi_s\,,\ (s = 1,2)$$
$$m\ddot{x} + (-c_1 + \hat{c}_1\dot{x}^2)\dot{x} + (k - \sum_{j=1}^{2} k_j \cos 2\varphi_j)(x + k_3 x^3) = \sum_{j=1}^{2} m_j r_j (\dot{\varphi}_j^2 \sin\varphi_j - \ddot{\varphi}_j \cos\varphi_j)\,. \qquad (1)$$

where x is oscillatory coordinate of vibrating body; φ_s is rotational coordinate of each DC motor; $\dot{\varphi}_s$ is rotational speed of rotors; J_s is a moment of inertia of each motor; m_s is unbalanced mass of each motor; r_s is eccentricity of each unbalanced mass; $L_s(\dot{\varphi}_s)$ is a controlled torque of each DC motor; $H_s(\dot{\varphi}_s)$ is a resistance torque of each DC motor.

To obtain the governing equations of the system in their dimensionless form, we define the non-dimensional time $\tau = \omega t$ and the non-dimensional displacement $X = \frac{M}{m_1 r_1} x$, where $\omega = \sqrt{\frac{k}{M}}$ is natural frequency of the system and $M = m + m_1 + m_2$. Hence, we will obtain:

$$\varphi_s'' = \hat{a}_s - \hat{b}_s\dot{\varphi}_s + \beta_s R_s X'' \cos\varphi_s \quad (s = 1,2)$$

$$X'' + (-\mu + q\dot{x}^2)\dot{x} + (1 - \sum_{j=1}^{2}\alpha_j \cos 2\varphi_j)(1 + pX^3)X = \sum_{j=1}^{2} R_j(\varphi_j'^2 \sin\varphi_j - \varphi_j'' \cos\varphi_j) \quad (2)$$

where $\hat{a}_s = \dfrac{a_s}{\omega^2 m_1 r_1^2 I_s}$, $\hat{b}_s = \dfrac{b_s}{\omega m_1 r_1^2 I_s}$, $R_s = \dfrac{m_s r_s}{m_1 r_1}$, $\beta_s = \dfrac{m_1}{M I_s}$, $\mu = \dfrac{c_1}{Mw}$, $q = \dfrac{\hat{c}_1 \omega m_1 r_1^2}{M^3}$, $\alpha_j = \dfrac{k_j}{k}$, $p = \dfrac{k_3 m_1 r_1^2}{M^2}$, $I_s = \dfrac{J_s}{m_1 r_1^2}$.

3. NUMERICAL RESULTS

Next, we carried out, a number of numerical simulations in order to observe the interaction between the two identical non-ideal DC motors and parametrically and self-excited structural system (as regular and irregular motion). Furthermore, we observe the self-synchronization and synchronization phenomenon in pre-resonance, resonance and post-resonance regions. The dimensionless nonlinear equations (2) were simulated using the block diagrams of SIMULINK™ that modeling the non-ideal parametrically self-excited system. To obtain different regimes in the system we varied the torques of each DC motor and we varied the initial rotational of second motor.

The first numerical result, shown in Fig. 2, we illustrate the development of self-synchronization by intervals when the torques of each DC motor are equal approximately $\hat{a}_1$ = 1 and $\hat{a}_2$ = 0.9.

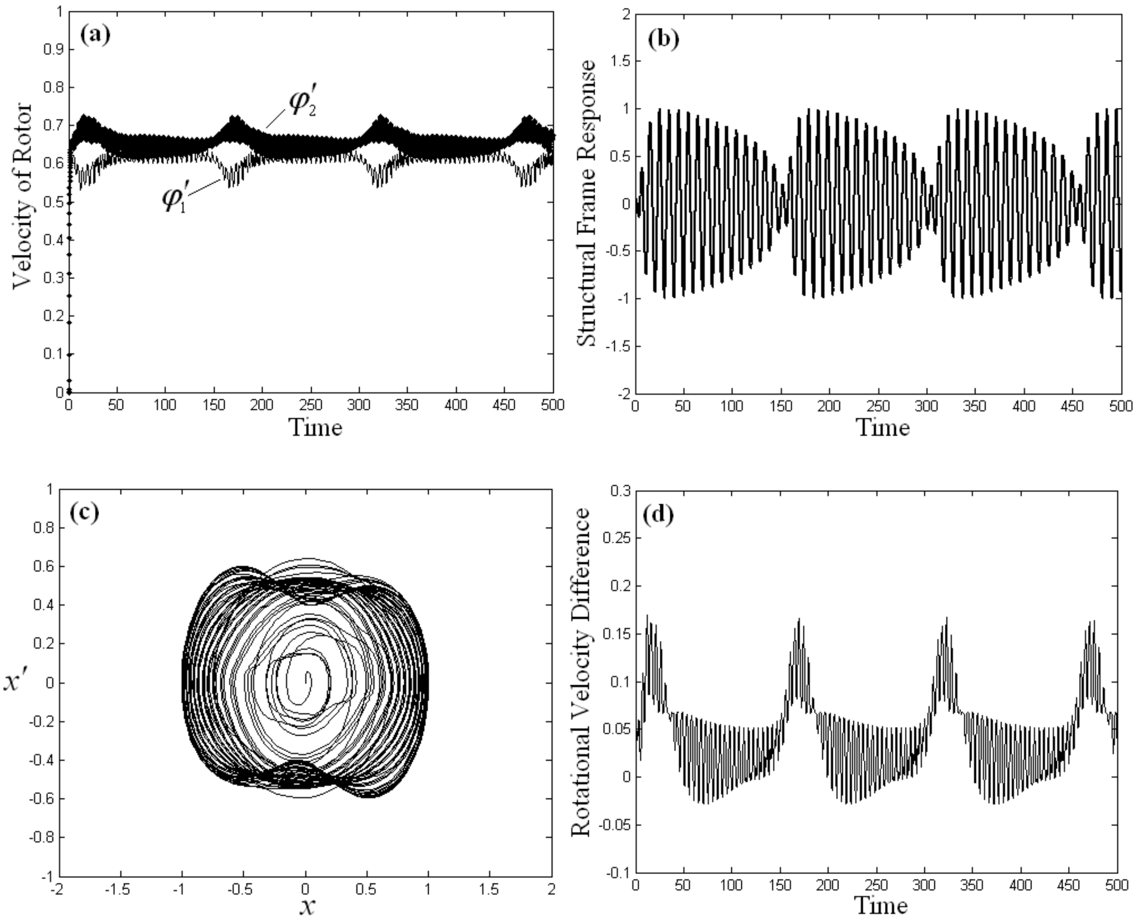

Figure 2. Detail dynamics of the system in the presence of self-synchronization by intervals for torques: $\hat{a}_1$ = 1 and $\hat{a}_2$ = 0.9

Figure 2(a), shows that the rotors turn in the same direction and arrive it at some average angular velocity in steady state motion where the angular velocities are in phase and anti-phase by intervals (the rotors synchronize phase and anti-phase), the velocities of rotors are below of resonance region (pre-resonance).

Figures 2(b) and 2(d) confirm this fact, by each interval of synchronization phase the velocity difference approaches at value zero and supporting structural response decreases. The phase plane, Fig. 2(c), shows the dynamical behavior of system. For this case, we consider the initial conditions for the rotors $\varphi_1'(0) = 0.0$, $\varphi_1(0) = 0.0$, $\varphi_2'(0) = 0.0$, $\varphi_2(0) = \pi$.

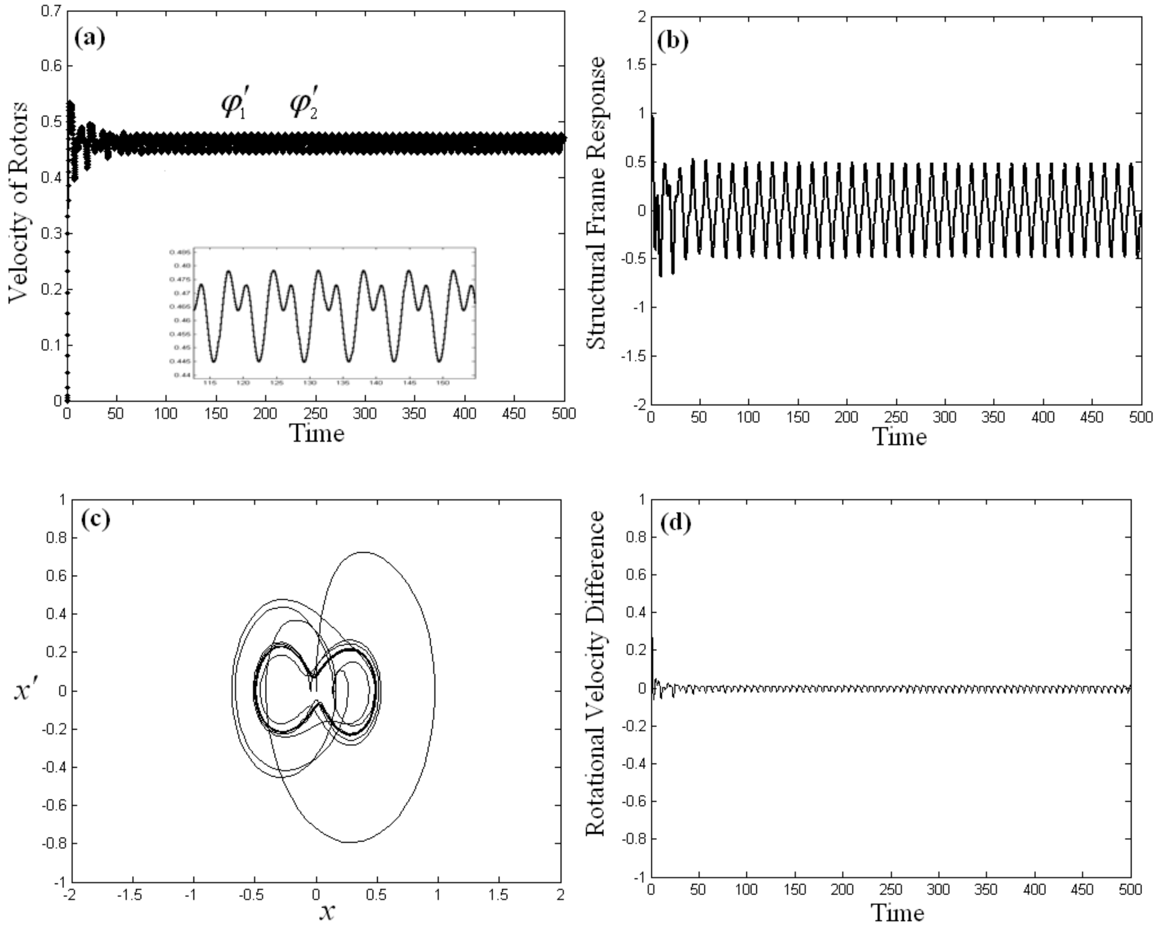

Figure 3. Detail dynamics of the system in the presence of self-synchronization for torques: $\hat{a}_1$ = 0.7 and $\hat{a}_2$ = 0.69

The second numerical result, shown in Fig. 3, we illustrates the development of self-synchronization when the torques of each DC motor are equal $\hat{a}_1 = 0.7$ and $\hat{a}_2 = 0.69$. Figure 3(a), shows that the rotors turn in the same direction and arrive it at some synchronous velocity in steady state motion, the velocities of rotors are below of resonance region (pre-resonance). Fig. 3(b) and 3(d) confirm this fact, by each interval of synchronization phase the velocity difference approaches at value zero and supporting structural response decreases. The phase plane, Fig. 3(c), shows the dynamical behavior of system. For this case, we consider the initial conditions for the rotors $\varphi_1'(0) = 1.5$, $\varphi_1(0) = 0.0$, $\varphi_2'(0) = 0.0$, $\varphi_2(0) = \pi/2$.

The third numerical result, shown in Fig. 4, the same dates of the torques of second numerical results, we illustrates the presence of self-synchronization and the synchronization in phase. For this case, we consider the initial conditions for the rotors $\varphi_1'(0) = 1.5$, $\varphi_1(0) = 0.0$, $\varphi_2'(0) = 0.5$, $\varphi_2(0) = \pi$.

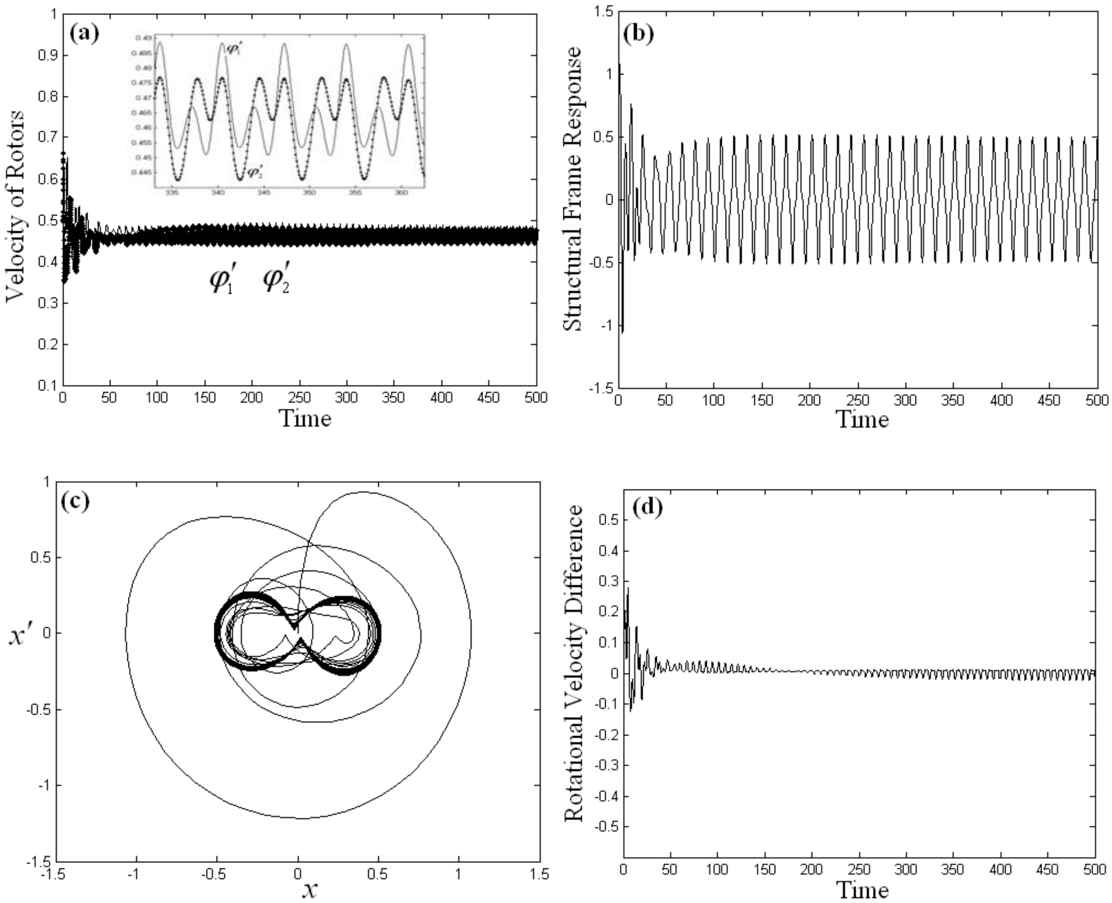

Figure 4. Detail dynamics of the system in the presence of self-synchronization for torques: $\hat{a}_1$ = 0.7 and $\hat{a}_2$ = 0.69.

The four numerical result, shown in Fig. 5, we illustrates the absence of self-synchronization when the torques of each DC motor are different $\hat{a}_1$ = 2.7 and $\hat{a}_2$ = 1.2. Figure 5(a), shows that the rotors turn in the same direction and arrive it at some average angular velocity in steady state motion, the velocities of rotors are captured in the resonance region (above) and synchronization anti-phase.

Figures 5(b) and 5(d) confirm this fact, the synchronization anti-phase in steady state motion the velocity difference creases and the average velocity difference not tending to at value zero and supporting structural response not decreases. The phase plane, Fig. 5(c), shows the dynamical behavior of system. For this case, we consider the initial conditions for the rotors $\varphi_1'(0)$ = 1.5, $\varphi_1(0)$ =0.0, $\varphi_2'(0)$ = 0.5, $\varphi_2(0)$ = $\pi/2$.

The five numerical result, shown in Fig. 6, we illustrates the slow presence of self-synchronization when the torques of each DC motor are different $\hat{a}_1$ = 3 and $\hat{a}_2$ = 2.5. Figure 6(a), shows that the rotors turn in the same direction and arrive it at some average angular velocity in steady state motion, the velocities of rotors are in the post resonance region and synchronization anti-phase.

Figures 6(b) and 6(d) confirm this fact, the synchronization anti-phase in steady state motion the velocity difference creases and the average velocity difference tending approximately to at value zero by above and supporting structural response has decreases small. The phase plane, Fig. 6(c), shows the dynamical behavior of system. For this case, we consider the initial conditions for the rotors $\varphi_1'(0)$ = 1.5, $\varphi_1(0)$ =0.0, $\varphi_2'(0)$ = 0.5, $\varphi_2(0)$ = $\pi/2$.

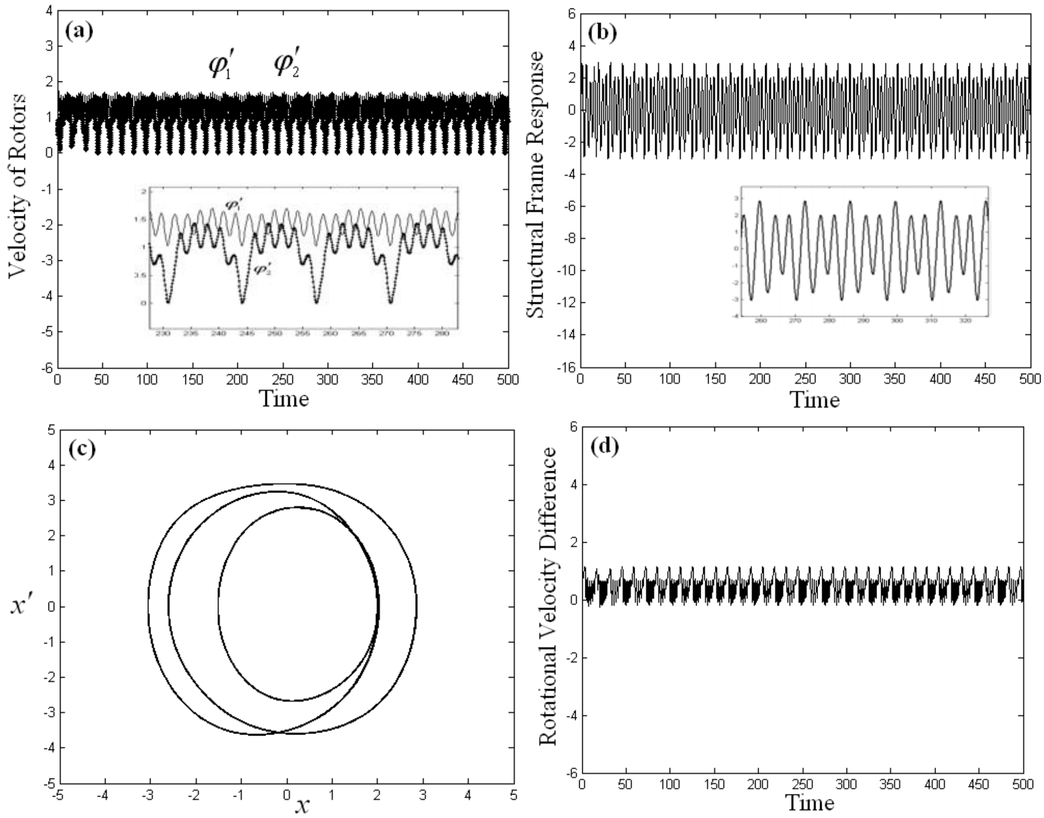

Figure 5. Detail dynamics of the system in the absence of self-synchronization for torques: $\hat{a}_1 = 2.7$ and $\hat{a}_2 = 1.2$.

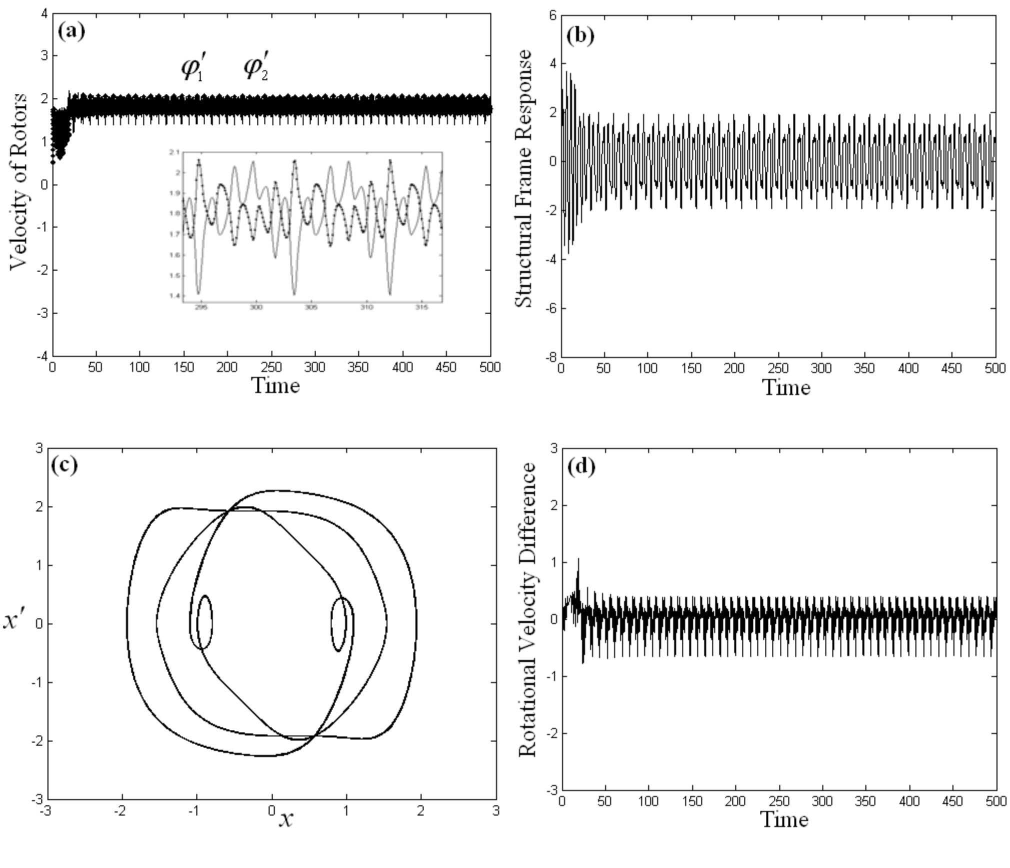

Figure 6. Detail dynamics of the system in the absence of self-synchronization for torques: $\hat{a}_1 = 3$ and $\hat{a}_2 = 2.5$.

Figure 7, shows different regimes, in the non-ideal parametrically and self excited system when we varied the parameter α_2 (see Eq. (2)) of the parametric excitation of the second DC motor. Other parameters are fixed. For α_2=1, we see a development chaotic, α_2=1.5 we see a development periodic with p-period, α_2=2 we see that is tending to a limit cycle with 1-period, α_2=3 we see a development periodic with 1-period,

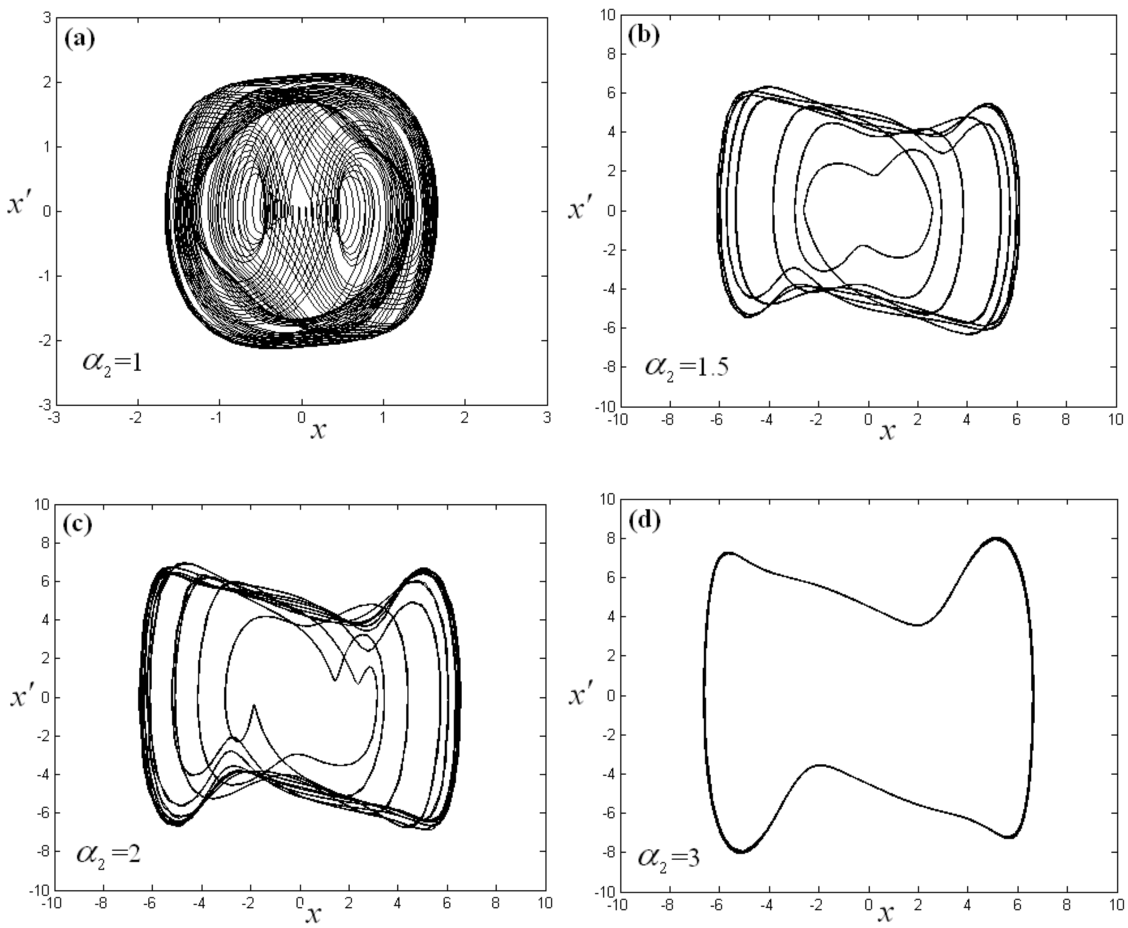

Figure 7. Phase plane for values different of the parameter α_2 of the parametric excitation of second DC motor: (a) α_2 = 1, (b) α_2 = 1.5, (c) α_2 = 2 and (d) α_2 = 3.

Finally, we present a numerical result in complete regimes of the system in the pre-resonance, resonance capture, post-resonance regions corresponding to constant torques increasing. We investigated the possibility of the presence, at same time, of the nonlinear phenomenon of self-synchronization, synchronization, Sommerfeld, and jump (see Fig. 8).

Figure 8(a), shows four resonance regions.

The region A corresponding to pre-resonance.

The region B corresponding to resonance capture were we observe the following: the angular velocities of rotors are captures, when increase the constant torques (additional power supplied to each motor) results that the rotors continues operating in the resonance region (small change in its angular velocities, see Fig. 8(a)) and a large increase in the amplitude of the response of the structure supporting (see Fig. 8(b)). In final of the region B and initial of the region C (post-resonance) was altered angular velocities of the non-ideal sources behavior the jump phenomenon. This is referred as the Sommerfeld effect, in honor of the first man who observed it (Sommerfeld, 1902).

The presence of self-synchronization and manifestations of a non-ideal energy source in the regions B and D (post-resonance) as is justified by the average velocity difference approaches the value zero but we observe that not arrive constants values and exhibits oscillations due to the structural response influence in the rotation of the rotors. In the Region B, the presence of self-synchronization, minimize the increasing of the amplitudes of the structure supporting.

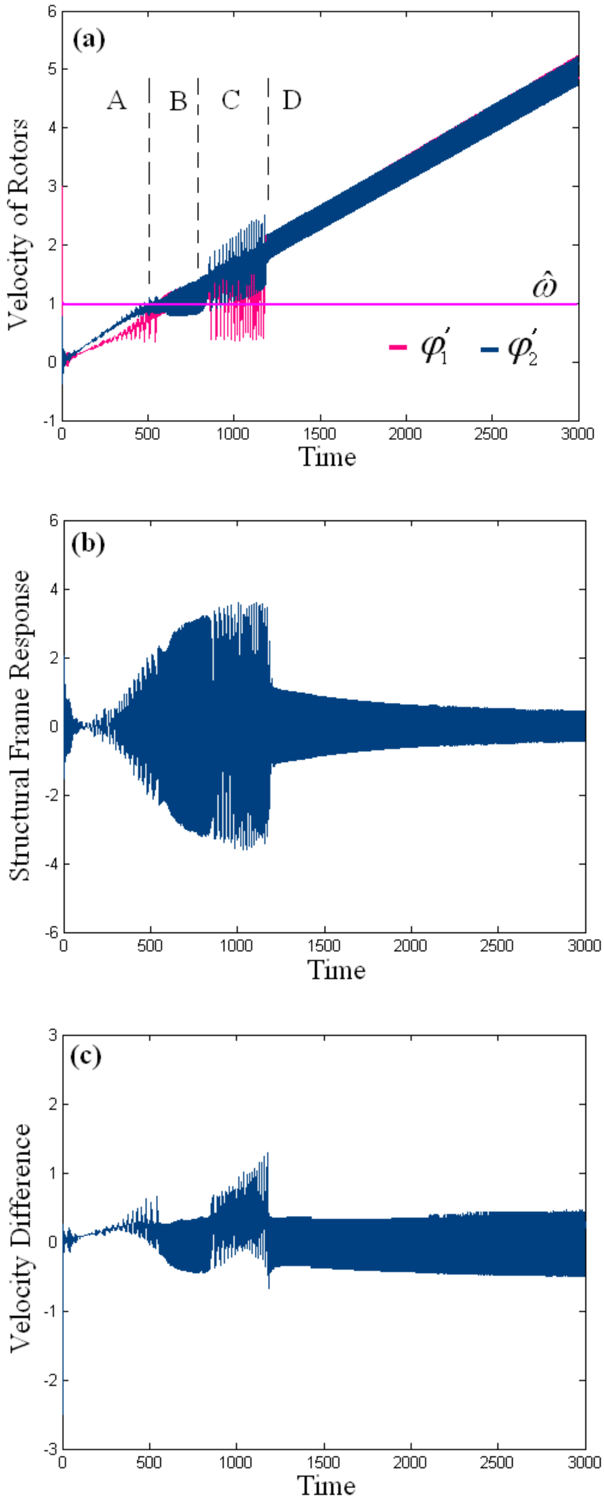

Figure 8. Detail dynamics of the system in the simultaneously presence of the nonlinear phenomenon: self-synchronization, Sommerfeld, non-ideal and jump.

4. CONCLUSIONS

A particular case of non-linear phenomenon of self-synchronization and synchronization in pre-resonance, resonance and post-resonance regions between the unbalanced dc motors that interacting with the parametrically and self-excited system has been analyzed through numerical simulation. In the presence of self-synchronization we observe that the amplitudes of the structure supporting decreasing which will be implemented as control technique in work future. From mathematical point of view was placed two different parametric excitations in the equation of motion. Moreover, we observe the Sommerfeld effect, regular and chaotic motion in non-ideal parametrically self-excited system.

5. REFERENCES

Balthazar J.M., Rente M.L. and Davi V.M., 1997, "Some Remarks on the Behaviour of a Non-Ideal Dynamical System", Nonlinear Dynamics, Chaos, Control and Their Applications to Engineering Sciences, Vol. 1, 97-104.

Balthazar, J.M., Mook, D.T., Weber, H.I., Brasil, R.M.L.R.F., Fenili, A., Belato, D. and Felix, J.L.P., 2003, " An Oerview on Non-Ideal Vibrations", Meccanica 38, pp. 613-621.

Blekhman, I.I., 1998, "Self-Synchronization in Science and Technology", ASME Press, New York.

Blekhman, I.I., 2000, "Vibrational Mechanics: Nonlinear Dynamic Effects, General Approach, Applications", World Scientific.

Brasil, R.M.L.R.F., 1999, "Multiple Scales Analysis of Nonlinear Oscillations of a Portal Frame Foundations for Several machines", RBCM-J. of the Brazilian Soc. of Mechanical Sciences, Vol. 21, No 4, pp. 641-654.

Dimentberg, M., Cobb, E. and Mensching, J., 2001, "Self-Synchronization of Transient Rotation in Multiple Shaft Systems", Journal of Vibration and Control, 7, pp. 221-232.

Kononenko, V.O., 1969, "Vibrating Systems with Limited Power Supply", Illife.

Palacios, J.L., 2002, "Teoria de Sistemas Vibratórios Não-Lineares e Não-Ideais", Tese de Doutorado, Universidade Estadual de Campinas, Campinas, SP, Brasil, 181 p. (in Portuguese).

Palacios, J.L., Balthazar, J.M. and Brasil, R.M.L.R.F., 2003, A Short Note on the Non-Linear Dynamics of a Non-Linear System Under Two Non-Ideals Excitations,. RBCM-J. of the Brazilian Soc. of Mechanical Sciences, Vol. 25, No 4, pp. 391-395.

Sommerfeld, A., 1902, "Beiträge Aum Dynamischen Ausbau Der Festigkeitslehe", Physikal Zeitschr, 3, pp. 266-286.

Warminski, J., Balthazar, J.M. and Brasil, R.M.L.R.F, 2001, "Vibration of Non-ideal Parametrically and Self-Excited Model", Journal of Sound and Vibration, Vol. 245, No 2, 363-374.

Warminski, J. and Balthazar, J.M., 2003, "Vibrations of a Parametricalllyand Self-Excited System with Ideal and Non-ideal Energy Sources", RBCM-J. of the Brazilian Soc. of Mechanical Sciences, Vol. 26, No 4, pp. 413-419.