SYSTEM FOR PERCUSSIVE DRILLING WITH ROCK MODEL

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Abstract. *New drilling techniques have been studied to increase the penetration rate in hard rock formations. One approach, that appears suitable for off shore drilling in deep seas, uses harmonic loads and, in some cases, impacts. Hard rocks present a resistance to drilling that can be modeled as a dry friction on the drill bit, which is rotating under static loading. The drilling is therefore a percussive penetration phenomenon, allowing the forward motion (with a drift) but in stick-slip condition due to the rock resistance and may be considered with and without impact. This paper focuses on numerical investigations and presents results using a novel way to change between the several phases that are possible in this nonlinear problem with two discontinuities. It is also shown that the behavior may vary from periodic to chaotic motion. Some engineering aspects are also analyzed.*

Keywords. *Drillstring, Vibro-impact, Dry-friction, nonlinear non-smooth dynamic system*

1. INTRODUCTION

Drilling hard rocks with three cone lobed bits produces a sinusoidal surface with an amplitude of 3 to 7 mm and frequency three or six times the speed of rotation [1]. This is a result of the interaction with the axial vibration of bottom hole assembly and may be used to create a self-excited vibration in this element. There is still some basic research needed to physically understand all the aspects of this phenomenon, on the other hand it is useful to know if this additional effect can increase the rate of penetration (ROP).

This paper uses a model for the resistance of the media in hard rocks coming from the literature [2,3], represented by a dry friction, resulting in a percussive penetration due to the possible stick-slip effect. The ROP is a consequence of the static loading (WOB – weight on bit), the dynamic loading due to the mentioned self-excitation and also the possible vibro-impact force. Shortly, the excitation coming from the lobes at the surface may resonate an internal hammer mass vibrating in clearances, where impacts are possible, to interact with the stick-slip of the percussive motion.

Several parameters have to be adjusted for an optimisation of the procedure and this is best done through a numerical simulation, after the selected model is validated. This is presented in [4]; in the actual work we focus the numerical technique which was used and analyse some results..

2. MATHEMATICAL MODEL

In this work we considered a physical model according to Fig. 1. The dynamics of the system corresponds to a progressive oscillatory movement along the drift. The mass m_1 is connected to the mass m_2, through a spring K and a viscous damper C. A preload B and a harmonic force are present in the external excitation applied to m_1.

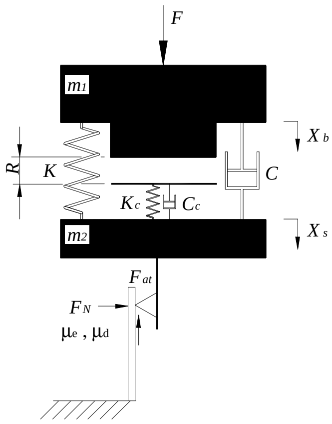


Figure 1 – Physical model.

As in the stick-slip phenomenon, the progressive movement occurs when the sum of the elastic and viscous forces applied on m_2 overcomes the friction force, F_{at}, between mass m_2 and the drift. Therefore, the system presents two different motion phases: *stick-phase*, without progression and *slip-phase*, with progression.

Then, for $F_{at} \geq K\,(X_b - X_s) + C\,(\dot{X}_b - \dot{X}_s)$, we don't have the progression and the system just corresponds to a spring-damper-mass (*stick-phase*):

$$\begin{aligned} & m_1\ddot{X}_b + C(\dot{X}_b - \dot{X}_s) + K(X_b - X_s) = A\cos(\omega t + \varphi) + B \\ & \dot{X}_s = 0 \end{aligned} \quad . \tag{1}$$

where B is the static load or preload, A the dynamic load amplitude and ω the excitation frequency. The phase angle, φ, is a phase shift between the overall drilling force and the progressive stationary motion.

On the other hand, when $F_{at} < K\,(X_b - X_s) + C\,(\dot{X}_b - \dot{X}_s)$, we have progression and the system equations are written as (*slip-phase*):

$$\begin{aligned} & m_1\ddot{X}_b + C(\dot{X}_b - \dot{X}_s) + K(X_b - X_s) = A\cos(\omega t + \varphi) + B \\ & m_2\ddot{X}_s + C(\dot{X}_s - \dot{X}_b) + K(X_s - X_b) = F_{at} \end{aligned} \quad . \tag{2}$$

The dry friction force is defined through a simple continuous model described as:

$$F_{at} = F_N[\mu_d + (\mu_e - \mu_d)\exp^{-\gamma\dot{X}_s}] \tag{3}$$

where F_N is the normal force, μ_e and μ_d are, respectively, the static and kinetic friction coefficients and γ is the decay parameter.

However, the m_1 displacement is limited in positive direction by the gap R. Thus, when numerically $X_b - X_s > R$, we have impact or contact. Note that the impulsive force resulting from the impact will only have a consequent penetration if the stick force is overcome and, therefore, it is necessary to know the contact force at each time. We use Hunt & Crossley's model [5]:

$$F_c(\delta,\dot{\delta}) = m\ddot{\delta} = -k_c\delta^n - C_c\delta^n\dot{\delta} = -k_c\delta^n(1+\lambda\dot{\delta}); \qquad \lambda = \frac{C_c}{k_c}; \tag{4}$$

where, δ is the indentation, m the relative mass ($m_1 m_2/m_1+m_2$), K_c the contact spring constant, λ the damping constant and the exponent n, which is often close to one, depends on the contact surface geometry . In addition there is a coupling between the impact and the dry friction, and the contact force can only be used to generate progression if it exceeds the damping force. The following discrete model, with the knowledge of the contact force is used in this paper:

$$F_{c\max} \le F_{at} \begin{cases} \dot{X}_b^+ = V_{rel}(t_{imp}) \\ \dot{X}_s^+ = 0 \end{cases} \quad \text{or} \quad F_{c\max} > F_{at} \begin{cases} \dot{X}_b^+ = \dfrac{m_1 \dot{X}_b^- + m_2 \dot{X}_s^- + m_2 V_{rel}(t_{imp})}{m_1 + m_2} \\ \dot{X}_s^+ = \dot{X}_b^+ - V_{rel}(t_{imp}) \end{cases} . \tag{5}$$

where $V_{rel}(t) = \dot{X}_b^+ - \dot{X}_s^+$ is the relative progression speed and t_{imp} the final impact time.

The ordinary differential equations of second order (1) and (2) can be transformed in a first order system of differential equations in an *autonomous system*, 5–*Dim*, through the following change of variables:

$$\tau = \omega_0 t;\ dx/d\tau = x';$$
$$x_1 = \frac{K}{\mu_e F_N} X_b,\quad x_2 = dx_1/d\tau = \frac{K}{\mu_e F_N} X'_b;$$
$$x_3 = \frac{K}{\mu_e F_N} X_s,\quad x_4 = dx_3/d\tau = \frac{K}{\mu_e F_N} X'_s;\ x_5 = \eta\tau. \tag{6}$$

Therefore,

$$\begin{cases} x_1' = x_2 \\ x_2' = a + b\cos(x_5 + \varphi) - 2\xi_1(x_2 - x_4) - (x_1 - x_3) \\ x_3' = P x_4 \\ x_4' = P(-2\xi_2(x_4 - x_2) - \varepsilon(x_3 - x_1) - \varepsilon f_{at}) \\ x_5' = \eta \end{cases} \tag{7}$$

where,

$$\omega_0 = \sqrt{\frac{K}{m_1}},\ \eta = \frac{\omega}{\omega_0},\ a = \frac{A}{\mu_e F_N},\ b = \frac{B}{\mu_e F_N},\ f_{at} = \left[\frac{\mu_d}{\mu_e} - \left(1 - \frac{\mu_d}{\mu_e}\right)\right]\exp^{-\gamma' x_4},\ \gamma' = \frac{\gamma\omega_0\mu_e F_N}{K}$$

$$\xi_1 = \frac{C}{2m_1\omega_0},\ \varepsilon = \frac{m_1}{m_2} \text{ and } \xi_2 = \frac{C}{2m_2\omega_0} = \varepsilon\xi_1. \tag{8}$$

When impact is identify, $(x_1 - x_3) > r$, the initial conditions of velocity are change, conform Eq. (5):

$$\begin{cases} x_2^+ = v, & f_c \max. \le f_{at} \\ x_2^+ = \dfrac{m_1 x_2^- + m_2 x_4^- + m_2 v}{m_1 + m_2}, & f_c \max. > f_{at} \end{cases} \quad \text{or} \quad \begin{cases} x_4^+ = 0, & f_c \max. \le f_{at} \\ x_4^+ = x_2^+ - v & f_c \max. > f_{at} \end{cases}, \tag{9}$$

where,

$$r = \frac{K}{\mu_e F_N} R,\ f_c = -k'\delta^n(1 - \lambda_c\dot{\delta}),\ k' = \frac{k_c}{\mu_e F_N} \text{ and } v = \frac{K}{\mu_e F_N}\dot{\delta}\ . \tag{10}$$

Here, P is *Heaviside* functions with respect to progression, which characterize the variable structure of the problem:

$$P(x_1, x_2, x_3, x_4) = \begin{cases} 0, & f_{at} \geq b + 2\xi_1(x_2 - x_4) + (x_1 - x_3) \\ 1, & f_{at} < b + 2\xi_1(x_2 - x_4) + (x_1 - x_3) \end{cases} \tag{11}$$

3. NUMERICAL SIMULATION

Numerical simulations are done using the fourth order *Runge-Kutta* method for numerical integration with fixed step. However, when a progressive movement is identified, a sub-routine (bisection) determines the exact instant of the progression start, τ_p (error=10^{-6}). The same procedure is applied when a contact is identified. In this case, the contact force profile (Fig. 2) is determined and applied in the system, identifying the new velocities. A typical steady-state time history, with impact and penetration, is presented in Fig. 3a, where: μ_e/μ_d = 0.6, γ' = 3.0, ε = 1.0, $\varphi = \pi/2$, ξ_1 = 0.12, λ_c = 0.6, n = 1.0, k'= 12 10^5 m^{-1}, a = 0.25, b = 0.7, η = 0.8 and r = 1.0. Because of the drift, the displacements of x_1 and x_3 are oscillatory with progression and *stick-slip*, respectively, Fig 3b. In the instant of impact the initial conditions are changed. Moreover due to the determination of the force profile and the determination of the final impact time (t_{imp}), these changes are done after this period of time. Fig. 2b presents a zoom of the impact zone, where one can see how the impact time was included in the problem.

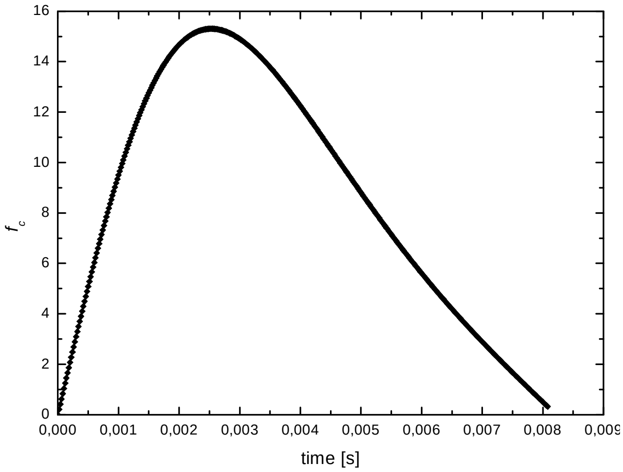
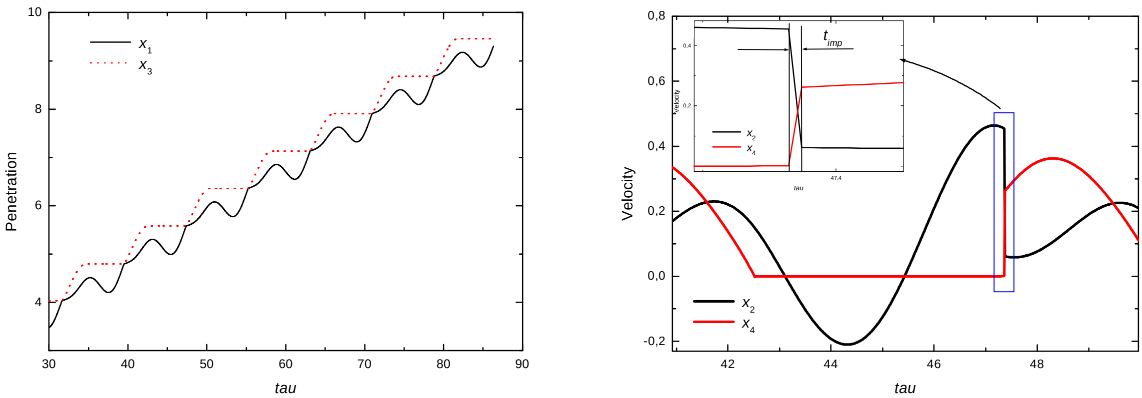


Figure 2. Force profile: impact force x time.



(a) (b)

Figure 3: Period-1: a) penetration x time; b) speed x time: a = 0.25, b = 0.7, η = 0.8 and r = 1.0.

One way to perform the analysis in nonlinear dynamic systems is through the phase space and the *Poincaré* map. The Phase space and *Poincaré* map of the relative motions ($x_1 - x_3$,x_2) obtained with same condition of Fig. 3, are presented in Fig. 4. It is observed a discontinuous phase space, when $x_1 - x_3 = r = 1.0$, being the principal characteristic of impact problems.

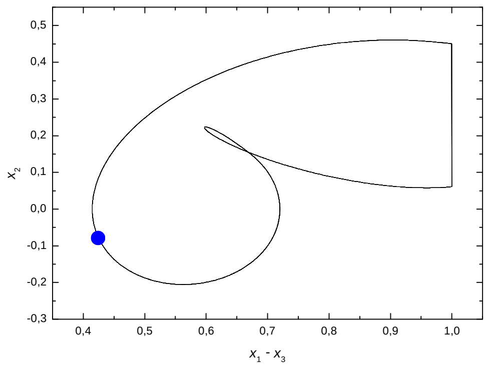
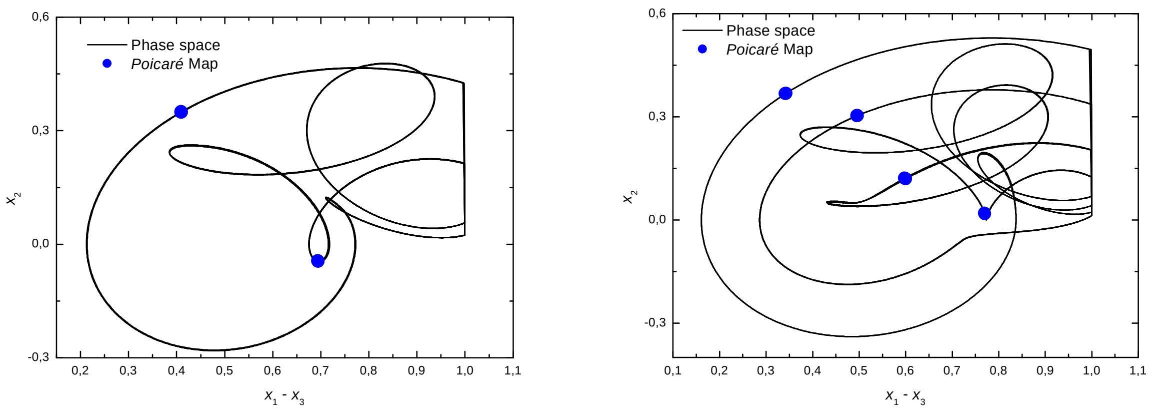


Figure 4. Phase space and *Poincaré* map: a = 0.25, b = 0.7, η = 0.8 and r = 1.0.

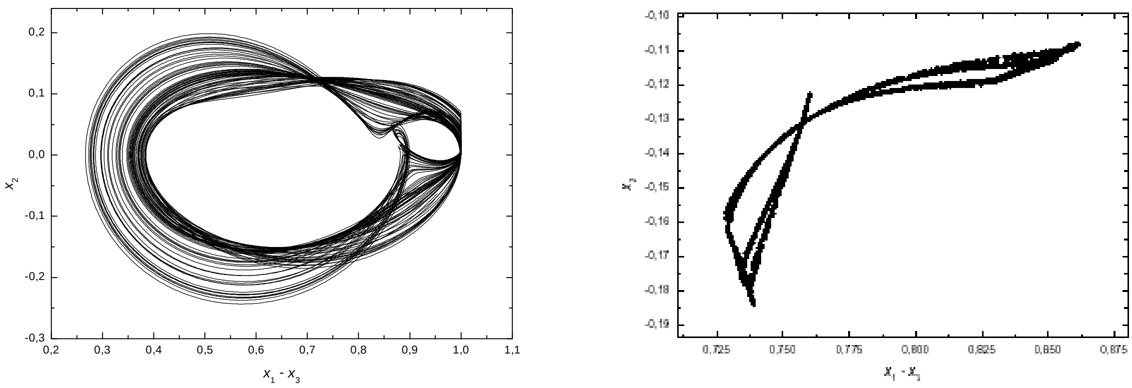
The behavior of the system can be changed by the action of a control parameter. Using η (frequency rate) as control parameter, a condition of period two is obtained with η = 0.6. In this case, we observe two points in *Poincaré* map and two impacts per cycle of external loading. With η = 0.63, the behavior of the system is period four, i.e., four points in *Poincaré* map, Fig. 5b.



(a) (b)

Figure 5. Phase space and *Poincaré* map: a = 0.25, b = 0.7 and r = 1.0: a) period two –η = 0.6; b) period four – η = 0.63.

Fig. 6a presents the phase space with η = 0.448. In this case, the behavior is chaotic and the strange attractor in *Poincaré* map is presented in Fig. 6b. However, the *Lyapunov* exponents could be a better indicator of chaos.



(a) (b)

Figure 6. Chaotic behavior, a = 0.25, b = 0.7 η = 0.448 and r = 1.0: a) phase space; b) *Poincaré* map.

CONCLUSIONS

This paper is concerned with a drilling problem using a self-excited vibro-impact mechanism acting on a penetration model represented by dry friction. Due to the coupling of both nonlinearities there is used a hybrid impact model. The results are consistent with experimental validations to be published [4]. The method showed a great efficiency in investigating as well periodic (period-1, period –2,…) as chaotic behavior, possible solutions for the percussive motion.

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