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GENETIC ALGORITHMS FOR IDENTIFICATION OF ELASTIC CONSTANTS OF COMPOSITE MATERIALS

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Abstract. *The aim of this work is to present a technique to identify elastic parameters of composite materials. The identification is based on the adjustment of coefficients in a optimization process in which the objective function is defined by the difference between the analytical natural frequencies and the measured ones. Such analytical natural frequencies are obtained by the finite element method while the experimental ones are determined by ordinary modal tests. The proposed technique is assessed by a number of different tests allows simultaneous identification of several global properties from a single test without damaging the structure. The proposed approach uses genetic algorithm to solve the optimization problem. Since genetic algorithms are not based on the gradient method, they do not require the expensive eigenvectors computations presented in gradient method.*

Keywords: *genetic algorithms, elastic constants, finite elements, composite materials.*

1. INTRODUCTION

Recently, composite materials have been used in many structural applications. They are formed by two or more different materials in order to obtain better engineering properties than conventional ones, like stiffness, strength, weight reduction and thermal properties (Reddy, 1991). In the design of structures, it is of extreme importance to have very precise estimate of the elastic constants which conventional techniques are not fully able to do. In Balasubramaniam at al (1998) the elastic constants were estimated using simulated ultrasonic phase velocity. Genetic algorithms are used in Cunha et al (1999) as a complementary technique to perform the initial estimation of the elastic parameters and then refining the solution by classical updating methods. In the present work, the elastic constants are identified by an optimization process, based on natural frequencies obtained by vibration tests. A genetic algorithm (GA) was implemented for the inverse problem, which consists of determining the elastic constants once experimental and analytical natural frequencies are known . The finite element method was used to solve the eigenvalue equations. Because it is not based on the gradient method, GAs do not require the expensive eigenvectors calculus which are used to compute the gradient. Moreover, two other advantages are also obtained: no initial guess is required and the optimization process could be more flexible, due to the fact that the search space begin from a set of elastic constants, corresponding to different chromosomes, rather than a single one. These characteristics are important to face on the local minimum problems. Due to these features, the GAs seem to be more robust and global than other techniques (Haupt, 1998).

In this paper, the experimental natural frequencies were obtained by tests for an aluminium plate (Bastos, 2001). In order to verify the capacity of the approach, simulated frequencies for aluminium, kevlar/epoxy and SCS-6/Ti-15-3 which were obtained by numerical methods were used. The stiffness properties of these materials were obtained from literature (Herakovich, 1998). In this manner, the estimated properties from GA were compared with the avaiable data.

2. CONSTITUTIVE EQUATION

This section introduces Hooke's law, which describes a linear elastic material subjected to small deformation (Eq.(1)), where ϵ is the strain vector and σ is the stress one (Reddy, 1991 and Herakovich, 1998).

$$\sigma_{\mathbf{ij}} = \mathbf{C_{ijkl}}\, \epsilon_{\mathbf{kl}} \tag{1}$$

In principal material coordinates of an orthotropic material, the plane stress constitutive equation has a simplified form:

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_{11} \\ \epsilon_{22} \\ \epsilon_{12} \end{Bmatrix} \tag{2}$$

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}} \qquad Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}} \qquad Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}} \qquad Q_{66} = 2G_{12} \tag{3}$$

where E_1 is the elastic modulus in the fibrous direction, E_2 is the elastic modulus in the transverse fibrous direction, ν_{12} and ν_{12} are the Poisson's ratio and G_{12} is the shear modulus. Only four out of the five material constant for plane stress of an orthotropic material are independent. In this work, the identified constants are E_1, E_2, G_{12} and ν_{12}. The Poisson's ratio ν_{21} is calculated by:

$$\nu_{21} = \frac{\nu_{12}\, E_2}{E_1} \tag{4}$$

The classical plate theory was used in this work (Shames, 1973). In the finite element model, the plate is discretized by triangular elements with three degrees of freedom per nodes, that is two rotations and the transversal displacement. The eigenvalue equation to find the natural frequencies is (Reddy, 1991):

$$(-\omega_i^2\,\mathbf{M} \,+\, \mathbf{K})\,\mathbf{u_i} = \,0 \tag{5}$$

where ω_i is the i^{th} natural frequency and u_i is the respective vibration mode. **M** and **K** are, respectively, the inertia and stiffness matrix of the finite element model.

3. GENETIC ALGORITHM

Genetic algorithms are search algorithms based on the mechanics of natural selection. This technique allows a population composed of many individuals to evolve according to some rules to a state that minimizes a cost function. Comparing with other random search techniques, the GA's are an intelligent way to find the global solution in the search space. These methods should be classified in some categories, analyzing some aspects such as (Haupt, 1998):

- multiple or single parameter;
- discrete or continuous;
- constrained and unconstrained;

In GA's, a finite number of candidate solution, the chromosomes, are randomly created forming the initial population. In this work, a binary code was used (Goldberg, 1998). Each chromosome represents a possible solution, divided in sub-strings that are decoded into their corresponding elastic constants. Those chromosomes will create the new generation, by natural selection and reproduction procedures. As the cost function has to be minimized, only a few of the best chromosomes (the members with lower errors) will be kept for breeding.

The natural selection is a procedure that decides which individual should survive, forming the *mating pool*. Individuals with lowest cost reproduces more often than highest cost ones. An overlapping population is permitted. In this case, the offsprings will replace the discarded chromosomes . Reproduction procedures consist in crossover and mutation. Two chromosomes, $parent_1$ and $parent_2$, are selected from the mating pool to produce two new offsprings, $child_1$ and $child_2$. A crossover point is randomly selected between the first and last bit of the parents, exchanging portions of their strings, in order to form the children. This operation is performed with a probability *pcross*, that is normally a high value. Mutation operation change a bit from "1" to "0" or vice versa, with a probability *pmutation*, normally a very low value. Increasing the number of mutations increases the algorithm's search outside the current region of parameter space. It also tends to distract the algorithm from converging on a solution. In order to propagate the best solution unchanged it is usual in GA to keep the fittest chromosome without mutation.

After that, the cost of the new generation is calculated and the described process is repeated, until a stopping criterion. The number of generations depend on whether an acceptable solution is reached or a number of iterations is exceeded. Figure (1) shows the present work's procedures.

In order to represent the four elastic constants(in MPa), a chromosome with 40 bits was used: 16 bits for E_1 and 8 bits for the others. Therefore, the search space for each constant is defined as:

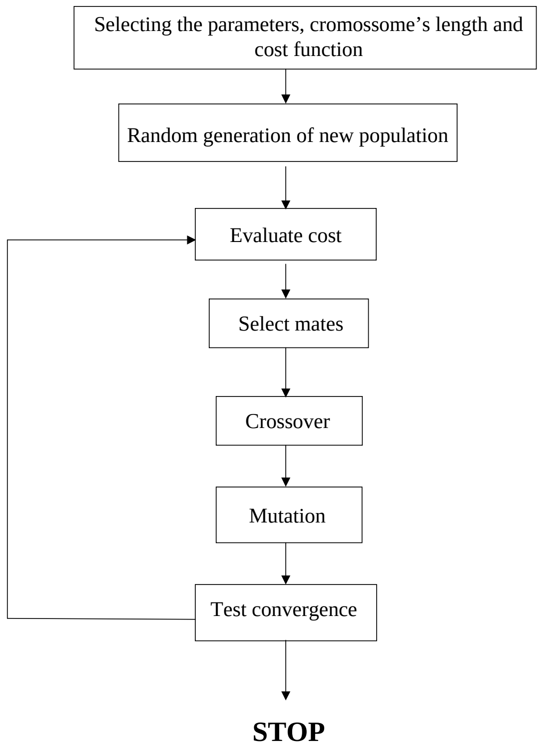


Figure 1: Flow chart

$$0 < E_1 < 256 \qquad 0 < \frac{E_2}{E_1} < 1$$
$$0 < \frac{G_{12}}{E_1} < 1 \qquad 0 < \nu_{12} < 0.5 \tag{7}$$

If the elastic modulus were greater than 256 MPa, the length of the chromosome should be increased, enlarging the range of the search space.

In the optimization problem, the objective function is defined by the difference between natural frequencies and calculated ones (Bledzki et al, 1999), stated as follows:

$$CF(\theta) = \sum_{i=1}^{N} \left[\frac{(f_{exp}^2 - f(\theta)^2)^2}{f_{exp}^4} \right] \tag{8}$$

where N is the number of modes and θ is a vector where each component is an elastic constants . After evaluating the fitness, the chromosomes are ranked from lowest to highest cost. Only the *nbest* chromosomes are kept to form the mating pool, while the others are discarded. If overlapping occurs, the new generations will be composed by the *keep* chromosomes and completed by the offsprings created by crossover and mutation operations.

Parameters' selection is different for each example. Small population size (*popsize*) should lead to premature convergence while a large one is commonly used to increase the variation within a population. However, the increase of the number of function evaluations results in increased computational cost.

4. RESULTS

The proposed approach is assessed by means of a number of applications. Both, isotropic and orthotropic plates are explored. The former, although it does not constitutes a composite structure, is used as far as it represents a test for the algorithm in which the detection of flaws is simple. The samples were hung upon two threads in order to simulate free-free boundary conditions.

The elastic constants were identified for each specimen using the data from Tab.(1).

Table 1: Parameters of the plate

Sample	a(m)	b(m)	h(m)	ρ(Kg/m^3)
Aluminium	0.6	0.4	0.0063	2700
Kevlar/Epoxy	0.6	0.4	0.004	1380
SCS-6	0.6	0.4	0.004	3860

4.1. Isotropic Material

In order to enlarge the number of situations that were analyzed, some non-experimental quantities were utilized. They will be referred to as simulated frequencies, which are obtained with an *a priori* choice of the elastic parameters and the use of a finite element model of the plate. The simulated and experimental natural frequencies are shown in Tab.(2) and Fig.(2). These two examples are represented by GAs and GAe, where the first case was calculated from the simulated frequencies and the second one was calculated from the experimental ones. The experimental frequencies were taken from (Bastos, 2001). In Tab.(3)-(4), the literature values of the elastic constants are placed in the second column. Those values were used to obtain the simulated frequencies.

Genetic Algorithm Parameters:
popsize = 80
keep = 10
nbest = 40
pcross = 0.95
pmutation = 0.03

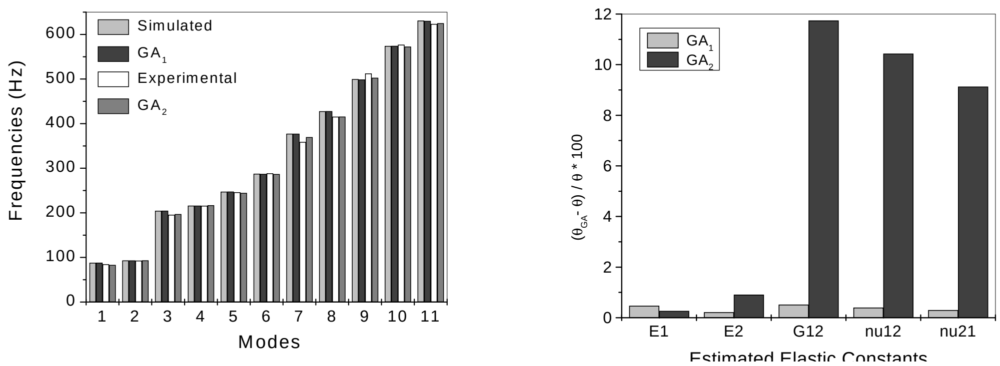


Figure 2: Frequencies on aluminium plate

Table 2: Aluminum plate frequencies in Hz

Modes	Simulated	GA_s	Experimental	GA_e
1	88.15	87.98	84.7	83.3049
2	93.26	93.2871	92.8	93.3378
3	204.73	204.565	195.7	197.141
4	216.05	216.061	215.9	217.076
5	247.66	247.486	246.2	244.686
6	287.20	287.576	288.5	286.82
7	377.27	377.338	359.1	369.781
8	427.97	427.75	415.6	415.814
9	498.86	499.814	512.5	502.916
10	574.30	574.218	577.0	572.796
11	630.31	631.078	623.5	625.124

Table (3) and Tab.(4) show the estimated elastic constants calculated by simulated and experimental frequencies respectively. Different tests were done and the results are represented in the columns 3,4,5,6 for GA_s and in columns 3,4,5 for GA_e. The mean value (μ) and the standard deviation (s) were calculated for the values obtained in these tests. In GA_e it is possible to note that the elasticity modulus are better estimated than the shear modulus and the Poisson's ratio (Fig. (2)). It is clear that the value of shear modulus estimated by GA_s is better than the one estimated by GA_e. This result was already expected inasmuch as the experimental data contain the corrupting effects of noise, filtering, analogue to digital conversion and etc.

In Tab.(5), the results obtained by genetic algorithm are compared with the results obtained from the least-squares method in (Bastos, 2001). In the reference, the sample under consideration was modeled as an isotropic plate thus only the elasticity modulus and the Poisson's ratio were calculated.

Table 3: Estimated elastic constants for an aluminium plate - GA_s

	Literature	1	2	3	4	μ	s
E_1 (GPa)	73	72.9804	73.2343	73.0156	74.1562	73.3466	0.22
E_2 (GPa)	73	72.6953	72.6621	72.7303	73.2872	72.8437	0.066
G_{12} (GPa)	28.0769	28.2228	28.0349	27.9512	27.5188	27.9319	0.066
ν_{12}	0.3	0.3027	0.2988	0.3066	0.2968	0.3012	$1.4 \cdot 10^{-5}$
ν_{21}	0.3	0.3015	0.2964	0.3054	0.2933	0.2991	$2.1 \cdot 10^{-5}$

4.2. Orthotropic Material

In this example, the plate thickness is shown in Tab. (1) and the thickness of each plie is $h = 0.001m$.

Table 4: Estimated elastic constants for an aluminium plate - GA_e

	Literature	1	2	3	μ	s
E_1 (GPa)	73	73.5546	73.0156	73.0156	73.1953	0.064
E_2 (GPa)	73	72.1180	72.4451	72.4451	72.3361	0.023
G_{12} (GPa)	28.0769	24.7097	24.8138	24.8138	24.7791	0.0024
ν_{12}	0.3	0.3339	0.33	0.33	0.3313	$3.38 \cdot 10^{-6}$
ν_{21}	0.3	0.3274	0.3274	0.3274	0.3274	0.0

Table 5: Comparison between GA and least-squares method

	Literature	Estimated elastic constants	Reference
E_1 (GPa)	73	73.1953	68.7517
E_2 (GPa)	73	72.3361	68.7517
G_{12} (GPa)	28.0769	24.7097	26.2616
ν_{12}	0.3	0.3313	0.3090
ν_{21}	0.3	0.3274	0.3090

4.2.1. Kevlar/Epoxy

The simulated and estimated natural frequencies are shown in Fig.(3). It is possible to verify, in the cost function graphic, that the algorithm escaped from a local minimum, looking for the global one. The estimated elastic constants obtained by GA are in good agreement with the literature values (Tab.(6)).

Genetic Algorithm Parameters:
popsize = 80
keep = 10
nbest = 40
pcross = 0.95
pmutation = 0.04

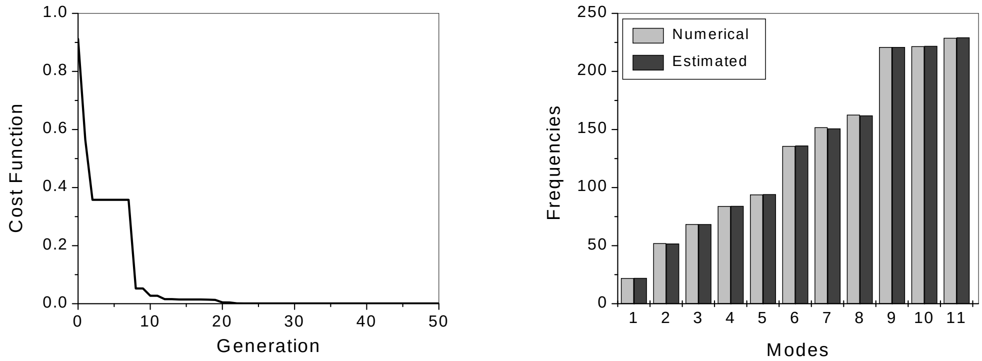


Figure 3: Cost function and frequencies on a Kevlar/Epoxy plate

Table 6: Estimated elastic constants for Kevlar/Epoxy

	Literature	1	2	μ	s
E_1 (GPa)	76.8	77.08	77.09	77.085	$0.25 \cdot 10^{-4}$
E_2 (GPa)	5.5	5.42	5.42	5.42	0.0
G_{12} (GPa)	2.07	2.1	2.1	2.1	0.0
ν_{12}	0.34	0.339	0.333	0.336	$9 \cdot 10^{-6}$
ν_{21}	0.024	0.0238	0.0234	0.0236	$4 \cdot 10^{-8}$

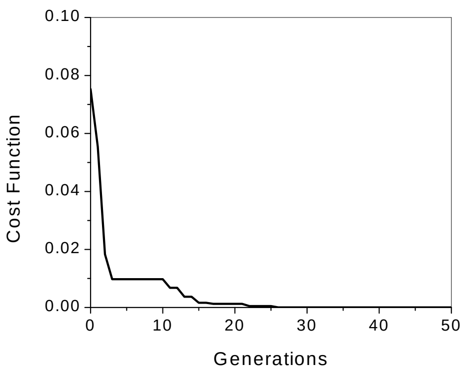
4.2.2. SCS-6/Ti-15-3

The simulated and estimated natural frequencies are shown in Fig.(4). As well as in the Kevlar/Epoxy example, the algorithm escaped a local minimum.

In Table 6, the estimated elastic constants are compared to the values of literature. Although the standard deviation has presented high values, the estimated mean values were very close to literature ones (Tab.(7)).

Genetic Algorithm Parameters:

popsize = 80
keep = 10
nbest = 40
pcross = 0.90
pmutation = 0.03



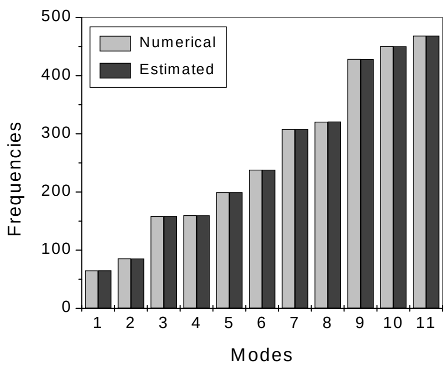


Figure 4: Cost function and frequencies on a SCS-6/Ti-15-3 plate

Table 7: Estimated elastic constants for SCS-6/Ti-15-3

	Literature	1	2	3	4	μ	s
E_1 (GPa)	221	224	216.69	220.28	224	221.243	9.21
E_2 (GPa)	145	146.1	143.05	144.55	145.24	144.735	1.24
G_{12} (GPa)	53.2	53.37	54.17	53.34	53.37	53.562	0.123
ν_{12}	0.27	0.24	0.294	0.27	0.25	0.263	$4 \cdot 10^{-4}$
ν_{21}	0.17	0.16	0.194	0.18	0.162	0.174	$1.9 \cdot 10^{-4}$

5. CONCLUSIONS

The proposed method for the identification of elastic constants has shown be effective for the examples that have been presented here. It is clear from the examples that the algorithm was able to skip local minimuns, i.e., it can be considered an efficient method to find the global minimum for the problem under study. It should be remarked that the experimental data are obtained out of non-destructive dynamics tests, the designer can perform several experiments considering the same composite structure.

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